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MASTER'S THESIS SUMMARY
Comparison between FRQI and NEQR quantum algorithms applied in digital image processing

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Quantum image processing represents a transformative approach to visual data analysis, leveraging the principles of quantum computing to overcome classical limitations. This work explores two prominent quantum image encoding methods: FRQI (Flexible Representation of Quantum Images) and NEQR (Novel Enhanced Quantum Representation). FRQI excels in qubit efficiency, making it suitable for hardware implementation, while NEQR offers superior precision in pixel intensity representation, ideal for complex image processing tasks. We detail the implementation of these algorithms, including preprocessing, quantum circuit design, and simulation, using platforms like Qiskit. The study highlights the potential of quantum image processing in fields such as medicine, industry, and environmental monitoring, while addressing challenges like qubit limitations and noise sensitivity. This research contributes to advancing quantum computing applications, paving the way for innovative and sustainable technological solutions.

Keywords: Quantum image processing, FRQI, NEQR, quantum computing, image encoding, Qiskit, qubit efficiency, pixel intensity, quantum simulation.

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1 Introduction

The foundations of quantum computation go back to the mathematical challenge posed by David Hilbert in his famous 1900 speech, in which he presented a list of 23 open problems that triggered the development of new theories. The classical approach to mechanics, based on Newton's and Maxwell's laws, proved to be an effective approach at the macroscopic scale. However, at the quantum scale, which encompasses individual atoms, electrons and photons, it becomes imprecise and makes the consideration of quantum physics necessary (Laverde Forero & Cárdenas Martínez, 2018). Milestones such as Heisenberg's uncertainty principle in 1927 underlined the need for a probabilistic framework instead of classical determinism.

In the 1980s and 1990s, Richard Feynman and David Deutsch laid the theoretical foundations of quantum computation by exploring how the principles of quantum mechanics could lead to more efficient computations than those performed by classical computers. These theoretical advances, along with the development of quantum hardware, led to the creation of qubits, which take advantage of phenomena such as superposition and entanglement to process large amounts of information more efficiently.

The relevance of quantum computing lies in its ability to tackle complex problems with performance superior to traditional methods, positioning it as an emerging technology with disruptive applications in various fields.

1.1 Motivations

Image and big data processing has a great impact on fields such as medicine and artificial intelligence. However, traditional computing faces limitations due to the exponential growth of data complexity and algorithmic requirements. Quantum computing offers a promising alternative, leveraging principles such as parallelism and quantum entanglement to provide substantial advantages in speed and efficiency.

Quantum image processing could revolutionize the way visual data is analyzed and managed, improving computational performance and scalability. Potential applications include solving optimization problems with large solution spaces, training machine learning algorithms to process large and complex data sets and developing advanced image processing techniques for object detection and classification.

This research aims to contribute to the development of quantum algorithms for image processing, addressing critical challenges in key areas such as healthcare, industry and environmental sustainability. Specifically, quantum image processing could:

- Accelerate diagnostics in medical imaging, enabling timely and effective treatments.
- Optimize quality control processes in industry by detecting defects efficiently.
- Enhance satellite image analysis for resource management and environmental conservation.

Moreover, this work aligns with global priorities such as the United Nations' 2030 Agenda for Sustainable Development, particularly Goals 3 (Good Health and Well-being), 6 (Clean Water and Sanitation), 9 (Industry, Innovation, and Infrastructure), 11 (Sustainable Cities and Communities), 13 (Climate Action), and 15 (Life on Land). It also supports national priorities within Mexico's PRONACES programs, promoting scientific and technological progress to address societal challenges.

By addressing these pressing issues, quantum computing in image processing could play a transformative role in advancing global and national objectives for societal benefit.

1.2 Description of the research problem

The exponential growth of data and images in the digital era has posed significant challenges for traditional computing, particularly in terms of speed and the ability to handle large volumes of information. Quantum computing, leveraging principles such as superposition and entanglement, promises to revolutionize image processing by enabling faster and more efficient analysis compared to classical methods.

However, this emerging field faces technological challenges, such as quantum image representation and the optimization of quantum circuits, which require a higher number of qubits and gates. Mechanisms like circuit compression or the fragmentation of images into smaller blocks are essential to overcome these limitations.

This work focuses on the development of quantum algorithms for image preprocessing, segmentation, and feature extraction, exploring how they can overcome the current limitations of classical computing while addressing the technological challenges associated with their implementation.

1.3 Objectives of the thesis

1.3.1 General Objective

To develop quantum algorithms for digital image processing by leveraging quantum computing techniques to optimize execution time and accuracy, paving the way for future image classification tasks.

1.3.2 Specific Objectives

- To analyze the state-of-the-art techniques in quantum-based image processing to identify their applicability in digital image processing.
- To select the appropriate technological tools for encoding, simulation, and implementation of quantum algorithms by comparing the available resources.
- To study the analogies and implementations of classical and quantum algorithms through reviews of comparative studies, enabling practical implementation of selected examples.
- To identify quantum algorithms with impactful and feasible applications for image processing through the review of previous works for practical use.
- To develop quantum algorithms for image segmentation and edge detection within the chosen technological framework.
- To generate simulations of these algorithms to evaluate their performance using quantum simulation environments aligned with the selected technology.
- To evaluate the performance of the implemented algorithms by analyzing execution time and accuracy metrics and comparing them with traditional computing methods.

1.4 Brief description of the contribution of the thesis

This thesis contributes to the advancement of quantum computing applied to digital image processing by proposing, developing, and evaluating quantum algorithms. These algorithms address key challenges in traditional computing, such as scalability and execution time, which limit the processing of large and complex datasets. The primary contributions of this research are:

1. The development of quantum algorithms for image segmentation and edge detection, demonstrating their feasibility and advantages over classical counterparts.

2. The implementation of a systematic evaluation framework to compare quantum and classical algorithms in terms of execution time and accuracy.
3. The generation of practical insights into the applicability of quantum computing for image processing, paving the way for future applications in fields such as medical diagnostics, industrial quality control, and environmental monitoring.

By bridging the gap between theoretical quantum computing and practical image processing applications, this work provides a foundation for future research and development in this interdisciplinary field.

2 Background

2.1 Quantum Computing: Principles and Relevance

Quantum computing is a revolutionary computational model that leverages the principles of quantum mechanics, such as superposition, entanglement, and interference. These properties allow quantum systems to process information in parallel and simultaneously, offering significant computational advantages over classical methods, particularly for high-complexity problems and volume datasets.

The fundamental concept of quantum computing is the ***qubit***, a unit of information that, unlike classical bits, can exist in a superposition of states (0 and 1). This means that a system with multiple qubits can represent an exponential number of states simultaneously. This principle, combined with quantum entanglement where qubits are strongly correlated regardless of their distance—and quantum interference, which amplifies correct probabilities while reducing incorrect ones, forms the foundation of quantum computational power.

Quantum computing has potential applications in optimization, machine learning, cryptography, and data analysis. Specifically, quantum image processing represents a promising advancement as it can overcome classical computing limitations related to scalability and execution time. These capabilities are essential for complex tasks such as segmentation, edge detection, and image classification, particularly in fields like medicine, artificial intelligence, and environmental monitoring.

However, current quantum hardware limitations, such as noise sensitivity, the limited number of qubits, and associated costs, present significant challenges. Despite this, the continuous development of quantum algorithms shows encouraging results and represents a critical area for technological advancement.

2.2 Quantum Image Processing: Formats and Techniques

Quantum Image Processing (QIP) is an emerging discipline that combines quantum mechanics with digital image processing. This approach explores how visual data can be represented, manipulated, and analyzed using qubits and quantum circuits, leveraging the unique properties of quantum systems to overcome classical limitations.

A key aspect of QIP is quantum image representation, which transforms classical visual data into quantum states. Some of the most notable formats include:

- **Flexible Representation of Quantum Images (FRQI):** Introduced by Le et al. (2011), this format uses qubits to encode pixel intensities and spatial positions into a quantum state. It is efficient for image manipulation but has limitations in terms of precision and resolution.
- **Novel Enhanced Quantum Representation (NEQR):** Proposed by Zhang et al. (2013), this format improves precision by encoding grayscale pixel values directly into quantum states, although it requires more qubits than FRQI.
- **Quantum Probability Image Encoding (QPIE):** Proposed by Yao et al. (2017), this format optimizes memory usage by representing data probabilistically, though it may lose fidelity in certain cases.

- **Normal Arbitrary Quantum Superposition State (NAQSS):** Introduced by Li et al. (2014), this format is suitable for multidimensional color images, offering flexibility but being complex to implement due to the required control of qubits.
- **Quantum Feature Map-based Encoding:** Described by Havlíček et al. (2019), this format is used in machine learning, transforming classical features into quantum states, making it suitable for tasks like pattern recognition.

These formats have facilitated the development of quantum image operations, such as geometric transformations, color transformation, edge detection, and compression. Algorithms like the quantum Fourier transform (QFT) (Chen et al., 2023) and hybrid methods, such as the Quantum Hybrid Edge Detection (QHED) algorithm (Shubha et al., 2024), have shown significant advantages over classical counterparts.

QIP presents major challenges, including hardware requirements, noise sensitivity, and the scalability of quantum operations. However, recent research continues to develop more efficient and practical techniques. For this thesis, the FRQI and NEQR formats were selected due to their balance between simplicity and image quality, enabling feasible implementation on current quantum simulators and hardware.

2.3 Related work

The field of Quantum Image Processing has significantly evolved since the introduction of the **Qubit Lattice** format by Venegas-Andraca & Bose (2003), which enable for the first representation of visual data in quantum systems. This work laid the foundation for more advanced formats, such as **FRQI**, proposed by Le et al. (2011), which improved image manipulation and representation.

Subsequently, Zhang et al. (2013) introduced the **NEQR** format, which stood out for its ability to handle grayscale images with higher precision. Anand et al. (2022) extended this approach to include applications in image classification and segmentation using quantum machine learning techniques.

In terms of applications, Yuan et al. (2020) implemented quantum segmentation algorithms using NEQR on the IBM Quantum Experience platform, demonstrating the feasibility of these techniques in real-world environments. On the other hand, Shubha et al. (2024) developed a hybrid quantum edge detection method (QHED) using the FRQI format, which is notable for its efficiency in processing large and complex images. Despite these advances, previous work faces significant limitations, such as reliance on specialized quantum hardware, noise sensitivity, and algorithmic complexity. This thesis aims to address these gaps by developing novel segmentation and edge detection algorithms optimized for quantum simulators and available hardware. It also proposes a systematic evaluation of performance compared to classical techniques, contributing to the practical advancement of QIP in applications such as medicine, industry, and environmental monitoring.

3 Proposed Solution Approach

In this section, authors should describe in detail the approach or methodology they have proposed to solve the research problem. They should explain the models, methods, algorithms, or techniques used and justify why this approach is appropriate for the described problem. A discussion of the advantages and limitations of the proposed approach may be included.

3.1 Introduction to the Approach

This work proposes a solution based on quantum computing to address current limitations in digital image processing, focusing on scalability, execution time, and precision. The methodology is built upon the design, implementation, and evaluation of quantum algorithms for fundamental tasks such as image segmentation and edge detection. To achieve this, the **Flexible Representation of Quantum Images (FRQI)** and **Novel Enhanced Quantum Representation (NEQR)** have been selected as the basis for encoding classical images into quantum states. These representations enable the exploitation of unique quantum properties, such as superposition and entanglement, to perform operations more efficiently than traditional methods.

3.2 Methodology

The proposed methodology is divided into three main stages**:** *a) image encoding, b) quantum algorithm design and c) performance evaluation.*

In the first *stage a*, the FRQI and NEQR representations are used to transform classical images into quantum states. The FRQI representation has been chosen for its ability to simplify quantum circuit design, requiring fewer qubits, by encoding spatial and pixel intensity information into superposition states. On the other hand, NEQR offers higher precision by encoding grey-scale pixel values directly into quantum states, making it particularly advantageous for tasks requiring greater detail, such as segmentation. Both representations will be implemented and validated on the Qiskit platform, enabling the simulation of quantum circuits and ensuring the fidelity of the processed data.

In the second *stage b*, specific algorithms will be developed to address key challenges in image processing. The first algorithm will be a quantum edge detection method based on Sobel, which exploits quantum parallelism to process multiple pixels simultaneously. This approach will be implemented using the FRQI representation to enable the construction of compact and efficient circuits. The second algorithm will be a double-threshold quantum segmentation method designed to identify specific regions within images with high precision. This segmentation algorithm will be developed using the NEQR representation, as its ability to handle grey scale details ensures more accurate results.

Finally, in the third *stage c*, the performance of the quantum algorithms will be evaluated against classical methods. This evaluation will include metrics such as Mean Squared Error (MSE) to analyze the accuracy of operations and execution time to assess computational efficiency. Simulations will be performed using both quantum simulators and real hardware available through IBM Quantum Experience, to ensure the practical viability of the designed algorithms.

3.3 Justification of the Approach

The proposed approach combines the strengths of the FRQI and NEQR representations with the capabilities of quantum algorithms specifically designed for image processing. The FRQI representation stands out for its simplicity in reducing the number of qubits needed for image encoding, facilitating implementation on limited quantum hardware. Meanwhile, NEQR enables high-fidelity pixel detail encoding, which is essential for more complex applications such as precise segmentation.

From an algorithmic perspective, the proposed quantum methods leverage the properties of superposition and entanglement, enabling parallel data processing that significantly reduces execution times compared to classical methods. Additionally, using Qiskit as the development platform ensures compatibility with current quantum technologies, guaranteeing the practical feasibility of the approach in both simulation and real hardware scenarios.

3.4 Advantages and Limitations of the Proposed Approach

The proposed approach offers several significant advantages. First, it provides a scalable solution for processing high-resolution images by leveraging quantum parallelism to handle large volumes of data efficiently. Additionally, data representation precision is ensured through NEQR, enhancing results in tasks such as segmentation and edge detection. Furthermore, the methodology's flexibility allows it to be adapted to various applications, such as image classification and complex pattern recognition.

However, there are inherent limitations to this approach. Current quantum hardware restrictions, such as the limited number of qubits and sensitivity to noise may impact the practical implementation of the algorithms. Moreover, although quantum simulators provide a robust testing environment, they do not always accurately reflect the behavior of algorithms on real quantum devices. Lastly, designing and optimizing quantum circuits requires significant effort in terms of time and technical expertise, which could hinder the immediate generalization of this approach.

4 Experimental results

In this work, two quantum image encoding algorithms were implemented with the aim of analyzing and comparing their efficiency in image processing tasks. As mentioned earlier, there are different standards for representing quantum images. Although there are various approaches to quantum image representation, this research project focuses on these two standards: FRQI (Flexible Representation of Quantum Images) and NEQR (Novel Enhanced Quantum Representation) for the following reasons:

- FRQI stands out for its efficiency in the use of qubits, requiring a small number, which facilitates its implementation in quantum hardware. It offers great flexibility, making it suitable for different image processing applications.
- NEQR allows for a more precise representation of pixel intensities, which represents a significant advantage when a higher degree of accuracy is required. It is better suited for representing and processing images with high complexity and variability in pixel intensity levels.

Below, in Figure 1, the presentation of the FRQI and NEQR algorithms is detailed through three stages: preprocessing, creation of the quantum circuit, and simulation and visualization of the quantum state.

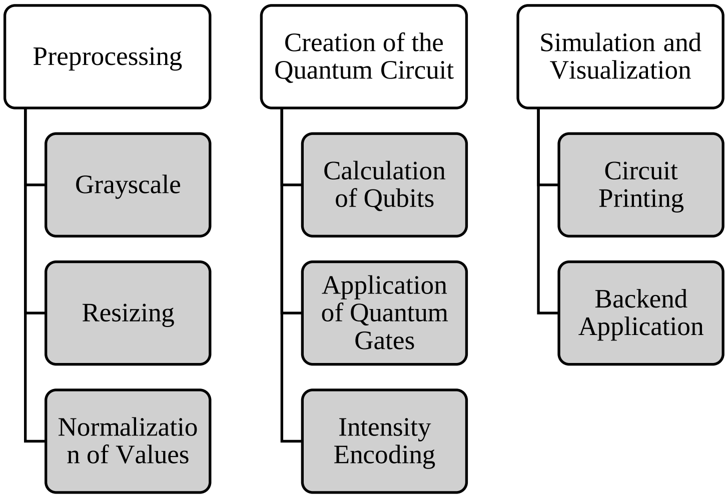

Fig. 1 Quantum Image Processing Stages

4.1 FRQI Encoding

The FRQI algorithm allows for the conversion of digital images into a quantum representation, divided into three main stages:

- **Preprocessing**. An image is taken from a specific local path and converted to grayscale to simplify operations and remove color information. The image is then resized to fit the FRQI format, which is typically used for small images. For this case study, the classic "Lena" image was used at 32x32 pixels. Finally, the pixel values are normalized to a range of [0, 1], facilitating their encoding as quantum states.
- **Creation of the Quantum Circuit.** The number of qubits required to represent pixel positions and one additional qubit for intensities is calculated. By example, for a 32x32 image (1024 pixels), 10 qubits are needed for the positions (5 for x, 5 for y) and 1 for intensity, totaling 11 qubits. It is worth mentioning that for this experiment, an additional qubit is added to handle complex calculations.

$$\begin{aligned} qubits &= log_2(1024) \\ qubits &= 10 \end{aligned} \tag{1}$$

The position qubits are placed in a quantum superposition state using Hadamard gates, representing all possible combinations of positions. Then, each pixel is associated with a specific binary position, such as (00), (01), (10), and (11) for a 2x2 image. "X" gates are applied to activate specific positions, and controlled rotations are used to encode the intensities on the "y" axis of the Bloch sphere.

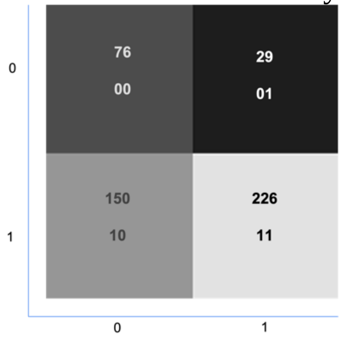

Fig. 2 Position and Intensity in a 2x2 Image for FRQI Encoding

- **Simulation and Visualization of the Quantum State.** Once the circuit is created, it is simulated using the *statevector_simulator* backend from Qiskit. A state vector is obtained and graphically represented, allowing for verification of the correct encoding of the image. This simulation helps validate the circuit's functionality and the quantum representation.

  In Figure 3, a fragment of the quantum circuit is shown with the implementation of the FRQI algorithm to encode the 32x32 pixel Lena image into a quantum state. In this algorithm, each pixel of the image is encoded into a superposition quantum state through the application of Hadamard gates, enabling a compact and efficient representation of the image.

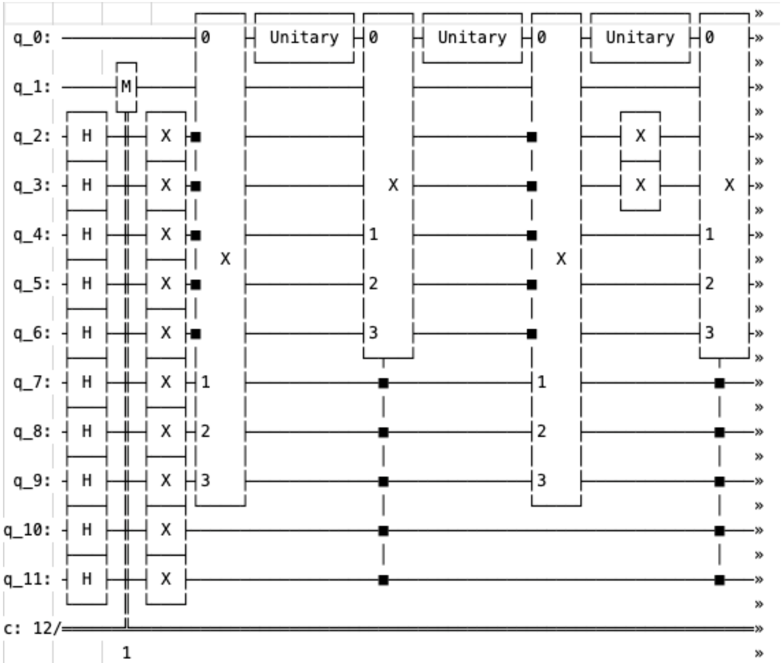

Fig. 3 FRQI Quantum Circuit (fragment) for Lena 32x32 Pixels

4.2 NEQR Encoding

This approach represents each pixel of the image as a quantum state, facilitating quantum image processing.

- **Preprocessing.** The NEQR algorithm begins with preprocessing the Lena image resized to 32x32 pixels. In this stage, the image is loaded from a specific path and converted to grayscale to handle the light intensity of each pixel, removing unnecessary color information. Unlike FRQI, in the NEQR approach, pixel intensities are directly encoded into a quantum register as binary states, explicitly preserving this information. The pixel values are normalized to the range [0, 255], maintaining the typical grayscale representation and preparing the data for encoding into the quantum circuit.
- **Creation of the Quantum Circuit.** The quantum circuit creation stage focuses on encoding the image in the NEQR format. To achieve this, the number of qubits required for pixel positions is calculated by taking the base-2 logarithm of the total number of pixels (1024 for a 32x32 image) and rounding to the nearest integer. Additionally, 8 qubits are added to represent the intensity of each pixel in binary format. Subsequently, the quantum circuit is initialized with the calculated qubits, dividing them into position qubits for the "x" and "y" coordinates and intensity qubits. The position qubits are prepared in a superposition state by applying Hadamard gates, allowing the representation of all possible pixel positions simultaneously. Finally, the intensity of each pixel (expressed in 8 binary bits) is encoded using multi-controlled gates (MCMT) that adjust the intensity qubits according to the corresponding binary values. These gates are controlled by the position qubits, and their efficient implementation is made possible by the MCMT class in Qiskit.
- **Simulation and Visualization of the Quantum State.** In the simulation and visualization stage, the Qiskit Aer quantum simulator is used to execute the circuit and measure the resulting quantum states. These measurements generate counts representing the frequency with which each state is observed after multiple circuit executions. From these frequencies, the probability distribution of the quantum states is calculated, providing information about the encoded pixel intensities. Additionally, graphs are generated to visualize both the probability distribution and the reconstructed image, facilitating the evaluation of

the representation's fidelity and validating the correct operation of the circuit. This process enables a precise and efficient representation of the Lena image in the NEQR scheme, leveraging the capabilities of quantum processing to handle both pixel positions and intensities.

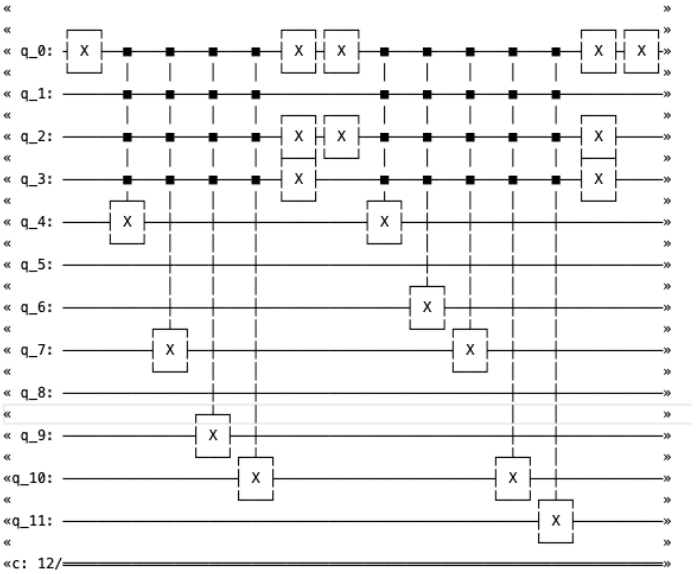

Fig. 4 NEQR Quantum Circuit (fragment) for Lena 32x32 Pixels

4.3 Operations on Quantum Images

4.3.1 Quantum Flip (Flip)

The flip is an inversion of the states along one of the axes, either x or y. This corresponds to reflecting the image around the selected axis. In the quantum domain, the FRQI encoding is leveraged, where there is already a distribution of qubits representing "x" and "y." This implies that to perform an Fx or Fy operation, it is necessary to apply NOT gates to the qubits corresponding to the positions of the opposite axis—either the Y axis for Fx or the X axis for Fy—directly to the image I (Dolciami, 2022).

$$F_I^X(|I\rangle) = \frac{1}{2^n}\sum_{k=0}^{2^{2n-1}} |c_k\rangle \otimes F^X(|k\rangle) \qquad (2)$$

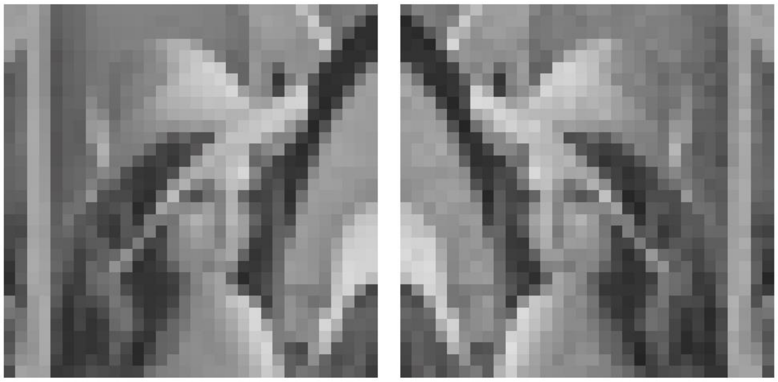

Fig. 5 Comparison Between Original Lena Image and Application of Quantum Flip

4.3.2 Coordinate Swap

According to (Dolciami, 2022), the coordinate swap "CI" reverses the position of the pixels between the two coordinate axes. When applied to |I⟩, the following results are obtained.

$$CI(|I\rangle = \frac{1}{2^n}\sum_{k=0}^{2^{2n-1}} |c_k\rangle \otimes \ C(|k\rangle \tag{3}$$

where: C_k represents the color information

This coordinate swap unlike the quantum flip, it is performed by applying swap gates. The swap gates apply a quantum operation that exchanges the state values of two qubits using the following operation matrix.

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

It is worth mentioning that swap gates can be constructed using three CNOT gates.

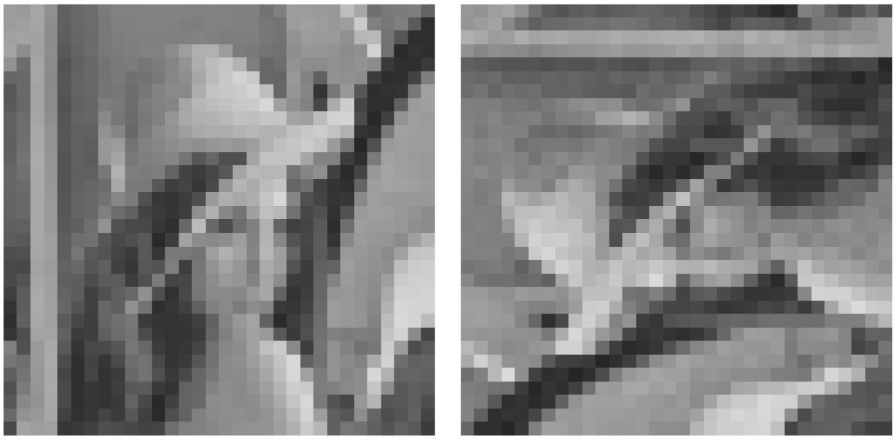

Fig. 6 Comparison Between Original Lena Image and Application of Quantum Coordinate Swap

4.3.3 Orthogonal Rotation

Orthogonal rotations are the result of combining the two techniques mentioned above. This implies that in quantum images, it is possible to apply rotations at angles of 90, 180, and 270 degrees. These rotations can be represented as follows.

$$R_{90}(|yx\rangle) = \ |xy\rangle \tag{4}$$

$$R_{180}(|yx\rangle) = \ |yx\rangle \tag{5}$$

$$R_{270}(|yx\rangle) = \ |xy\rangle \tag{6}$$

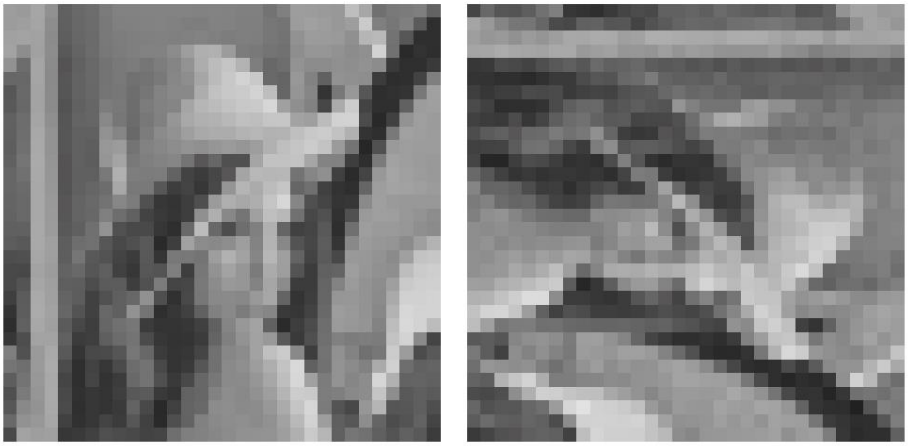

Fig. 7 Comparison Between Original Lena Image and Application of Quantum Orthogonal Rotation

5 Conclusions

Quantum image processing represents a revolutionary advancement in analysis and manipulation of visual data. Throughout this work, it’s have been explored two main approaches for quantum image encoding: FRQI (Flexible Representation of Quantum Images) and NEQR (Novel Enhanced Quantum Representation). Both methods offer unique advantages, such as efficiency in qubit usage and precision in representing pixel intensities, respectively. However, there have been identified significant challenges, including limitations in the number of qubits available for implementation on real quantum hardware and sensitivity to noise in current quantum systems.

Beyond the technical aspects of quantum image processing, this work invites us to reflect on the potential impact it could have in critical areas such as medicine, industry, and environmental monitoring. Imagining a future where medical diagnoses are faster and more accurate, or where industrial processes can be optimized with unprecedented efficiency, is exciting. However, it also serves as a reminder that technology is not an end but a tool to improve people's lives and address the most pressing global challenges.

The path toward practical quantum computing is filled with challenges, but it also holds the potential to transform technology in ways worth exploring. This work is a small step in that direction, but every step counts in building a more innovative and sustainable future.

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