

A Novel Constructive Routing Algorithm for Fleet Size and Mix Vehicle Routing Problem

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Abstract. In this study, a new constructive routing algorithm for fleet size and mix vehicle routing problem is proposed in which residual costs rather than vehicle types are considered for route selection. The algorithm of the proposed routing approach is given and then the solution phases of a sample problem are shown by using the given algorithm. In order to highlight the performance of the routing approach, Golden's 12 test problems (Fleet Size and Mix Vehicle Routing Problem with Fixed Cost) are used. It is seen that the proposed method has better average time complexity and cost performances than Ochi's routing approach. Therefore, the solutions of the proposed method that uses vehicle type information are better than those of the methods that use residual cost based on the vehicle type information.

Keywords : Fleet Size and Mix Vehicle Routing Problem, Constructive Heuristics, Vehicle Routing Problem, Routing Algorithm, Ochi's routing approach

1. Introduction

In fleet size and mix vehicle routing problem (HFVRP) each customer is visited by exactly one route. HFVRP consists of designing a number of feasible paths having minimum total cost / total distance. Aim of HFVRP is mainly to determine the best fleet composition as well as the set of paths that minimize the sum of fixed and travel costs in such a way that:

- (a) every route starts and ends at the depot and is associated to a vehicle type;
- (b) each customer belongs to exactly one route; and
- (c) the vehicle's capacity is not exceeded.

The HFVRP is an NP-hard problem and numerous methods have been proposed as it is a natural generalization of the travelling salesman problem (TSP) and as it includes the classical vehicle routing problem (VRP). [1] [2] [3] [4]

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Some researchers developed algorithms such as the savings algorithm of Clarke and Wright [5], the sweep algorithm of Gillett and Miller [6] and the generalized assignment of Fisher and Jaikumar [7]. Matching based saving algorithms were also proposed by Desrochers and Verhoog [8], Salhi and Rand [9] and Osman and Salhi [10]. Evolutionary algorithms have been attempted by Ochi et al. [1] and Lima et al. [2] on FSMF (Fleet Size and Mix Vehicle Routing Problem with Fixed Cost). However, the vehicle type information is always ignored in these methods [11]. In our study, a new constructive routing algorithm is proposed incorporating the vehicle type information.

2. Ochi and Proposed Constructive Routing Algorithms

A undirected graph is defined by $G = (V, A)$ where $V = \{0, 1, 2, \dots, n\}$ is a set composed of $n + 1$ vertices, and $A = \{(i, j) : i, j \in V, i \neq j\}$ is the set of arcs. The vertex 0 denotes the depot, where the vehicle fleet is initially located, while the set $V' = V - \{0\}$ is composed of the remaining vertices that represent n customers.

It is assumed that each customer $i \in V'$ has a positive demand q_i and depot's demand is always zero. $C = [c_{ij}]$ is the distance matrix where the parameter c_{ij} represents a positive cost or distance between vertices i and j . A heterogenous fleet of vehicles must be used to supply the customers. The vehicle fleet is composed by a set $\Psi(k) \in \{1, 2, \dots, t\}$ of different vehicle types where t is the number of vehicle types associated with the route, and it is assumed that each vehicle type is available at unlimited numbers. For each vehicle type $i \in \Psi$, Q_i is the capacity, f_i is the fixed cost to be paid, and D_i is the amount of demand collected from or loaded to the vehicle. It is assumed that the fixed costs are increasing with the capacity i.e. $Q_1 < Q_2 < \dots < Q_t$ and $f_1 < f_2 < \dots < f_t$ [1] [2] [3] [4].

A route for vehicle type k is defined by the pair $(R, \Psi(k))$ where $R = (i_1, i_2, \dots, i_{|R|})$, with $i_1 = i_{|R|} = 0$ and $\{i_2, i_3, \dots, i_{|R|-1}\} \subseteq V$, is a simple circuit in G containing the depot. Here, R will be used to refer both to visiting sequence and to the set of customers (including depot) of the route. A route $(R, \Psi(k))$ is feasible if the total demand of the customers visited by the route does not exceed the vehicle capacity Q_k , that is, $\sum_{h=2}^{|R|-1} q_{i_h} \leq Q_k$.

The cost of a route corresponds to the sum of the costs of the edges forming the route, plus the fixed cost of the associated vehicle, that is, $\sum_{h=1}^{|R|-1} c_{i_h i_{h+1}}^k + f_k$ [3].

The route configuration proposed by Ochi et al is achieved by selecting the minimum from the alternatives obtained by the constraint $(Q_k - D_k) * f_k$. However, our study is based on $(Q_k - D_k)$ constraint for route configuration and then selecting the minimum from the alternatives obtained. The constraints of the related routing strategies are given in Table 1.

Table 1: Proposed Approach and Ochi Approach routing constraint for HFVRP

	Ochi(1998)	Karagul
Route Construction / Route Selection	$\min_k(Q_k - D_k) * f_k$	$\min_k(Q_k - D_k)$

In this study, the routing approach in [1] is denoted as Ochi Minimum Distance Minimum Vertex Algorithm (Ochi MinDis-MinVer Algorithm) and the proposed routing approach is denoted as Karagul Minimum Distance Minimum Vertex Algorithm (Karagul MinDis-MinVer Algorithm). In Figure 1, the Ochi MinDis-MinVer algorithm is demonstrated. In Figure 1, the constraint $(Q_k - D_k) * f_k$ is defined with (Residuals*FixedCost). For equal minimum (Residuals*FixedCost) of any two paths, the first obtained path is used.

```

start with the Initial Solution Space (TSP_order)
for each TSP_order
  while {end of the TSP_order}
    while {end of the number of vehicle_type}
      Construct temporary routings for each vehicle_type
    end {of while}
    Find the minimum(Residuals*FixedCost) that is the temporary Path
    Assign the vertex and vehicle type to the Path
    Calculate Routing_Cost, TSP_order, vehicle_type
  end {of while}
  TSP order solution: [Total_Cost Routings Type_of_Vehicles TSP_order]
end {of for}

Solution Space:[TSP_order solution [Total_Cost Routings Type_of_Vehicles TSP_order]
]
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Figure 1 : Ochi Minimum Distance Minimum Vertex Algorithm (Ochi MinDis-MinVer Algorithm)

In Figure 2, Karagul MinDis-MinVer algorithm is given. For this algorithm, the constraint $(Q_k - D_k)$ is defined with (Residuals). Similar to Ochi MinDis-MinVer Algorithm, for equal minimum (Residuals) of any two paths, the first obtained path is used.

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start with the Initial Solution Space (TSP orders)
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for each TSP order
  while {end of the TSP order}
    while {end of the number of vehicle type}
      Construct temporary routings for each type of vehicle
    end {of while}
    Find the minimum(Residuals) that is the temporary paths
    Assign the vertex and vehicle type to Path
    Calculate Routing Cost, path part of TSP order, TSP order part, vehicle
type
  end {of while}
  TSP order solution: [Total Cost Routings Type of Vehicles TSP order]
end {of for}

Solution Space:[TSP order solution [Total Cost Routings Typeof Vehicles TSP order]]

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Figure 2 : Karagul Minimum Distance Minimum Vertex Algorithm (Karagul MinDis-MinVer Algorithm)

3. Sample Problem and Solution Phases

In Figure 3, an HFVRP problem is defined in order to show the effectiveness of the proposed method. The problem is composed of a depot, two types of vehicles (t_1, t_2), and 6 customers. In Figure 4, the parameters defining the problem are given.

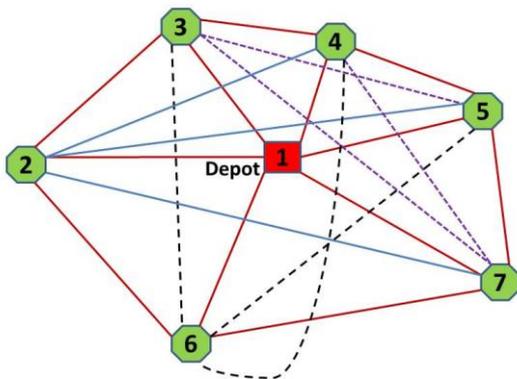


Figure 3 : Representation of sample problem with vertices and all connections

$$C = \begin{pmatrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{matrix} 0 & 60 & 65 & 50 & 50 & 70 & 60 \\ 60 & 0 & 20 & 70 & 73 & 118 & 122 \\ 65 & 20 & 0 & 50 & 50 & 110 & 120 \\ 50 & 70 & 50 & 0 & 47 & 80 & 90 \\ 50 & 73 & 50 & 47 & 0 & 80 & 90 \\ 70 & 118 & 110 & 80 & 80 & 0 & 70 \\ 60 & 122 & 120 & 90 & 90 & 70 & 0 \end{matrix} \end{pmatrix}$$

Depot/Customer	1	2	3	4	5	6	7
Demands	0	30	28	40	30	40	50

Vehicle Types	1	2
Fixed Costs	40	70

Vehicle Types	1	2
Capacity	50	90

Figure 4 : Sample problem's distance matrix, customer demands, vehicle types fixed costs and vehicle types capacity

In Figure 5, the solution routes of Ochi MinDis-MinVer algorithm for a random TSP order {2, 3, 4, 5, 6, 7} is shown step by step.

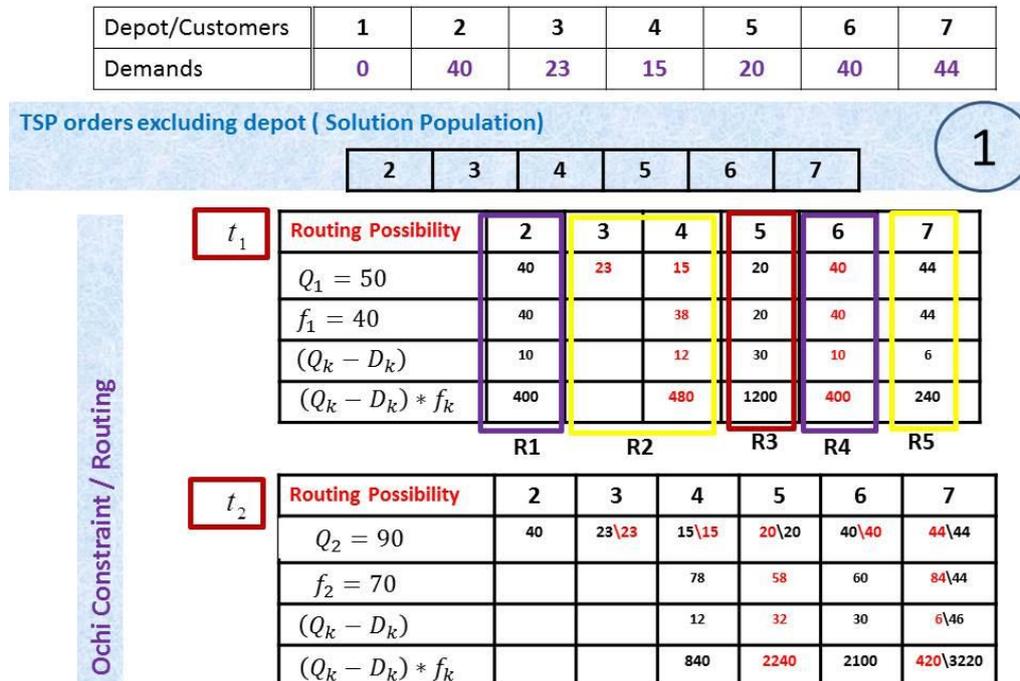


Figure 5 : Ochi Minimum Distance Minimum Vertex Algorithm solution phases for routing TSP order {2, 3, 4, 5, 6, 7}

The vehicle routes are constructed with respect to $(Q_k - D_k) * f_k$ constraint and vehicles with smaller capacities are considered first. As shown in Figure 5, when customer 2 is considered for t_1 , the demand is 40 units. As the vehicle capacity will exceed 50, customer 3 cannot be added. Therefore, for vehicle t_1 the temporary route is {2}, the total load quantity is 40 units and residual is $50-40=10$ units, and the residual cost is $10*40=400$ unit cost.

Similar to t_1 , when the demands of 40 units from customer 2, 23 units from customer 3 and 15 units from customer 4 are loaded to the vehicle t_2 not to exceed 90 unit capacity, 78 unit loading is made in total. The temporary path {2, 3, 4} is obtained. The residual for t_2 is $90-78=12$ units and the residual cost is $12*70=840$ unit cost.

When the residual costs of two vehicles are considered, it is seen that there is 400 unit cost for t_1 and 840 unit cost for t_2 . Therefore, the first constructed path $R1=\{(1-2-1), t_1\}$ is taken as it has minimum residual cost. Then, customer {2} is discarded from TSP order.

The unrouted customers {3, 4, 5, 6, 7} are reconstructed for temporary routes. As can be seen from the second phase in Figure 5, firstly for vehicle t_1 , 23 units from {3}, 15 units from {4} are loaded which in total compose 38 unit load. The residual is 12 units and the residual cost is 480 unit cost. Similarly, for vehicle t_2 , 23 units from {3}, 15 units from {4} and 20 units from {5} are loaded which in total compose 58 unit load. The residual is 32 units and the residual cost is 2240 unit cost. When the residual costs of two vehicles are considered for the second phase, it is seen that there is 480 unit cost for t_1 and 2240 unit cost for t_2 . Therefore, the second constructed path $R2=(\{1-3-4-1\}, t_1)$ is taken as it has minimum residual cost.

Then, customer {3,4} are discarded from TSP order. The unrouted customers {5, 6, 7} are reconstructed for temporary routes. As can be seen from the second phase in Figure 5, firstly for vehicle t_1 , 20 units from {5} is loaded. The residual is 30 units and the residual cost is 1200 unit cost. Similarly, for vehicle t_2 , 60 units from {5,6} are loaded which in total compose 58 unit load. The residual is 30 units and the residual cost is 2100 unit cost. When the residual costs of two vehicles are considered for the second phase, it is seen that there is 1200 unit cost for t_1 and 2100 unit cost for t_2 . Therefore, the third constructed path $R3=\{1-5-1\}, t_1)$ is taken as it has minimum residual cost. Then, customer {5} is discarded from TSP order and the unrouted customers {6, 7} are reconstructed for temporary routes.

When the same process is executed for remaining customers {6, 7}, the routes and assigned vehicles are t_1 and $R4=(\{1-6-1\}, t_1)$ and $R5=(\{1-7-1\}, t_1)$. Thus, for Ochi MinDis-MinVer algorithm the routing process for the TSP orders is completed. The summary table for the routings and costs are given in Figure 7.

In Figure 6, the solution routes of Karagul MinDis-MinVer algorithm for the same TSP order given in Figure 5 for Ochi MinDis-MinVer algorithm, is shown step by step.

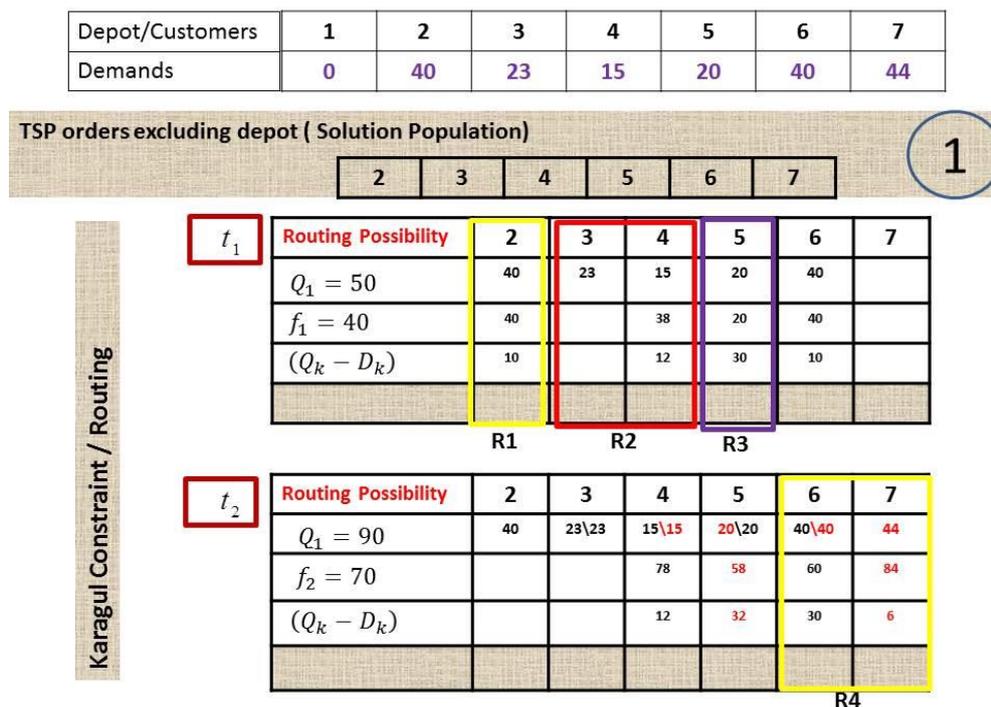


Figure 6 : Karagul Minimum Distance Minimum Vertex Algorithm solution phases for routing TSP order {2, 3, 4, 5, 6, 7}

The vehicle routes are constructed with respect to $(Q_k - D_k)$ constraint and vehicles with smaller capacities are considered first. As shown in Figure 6, when customer {2} is considered for t_1 , the demand is 40 units. As the vehicle capacity will exceed 50, customer {3} cannot be added. Therefore, for vehicle t_1 the temporary route is {2}, the total load quantity is 40 units and residual is $50-40=10$ units.

Similar to t_1 , when the demands of 40 units from customer {2}, 23 units from customer {3} and 15 units from customer {4} are loaded to the vehicle t_2 not to exceed 90 unit capacity, 78 unit loading is made in total. The temporary path {2, 3, 4} is obtained. The residual for t_2 is $90-78=12$ units.

When the residuals of two vehicles are considered, it is seen that there is 10 unit for t_1 and 12 unit for t_2 . Therefore, the first constructed path $R1=\{1-2-1, t_1\}$ is taken as it has minimum residual. Then, customer {2} is discarded from TSP order.

The unrouted customers {3, 4, 5, 6, 7} are reconstructed for temporary routes. As can be seen from the second phase in Figure 6, firstly for vehicle t_1 , 23 units from {3}, 15 units from {4} are loaded which in total compose 38 unit load. The residual is 12 units. Similarly, for vehicle t_2 , 23 units from {3}, 15 units from {4} and 20 units from {5} are loaded which in total compose 58 unit load. The residual is 32 units. When the residual of two vehicles are considered for the second phase, it is seen that there is 12 unit for t_1 and 32 unit for t_2 . Therefore, the second

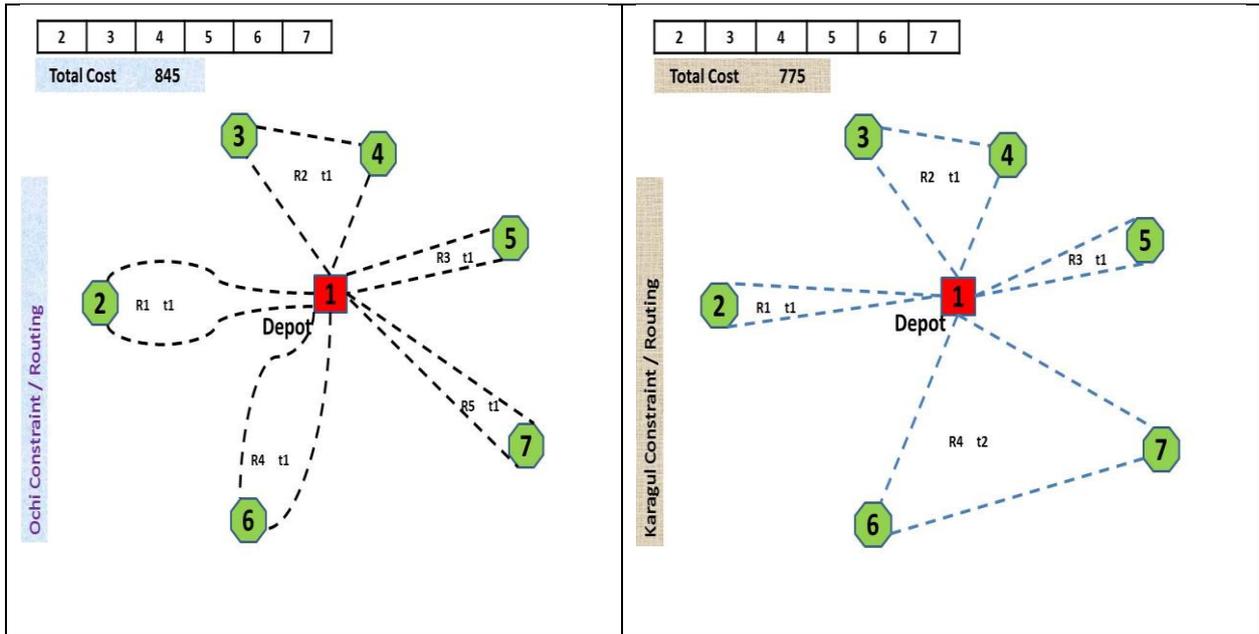


Figure 8 : Ochi and Karagul Routing Algorithms solutions results for TSP order {2, 3, 4, 5, 6, 7}

Then, Ochi and Karagul routing approaches are compared for the same sample problem with different initial TSP orders. Solutions are obtained based on Ochi and Karagul routing approaches for two random TSP orders that are {3, 4, 2, 6, 5, 7} and {2, 7, 6, 5, 3, 4}. These solutions are given as follows:

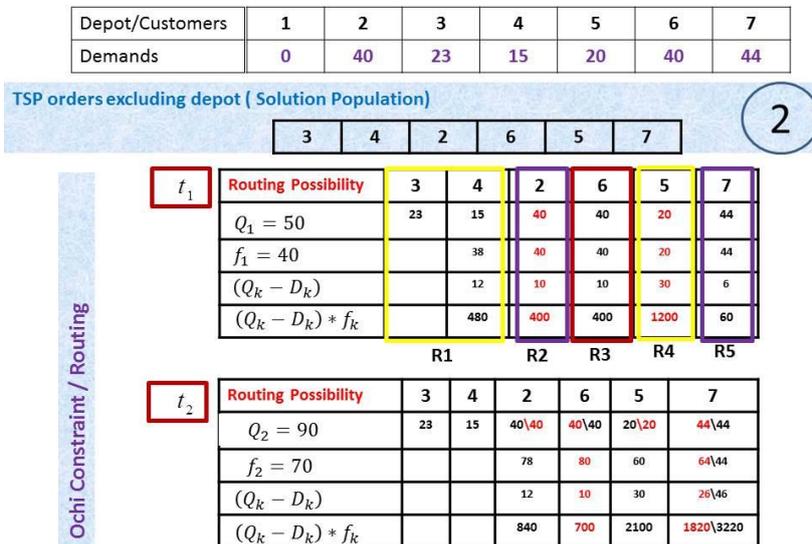


Figure 9 : Ochi MinDis-MinVer Algorithm solutions for {3, 4, 2, 6, 5, 7} TSP order

As can be seen from Figure 9, the constructed routes and the types of vehicles assigned to each route with Ochi MinDis-MinVer Algorithm are R1=({1-3-4-1}, t_1), R2={1-2-1}, t_1), R3=({1-6-1}, t_1), R4=({1-5-1}, t_1), R5=({1-7-1}, t_1), respectively.

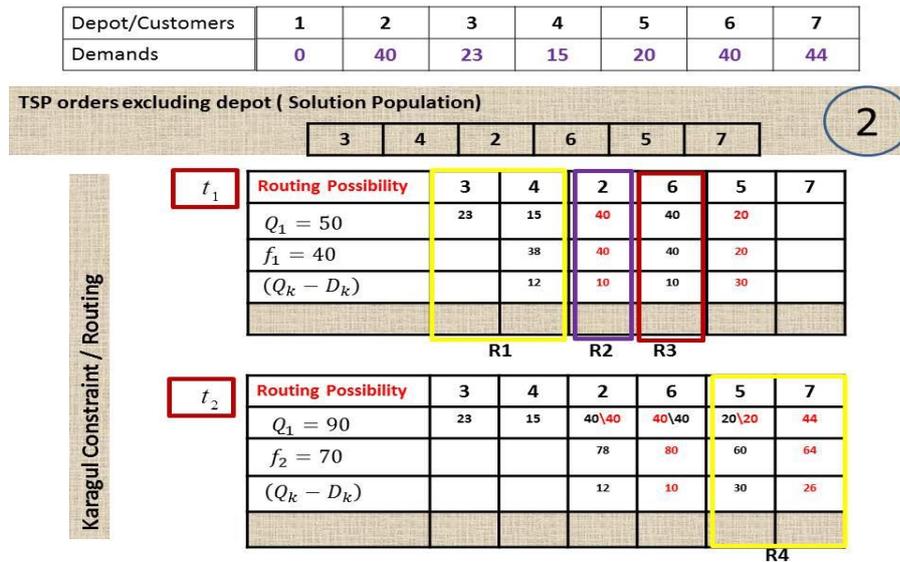


Figure 10 : Karagul MinDis-MinVer Algorithm solutions for {3, 4, 2, 6, 5, 7} TSP order

As can be seen from Figure 10, the constructed routes and the types of vehicles assigned to each route with Karagul MinDis-MinVer Algorithm are R1={1-3-4-1}, t_1 , R2={({1-2-1}), t_1), R3={({1-6-1}), t_1), R4={({1-5-7-1}), t_2), respectively. In Figure 11, the solutions and costs obtained from the algorithms are shown.

TSP orders excluding depot (Solution Population)

3	4	2	6	5	7
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	R Name	Route	T	Fixed Costs	Route Distance	Route Cost
Ochi Routing	R1	1-3-4-1	t_1	40	165	205
	R2	1-2-1	t_1	40	120	160
	R3	1-6-1	t_1	40	140	180
	R4	1-5-1	t_1	40	100	140
	R5	1-7-1	t_1	40	120	160
						845
Karagul Routing	R1	1-3-4-1	t_1	40	165	205
	R2	1-2-1	t_1	40	120	160
	R3	1-6-1	t_1	40	140	180
	R4	1-5-7-1	t_2	70	200	270
						815

Figure 11 : Ochi and Karagul Routings Algorithms solutions results for {3, 4, 2, 6, 5, 7} TSP order

The number of routes constructed with Karagul MinDis-MinVer Algorithm is less than Ochi MinDis-MinVer Algorithm. Also total costs are 815 and 845 money unit for Karagul and Ochi MinDis-MinVer Algorithms, respectively. In Figure 12, the graph representation of constructed routes of both algorithms are given.

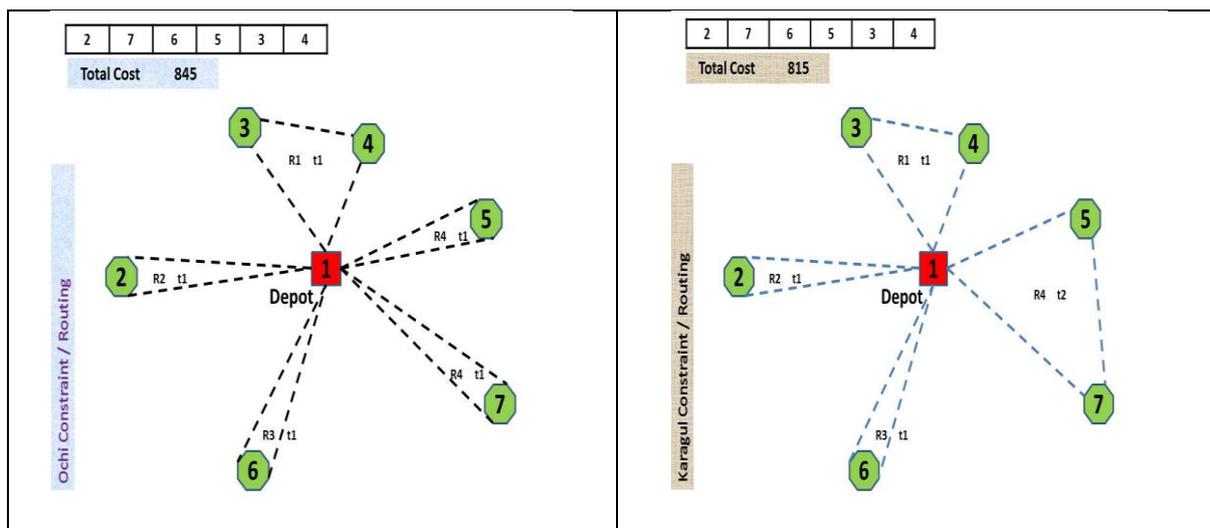


Figure 12 : Ochi and Karagul Routings Algorithms solutions graphs for {3, 4, 2, 6, 5, 7} TSP order

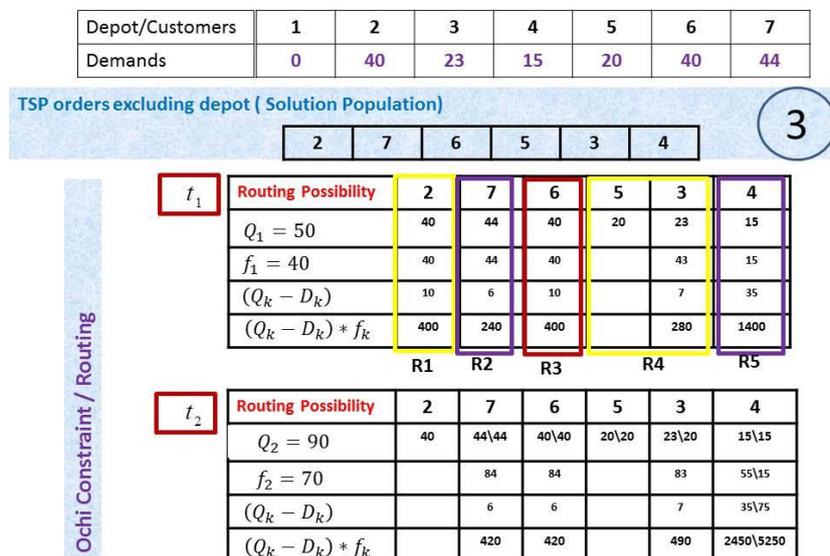


Figure 13 : Ochi MinDis-MinVer Algorithm solutions for {2, 7, 6, 5, 3, 4} TSP order

As can be seen from Figure 13, the constructed routes and the types of vehicles assigned to each route with Ochi MinDis-MinVer Algorithm are $R1 = (\{1-2-1\}, t_1)$, $R2 = (\{1-7-1\}, t_1)$, $R3 = (\{1-6-1\}, t_1)$, $R4 = (\{1-5-3-1\}, t_1)$, $R5 = (\{1-4-1\}, t_1)$, respectively.

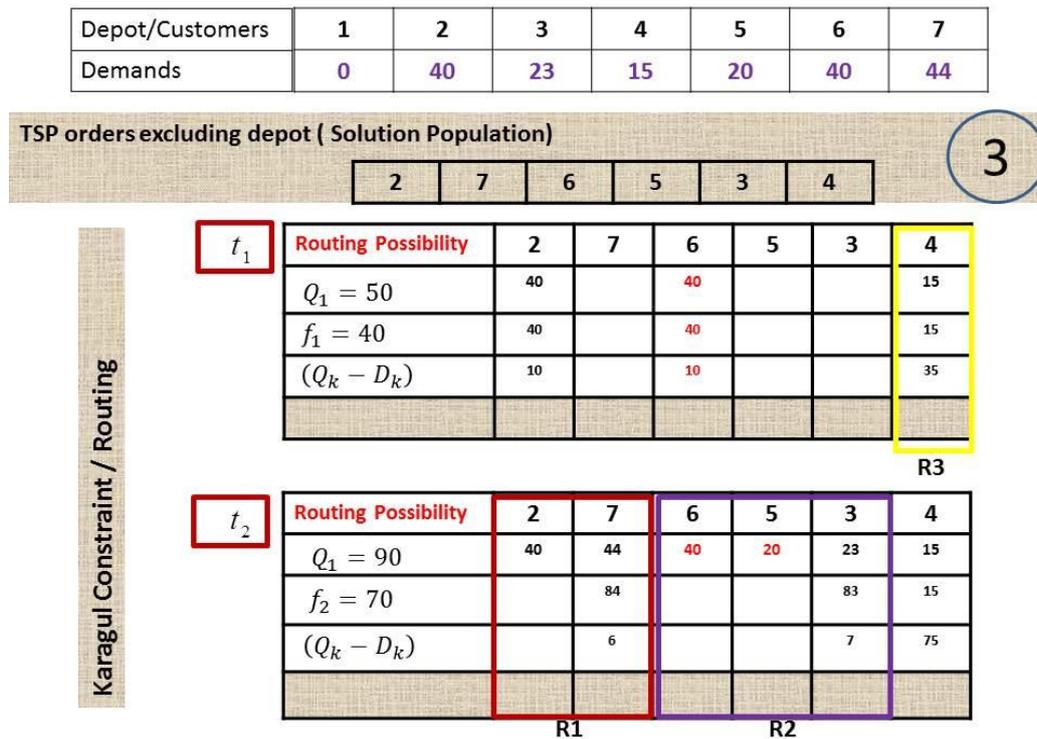


Figure 14 : Karagul MinDis-MinVer Algorithm solutions for {2, 7, 6, 5, 3, 4} TSP order

As can be seen from Figure 14, the constructed routes and the types of vehicles assigned to each route with Karagul MinDis-MinVer Algorithm are R1=({1-2-7-1}, t_2), R2=({1-6-5-3-1}, t_2), R3=({1-4-1}, t_1), respectively.

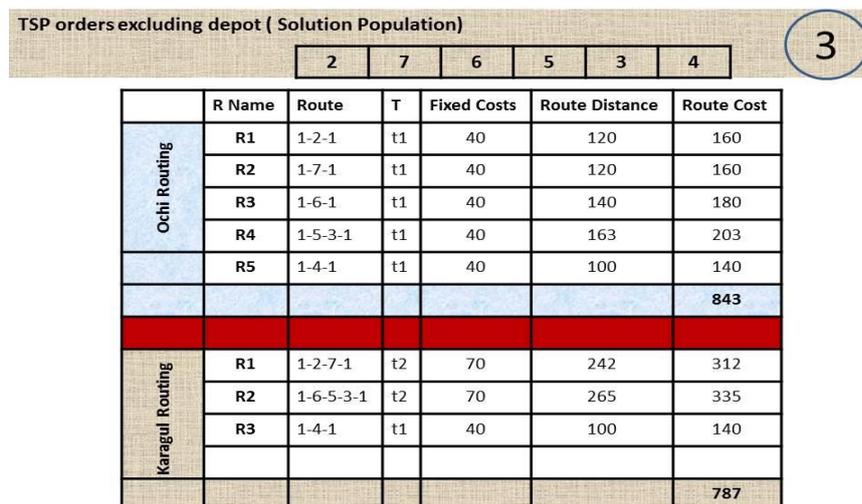


Figure 15 : Ochi and Karagul Routings Algorithms solutions results for {2, 7, 6, 5, 3, 4} TSP order

When Figure 15 is reviewed, it is seen that while R1, R2, R3, R4 and R5 routes are obtained with a total cost of 843 money unit from Ochi MinDis-MinVer Algorithm, R1, R2, R3 routes are obtained with a total cost of 787 money unit from Karagul MinDis-MinVer Algorithm.

Both the number of routes and total cost are less for Karagul approach than Ochi approach. The graph representation of constructed routes for both methods are given in Figure 16.

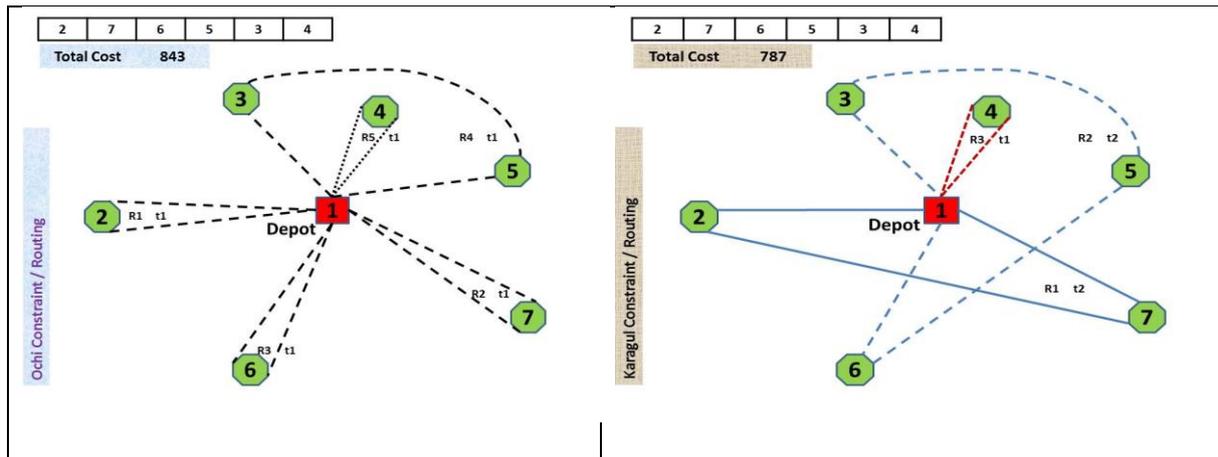


Figure 16 : Ochi and Karagul Routing Algorithms solutions graphs for {2, 7, 6, 5, 3, 4} TSP order

From the sample routing problem, the difference of the proposed method and the difference of the solutions are analyzed. In the next section, to see the performance of the proposed method on some known test problems from the literature, Golden’s test instances are used.

4. Computational Results

The proposed method is tested by using 12 sample problems obtained by Golden et al. [12] and extensively used in the literature for FSMF.

The calculations are constructed from two phases: the first step is obtaining the initial solution space, and the second step is the route configuration and the selection of the appropriate constraint. The initial solution space is generated based on the method presented by Liu et al. [13] where the initial solution space is composed of 3 parts: first part from the Savings Algorithm, second part from the Sweep Algorithms and the rest of the individuals are generated randomly. In our study, on the other hand, the randomly generated individuals are not used. The solutions of the Savings and Sweep algorithms are obtained by using “Matlog: Matlab Logistic Engineering Toolbox” [14].

The problems are tested on a computer with Pentium Core Duo i7 processor and 4 GB RAM.

The results obtained on the basis of the initial solutions from Sweep and Savings algorithms are listed in Table 2 where **P.No** is the problem number as given by Golden et al. [12], **BKS** is the best known solution in the literature, **Solution** is the Karagul and Ochi solutions obtained for the given problems with this study, **Deviation** is the percent deviation from the best known solution, **Time** is the solution time in seconds and **S.S.** is the dimension of the initial solution space. The initial solutions are obtained excluding the depot in the form of **TSP order**. Then the routes are configured with respect to the related methods. From the alternative route solutions, the type of vehicle that provides the minimum condition is selected as the optimal route. For the solution times in Table 2, the period for obtaining the initial solutions are not considered. Therefore, the solution times are solely giving the execution times of the algorithms.

Table 2: Ochi MinDis-MinVer Algorithm and Karagul MinDis-MinVer Algorithm computational results for FSMVRP with fixed cost (FSMF) on 12 test problems

P.No	BKS	Ochi MinDis-MinVer Algorithm			Karagul MinDis-MinVer Algorithm			S. S.
		Solution	Deviation	Time	Solution	Deviation	Time	
3	961,03	1.088,70	-13,28	0,1033	977,18	-1,68	0,0973	4
4	6.437,30	7.324,70	-13,78	0,0876	7.324,70	-13,78	0,0872	6
5	1.007,10	1.183,60	-17,53	0,0755	1.116,10	-10,82	0,0779	4
6	6.516,50	7.031,40	-7,90	0,0586	7.031,40	-7,90	0,0646	6
13	2.406,40	2.830,50	-17,63	0,2484	2.638,50	-9,65	0,1922	8
14	9.119,00	9.214,40	-1,05	0,0689	9.214,40	-1,05	0,0761	6
15	2.586,40	2.795,30	-8,08	0,0905	2.856,10	-10,43	0,0828	6
16	2.720,40	3.063,80	-12,62	0,0716	2.899,00	-6,56	0,0915	4
17	1.734,50	2.088,90	-20,43	0,1554	1.954,10	-12,66	0,1151	8
18	2.369,70	2.992,40	-26,28	0,3118	2.846,00	-20,10	0,2099	10
19	8.661,80	9.599,20	-10,82	0,0904	9.649,30	-11,40	0,1191	6
20	4.039,50	4.459,10	-10,39	0,1224	4.446,20	-10,07	0,1189	6
Average	4.046,64	4.472,67	-13,32	0,1237	4.412,75	-9,68	0,1110	4

When Table 2 is reviewed, for 7 of 12 test problems Karagul MinDis-MinVer Algorithm has better total cost values. Also, for 3 problems it has same total costs with Ochi MinDis-MinVer Algorithm. With Ochi MinDis-MinVer Algorithm only 2 of 12 test problems have best total cost values. When the average performances are compared, the proposed method has better characteristics from time complexity and total cost point of view. Based on the given tests, Karagul MinDis-MinVer Algorithm can be proposed as a new constructive routing algorithm for HFVRP.

5. Conclusions and Discussion

In this study, a new constructive route configuration different from the method recommended by Ochi et al and an approach certainly competitive with their method are proposed. The problems in the literature are solved using a seeding with Sweep and Savings algorithms proposed by Liu-Huang-Ma [13]. When the proposed method is logically compared for different situations, it gives better results than the approach of Ochi et al [1]. Thus, the new method can be suggested both for route configuration and route selection in heterogeneous VRPs. The solution given in this study can be enriched using different initial solution generation methods and new hybrid solution methods can be obtained by combining with heuristic search methods.

6. Acknowledgements

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