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Forecasting 3PL demand of warehousing services with interval type-3 fuzzy logic and GM (1,1)

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Abstract. Abstract. Real-world information can be vague or imprecise, not reliable, where the information is presented in fragments, ambiguity in the data, or even contradictory information, these can lead to uncertainty, but even this uncertainty we need to take decisions [1]. Part of this uncertainty can be handled by different un-certainty models, such: grey systems [2-3], type-1, type-2 [4-7] or typ-3 fuzzy systems, all used represent this uncertainty with numbers. But some-times, there more complex situations are, it is extremely difficult to find the precise numeric value or model to provide accurate value for any uncertain entity. This paper pro-vides a state-of-the-art review on the different applications of the type-3 [8-18]and grey systems [19-30] and proposes the application of Type-3 fuzzy logic for demand forecasting within the supply chain combined with the preciseness GM (1,1) of the grey systems theory (GST), applied for predicting the demand of warehousing services (3PL) in the industry. While utilizing interval type-3 fuzzy logic helps handling the uncertainty in the decision-making when forecasting re-duces its effect, the GM (1,1) improves its accuracy. Type-3 Fuzzy Logic is a robust and capable model to cope with high-order uncertainties associated with non-stationary time-dependent features of the GM (1,1). The main objective this paper is to present Type-3 fuzzy logic, denoted as A3, combined with a GM (1,1) model, which provides better results than its individual application. The hybrid approach is formed by an interval type-3 fuzzy model structured by fuzzy if then rules that, utilize as inputs the linear GM (1,1) equation. The contribution is the new scheme based on interval Type-3 Fuzzy Logic and the linear GM (1,1), which has not been proposed before, aiming to achieve and accurate forecast of multivariable forecasts time series, while reducing the mathematical complexity of the model. The fuzzy rules can be established with the Mandami reasoning meth-od, but in this paper were established with the Sugeno-Takagi-Kang approach. The proposed method has been compared with previous works and its results confirm the superiority of the Type-3 Fuzzy Logic combined with the GM (1,1).

Keywords: Type-3 Fuzzy model, Forecasting, Grey Systems, Supply Chain.

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1 Introduction

Humans are not designed to be perfect decision-makers, and it can be shown that under certain circumstances, a group is often smarter than the individuals in it. One does not have to be exceptionally intelligent or completely master an issue for a group decision to be made in an informed and rational manner. We usually find ourselves in possession of less information than we would like or require for a problem. We have an incomplete, blurred, or fuzzy view of the future. Instead, we are faced with the need to make the best possible decision that we can deem acceptable enough. On many occasions, we allow our decisions to be affected by emotions and biases; however, despite this, when our imperfect judgments are aggregated in the right way, our collective intelligence is usually excellent and provides us with a good enough solution. While this could be considered as: "the wisdom of the crowd", as Francis Galton [31] had already expressed in 1906, it is not about finding the right person who will have that answer, but how as a whole certain people or tools, grouped or coordinated in the right way, will have it; looking for the expert could be a mistake. This paper is about just that. "Combining multiple forecasting methods for a given time series improves accuracy through integration obtained from different sources, thus avoiding the need to identify a best method" [32]. Likewise, this work focuses on the importance of assets in organizations, specifically the value that inventories represent for them, where a demand with a high degree of volatility, but above all with a high degree of uncertainty in supply, can represent more than 80% of the money invested in organizations. In this paper, we focus on tangible goods, specifically those that are stored in a third-party warehouse (3PL), either as finished product or raw materials. The enormous volatility in demand, but above all, the enormous uncertainty about it, has forced organizations to question the validity of traditional forecasting models, and that both the experience of the organization must be considered when making decisions to maintain or not such inventory.

The proposed model combines Type-3 fuzzy logic T3-FS and the general first-order, one-variable model of GM (1,1) gray systems [2-3] and proposes that combined they can result in a better approach to demand forecasting within the supply chain. This combination is a novel alternative to the various current and most popularly used methods, with the difference that the proposed model, by making use of type-3 fuzzy logic and the GM (1,1) model, integrates the user's experience precisely, in an environment of high level of uncertainty, and limited available information (>20 data), to perform demand forecasting with a higher level of accuracy.

The proposed model makes use of a Sugeno-Takagi-Kang controller, and the exponential function of GM (1,1) gray systems, by transitioning smooth linear functions. This fuzzy controller: 1) merges, 2) cuts, 3) joins, but does not require a defuzzifier, i.e., it simplifies the result, by only considering the inputs to the controller, in this case as: a) the current demand behavior, b) the level of uncertainty perceived by the organization and c) the degree of experienced temporality perceived by the user. These data are evaluated on the fuzzy sets of each if-then rule, i.e., the incoming values are evaluated based on a membership function μ_{A1}, μ_{B1} and μ_{C1} and merged; then the consequent sets of each rule are cut to the height of the merging value that corresponds to it, to subsequently join to calculate the set resulting from the inference μ_F, a single scalar value resulting from the controller is obtained, and this is repeated every certain time interval, in this work, is used instead of a defuzzifier (as opposed to a Mandami controller), as many linear functions required to represent the exponential function of the GM model (1,1), as a means of inference to obtain the value of the forecast. This process simplifies the calculations, the inference rules and speeds up the sensitivity analysis of the system. The model used is the one described by Sugeno and Takagi [33].

2 Fuzzy Logic

Fuzzy logic provides an effective way to deal with problems involving imprecise, vague or fuzzy concepts. They allow evaluators to use linguistic terms to evaluate indicators in natural language expressions, and each linguistic term can be associated with a membership function. Fuzzy logic has evolved since Zadeh makes it known in 1965 [4] and subsequent contributions of type-1, type-2 fuzzy logic [5-7]. This paper provides a brief review on the state of the art specifically of type-3 fuzzy logic (T3-FS) where different applications of it are shown.

Within the supply chain [8], fuzzy logic can be found many applications in the literature, where fuzzy set theory has been used, including production and quality planning and management. The existence of imprecision and vagueness of these problems are the main reason for its application. In addition, the unavailability of complete information, accurate references and reliable data makes the application of fuzzy logic highly applicable. Fuzzy logic works successfully with models that are fuzzy or vague, unlike other methods. The elements of a fuzzy set are taken from a universe of discourse, or universe for short, where fuzzy set theory deals with the imprecision associated with many variables by allowing a degree of membership to be defined in the interval [0, 1], called a membership function or as $\boldsymbol{\mu_x}$ where it will be referred to throughout this paper [9]. Also, there are many studies using some well-known data to propose Fuzzy Time Series FTS models that provide better forecasting results [10]. FTS were proposed to handle the vagueness and imprecision of time series and have become very competitive forecasting methods [11] and methods have been proposed to handle stationarity and non-stationarity of demand data, additionally fuzzy time series models can be applied to nonlinear problems [12], Likewise, there have been different fuzzy methods proposed to solve time series problems, where systems of linear equations are obtained that can be solved through linear programming [34], where the fuzzy linear functions are defined by the Zadeh equation extension principle, where the deviations between the observed values and the estimated values are due to measurement errors. Under fuzzy logic, on the contrary, it assumes that these deviations depend on the indefiniteness of the system structure [35]. Fuzzy time series can model the forecasting problem where historical information is represented through linguistic values. Type-2 fuzzy logic models are robust models, able to cope with high order uncertainties associated with non-stationary time-dependent characteristics [13]. Other models propose the simultaneous use of linear and nonlinear relationships and produce satisfactory predictions. On the other hand, statistical models are well known and widely used for time series modeling, but they need to satisfy some strict assumptions. This is not the case for non-probabilistic models [14-18].

2.1 Fuzzy Logic Type-3

The nature of data generated over time is broadly classified as stationary, seasonal, trend and strictly non-stationary. However, most historical data generated over time in the real world is not as statistically consistent as it is considered for data and forecasting purposes. Traditional algorithms based on mathematical and statistical forecasting models cannot address the problem in the real world, where historical data are imprecise [36-37] and vague [38]. Type-3 interval fuzzy logic can be seen as an extension of type-2 models. The basic terminology of type-3 fuzzy sets is described to show how it differs from its type-2 counterparts [16].

A type-3 fuzzy set (T3-FS), denoted $\mathbf{A^3}$, is represented by a trivariate function, called the membership function (MF) of $\mathbf{A^3}$, as the Cartesian product X x [0,1] x [0,1] in [0,1], where X is the universe of discourse of the primary variable of $\mathbf{A^3}$, x. This MF of $\boldsymbol{\mu_A^3}$, is formulated by $\boldsymbol{\mu_A^3\ (x, u, v)}$, such that:

$$\mu_A{}^3: X \text{ x } [0,1] \text{ x } [0,1] \rightarrow [0,1]$$

$$A^3 = \{(x, u(x), v\,(x, u), \mu_A{}^3\,(x, u, v)) \mid x \in X, u \in U \subseteq [0,1], v \in V \subseteq [0,1]\} \quad (1)$$

where $\boldsymbol{U}$ is the universe of discourse for the secondary variable $\boldsymbol{u}$ and $\boldsymbol{v}$ is the universe of discourse for the tertiary variable $\boldsymbol{v}$. Castillo, Castro and Melin [9]. If the tertiary MF is uniformly equal to 1, then we have a fuzzy interval set type-3 (IT3 FS) with interval type-3 MF (IT3MF). These membership functions can take basically any function that the user defines, based on the historical performance of the data, or based on experience. The most used typical functions are generalized bell, Gaussian bell, triangular, sigmoid, Z, and trapezoid, also the product of the combination of any of these, such as, for example, the product of the S and Z function, resulting in the Pi function, or a custom membership function [39].

If $\mu_A{}^3\ (x, u, v) = 1$, the T3-FS, A^3, reduces to a type-3 interval denoted by: $\mathbb{A}$, defined by:

$$\mathbb{A} = \int_{x \in X} \left[\int_{u \in [0,1]} \left[\int_{v \in [\mu - A(x,u), \mu + A(x,u)]} 1/\mathrm{v} \right] /\mathrm{u} \right] /\mathrm{x} \quad (2)$$

$$\mathbb{A} = \int_{x \in X} \mu\, \mathrm{A}\,(\mathrm{u}, \mathrm{v})/\mathrm{x} \quad (3)$$

An interval T3-FS, denoted by $\mathbb{A}$, is an isosurface with a bivariate function, called MF of $\mathbb{A}$, over the Cartesian product X x [0,1] in [0,1], where X is the primary membership function of the variable of $\mathbb{A}$, x. The membership function, MF of $\mathbb{A}$, is denoted by $\mu\mathcal{A}\ (x, u)$ and is called the interval of the type-3 membership function or MF (IT3 MF), that is:

$$\mathcal{A} = \{(x, u, \mu\mathcal{A}\ (x, u)) \ / \ x \in X, u \in U \equiv [0,1] \quad (4)$$

Where $\mu\mathcal{A}\ (x, u) \subseteq [0,1]$; U is the universe of discourse of the secondary variable u, and in this paper, U is always assumed to be [0,1].

2.2 The grey systems GM (1,1)

The theory of gray systems is a relatively young discipline of knowledge, which has demonstrated its usefulness in solving many real problems. In recent years it has been used to explain and forecast several phenomena in many aspects of reality, such as in the analysis of multiple social phenomena, economic, technical models and natural disasters, accidents or even tourism [19-30], [40-48].

However, it appears that there is little or no application of gray systems as a forecasting method in the supply chain, from what has been found in the literature.

Using a first order differential equation, with one variable, characterized by an unknown system, i.e., one in which there is no additional information that defines it and for which there is little information, including few records (>20). The GM (1,1) model is suitable for competitive forecasting in an environment where decision makers can refer to a limited amount of historical data [42], obtaining, however, highly accurate results, above the traditional models of: simple moving averages, weighted, exponential, simple linear regression, etc. The Markov chain forecasting model can be used to forecast a system with time series that vary randomly, with no causal reasons identified to justify this behavior. It is a dynamic system that forecasts the development of the system according to the transition probabilities between states reflecting the influence of all random factors [42].

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The GM (1,1) model, is defined as:

$$x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\} \tag{5}$$

where $x^{(0)}$ is assumed to be the original data sequence. To subsequently obtain the one-time accumulated generating operator **1-AGO** (one-time accumulated generating operator **1-AGO**), as follows:

$$x^{(1)}(k) = \{x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \ldots, x^{(1)}(n)\} \tag{6}$$

where:

$$x^{(1)}k = \sum_{i=1}^{k} x^{(0)}i \tag{7}$$

The GM differential equation (1,1) and its "whitening" equation are obtained respectively:

$$x^{(0)}(k) + ax^{(1)}(k) = b, \; k= 1, 2, 3, \ldots, n \tag{8}$$

$$\frac{dx^{(1)}}{dt} + ax^{(1)}(k) = b \tag{9}$$

Where ***a*** represents the development coefficient and ***b***, denotes the uncertainty factor or input error, or "grey factor". Let û now be the parameters of the vector:

$$\hat{u} = (\hat{a}, b)^{T} = (B^{T} B)^{-1} B^{T} Y_{N} \tag{10}$$

Where **B** denotes the cumulative matrix and, **Y** is the constant vector, such that ***a*** and ***b*** can be obtained using the solution of the equation by the least squares method, where:

$$z^{(1)}(k) = \frac{x^{(1)}(k) + x^{(1)}(k-1)}{2} \; (k = 2, 3, 4, \dots, n) \tag{11}$$

And the matrix **B** and $\mathbf{Y_N}$ are defined by:

$$\boldsymbol{B} = \begin{bmatrix} -z^{(ee1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \; \boldsymbol{Y_N} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{12}$$

The solution of the differential equation is obtained as:

$$x^{\wedge(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a} \tag{13}$$

Re-expressing it:

$$x^{\wedge(1)}(k) = (x^{(0)}(1) - \frac{b}{a}) e^{-a(k-1)} \tag{14}$$

To obtain the forecast data, the equation obtained in the previous step is applied and the cumulative inverse operation is performed:

$$x^{\wedge(0)}(k+1) = (1 - e^{a}) \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} \tag{15}$$

3 Combined forecasting model T3-FS y GM (1,1)

In this paper we establish the membership function of a fuzzy set $\mathbb{A}$, as defined in (4). All fuzzy sets are associated with membership functions are linear. Thus, a membership function is characterized by two parameters that give higher degree 1 and lower degree 0 of membership respectively, i.e., μ_n: $U \rightarrow [0,1]$. Likewise, the truth value of a proposition "x is A and y is B" is expressed by:

$$| x \text{ is } A \text{ and } y \text{ is } B | = A(x) \wedge B(y) \tag{16}$$

This has the following implications C:

$$\text{C: if f } (x_1 \text{ is } A_1, \dots, x_n \text{ is } A_n), \text{ then } y = g\,(x_1, \dots, x_n) \tag{17}$$

Where:
y, is the consequent variable of the inferred values.
$x_1 - x_n$, are the variables of the premise that also appear in the consequent part.
μ_n, is the membership function n of each fuzzy set An and for the case of T3-FS, the one defined by $\mathbb{A}$.
$A_1 - A_n$, are fuzzy sets with linear membership functions that represent a fuzzy subspace in which the implication C can be applied for reasoning, i.e., decision rules can be established.

f_n, is a logical function that connects the propositions in the premises.
g, is a function that implies the value of y when the premises $x_1 - x_n$ are satisfied.

Where if premise A_i is equal to X_i, for a given i, where X_i is the universe of discourse of x_i. Thus, for example if x_1 is high and x_2 is low, then y= 2x1 -3x2 + c, i.e., the value of y would be, twice x_1 minus 3 times x_2 plus a value c. This can be defined by virtually any means, whether by user experience, historical data, an econometric model, temporality data or analytical geometry. So, an implication would be written as:

$$C\text{: If } x_1 \text{ is } A_1 \text{ and } x_2 \text{ is } A_2 \text{ and } \ldots x_n \text{ is } A_n, \text{ then } y=p0+p_1x_1+\ldots+p_nx_n$$

The model establishes the following elements:
The fact, i.e., the input to the system for the forecast is given from:

$$x \text{ is } A' \text{ and } y \text{ is } B'$$

then come the rules and their antecedents:

$$\begin{array}{c} \text{If } x \text{ is } A_1 \text{ and } v \text{ is } B_1, \text{ then } z=f_1(x,y) \\ \text{If } x \text{ is } A_2 \text{ and } v \text{ is } B_2, \text{ then } z=f_2(x,y) \\ \vdots \\ n \\ \hline z=z_0 \end{array}$$

This model has no fuzzy set consequent, but the function f_1, f_2, … f_n. that depend on the inputs of the system, which are x, u and v, and a conclusion z, which is not a fuzzy set, but a scalar quantity represented by z_0, this is because the consequents of the rules are functions.

Where z_0 is calculated:

$$z_0 = \frac{\omega_1 f_1(x_0, y_0) + \omega_2 f_2(x_0, y_0) + \ldots}{\omega_1 + \omega_2 + \ldots} \tag{18}$$

That is, each of the functions is evaluated at the fact entries x_0, y_0, which are the value of the Singleton sets for A' and B', in such a way that they are weighted and a linear combination of f_1, f_2, … f_n is made. The values ω_n, will be calculated as the minimum values resulting from the merging of each rule. Thus:

$$\omega_n = \min\left(\mu_{An}(x_o), \mu_{Bn}(y_o)\right) \tag{19}$$

The output is a linear combination of the functions that are in each rule.
Using the Sugeno-Takagi-Kang controller [33], it is considered that the control function or surface is known, in this case, the behavior of the demand in a given period of time is based on the control function or surface and from this, the control rules are inferred, this premise is fundamental for the application of this model, it also considers that all the functions f_n, are convex.

The determination of the optimal number of intervals of linear functions can be established in different ways: a) from the simple analysis of the plotted data, where the grouping of data is evident, b) through the model established by Shah [49], and the resultant of the GM model (1,1) shown in equation (14), where the universe of discourse is divided into equal intervals of uniform length, and the total number of intervals is defined as that ≤ integer (N/2 +1), where N is the total number of data. Obtaining the percentage of variation Y of a time series X between two consecutive time periods [50]:

$$Y = Y_t, Y_{t+1}, Y_{t+2} \ldots \text{where } Y_t = \frac{X_{t+1} - X_t}{X_t} * 100, \ t=1, 2, \ldots, n-1 \tag{20}$$

Where the data series t will belong to the same group if: a) Yt is descending or ascending but does not change sign and will not belong in the opposite case, or if Yt remains constant.
This results in an inference system as shown in Figure 1.

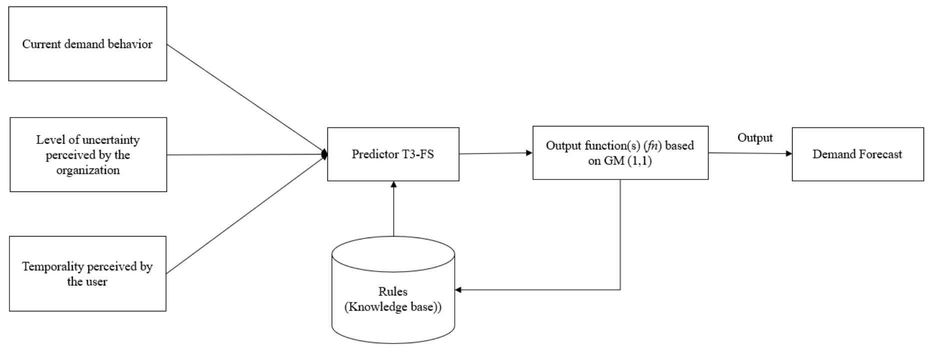


Fig. 1. Representation of the proposed inference system for demand forecasting (T3-FS), based on the combination of fuzzy logic type-3 and GM (1,1), applying the Sugeno-Takagi-Kang model.

3.1 Algorithm of the combined forecasting model T3-FS and GM (1,1)

1. Development of the T3-FS+GM (1,1) algorithm for demand forecast determination.
2. Given the time series $x^{(0)}$
3. Obtain the preliminary forecast of the GM (1,1) model.
4. Determine the resulting linear functions f_n, based on that GM (1,1)
5. Define the input parameters of the MFs to define A^3, in this case select **a)** a Gaussian function for the demand behavior, **b)** a Triangular function for the level of uncertainty perceived by the organization and finally, **c)** a Sigmoid function for the temporality of the demand, for input values by the user.
6. Given the input $x^{(0)}$ and the level of knowledge of the organization expressed by the corresponding membership functions, the demand forecast is computed.
7. Having the Sugeno-Takagi-Kang model: a) control rules, b) semantics with linguistic variables, c) membership functions, through the Fuzzy Logic Designer tool of Matlab R2023b, this model is applied and.
8. The new demand forecast is obtained, considering the different parameters of the input functions: demand behavior, uncertainty level and temporality. This result can be expressed through different control surfaces, where the expert or novice user can find the forecast according to his uncertainty level.

4 Example of the application of the Sugeno-Takagi-Kang type-3 (T3-FS) fuzzy combined model and the GM (1,1) model for demand forecasting in a 3PL warehousing company

The combination of the T3-FS model and the GM (1,1) will be exemplified with real data on the storage demand level of a logistics service provider (3PL) in the city of Tijuana BC, Mexico. The data available are 24 records, corresponding to two full years.

Step 1. We start with the known demand data d and its behavior over time. The monthly demand data for the last 24 months are shown in **Table 1** and the resulting graph of the same data in **Figure 2**.

Table 1. Demand data for a customer of a 3PL warehousing company.

MM-YY	Period	Observed Data	MM-YY	Period	Observed Data
Jan-21	**1**	143	Jan-22	**13**	206
Feb-21	**2**	152	Feb-22	**14**	193
Mar-21	**3**	161	Mar-22	**15**	207
Apr-21	**4**	139	Apr-22	**16**	218
May-21	**5**	137	May-22	**17**	229
Jun-21	**6**	174	Jun-22	**18**	225
Jul-21	**7**	142	Jul-22	**19**	204
Aug-21	**8**	141	Aug-22	**20**	227
Sep-21	**9**	162	Sep-22	**21**	223
Oct-21	**10**	180	Oct-22	**22**	242
Nov-21	**11**	164	Nov-22	**23**	239
Dec-21	**12**	171	Dec-22	**24**	266

Given the time series, as the input data x $^{(0)}$, set in (5), the forecast equation (14) and the forecast values are determined based on (15).

$$x^{\wedge(1)}(k) = (x^{(0)}(1) - \frac{b}{a})e^{-a(k-1)}$$

$$x^{\wedge(1)}(k) = 4941.49324e^{0.02768983(k-1)} \tag{21}$$

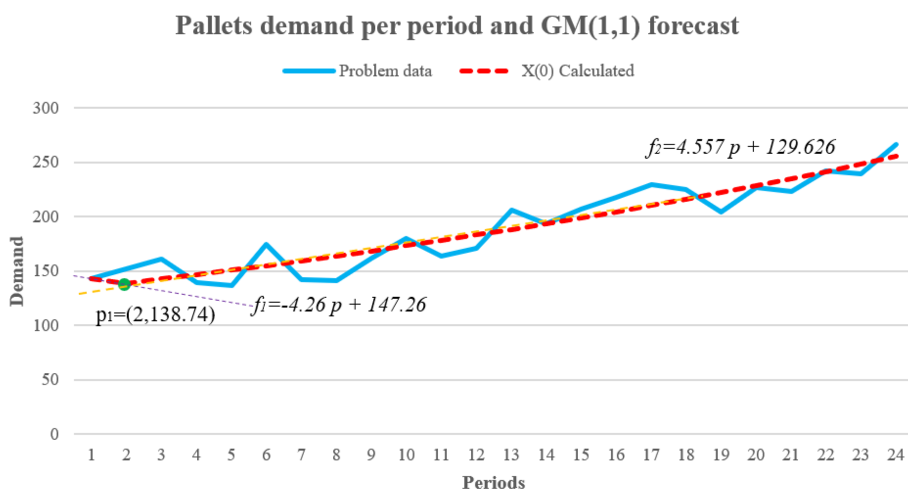


Fig. 2. Graph of a 3PL warehousing company's customer's last 24 months storage space demand and GM demand forecast (1,1).

Step 2. Let now be the resulting functions of this demand f_1 y f_2,, shown in equations (22 and 23), applying simple analytical geometry, as shown in **figure 2**, the functions representing the demand intervals. Where the transition points need to be determined, in this case: p1, only. The coordinate of this point is:

$\boldsymbol{p_1}$**= (2, 138.74)**

$$f_1=-4.26\ p + 147.26 \tag{22}$$
$$f_2=-4.557\ p + 129.626 \tag{23}$$

Step.3 Let now be the control rules that represent these linear functions, starting from the form: If x belongs to DB, then y= a1 x + b1, so for each of the rules, applying analytic geometry, are as follows:

If d is DB, then d=f_1 (DB) = -4.26 d +147.26 (24)
If d is DA, then d=f_2 (DA) = 4.557 d +129.626 (25)

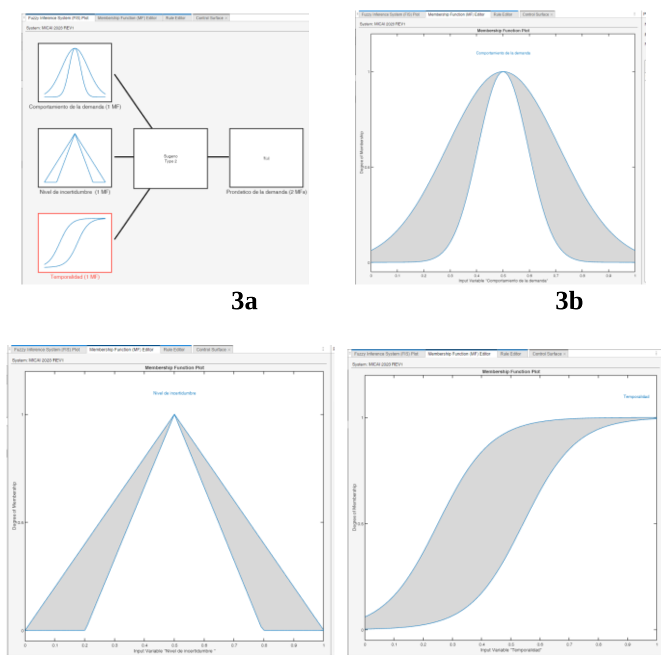
The application of these equations on the demand curve can be seen in **Figure 2**.

Step 4. Define the semantics of the **linguistic terms**, for this we choose the membership function that best fits the behavior of the data. In this case, it is observed that f_1 belongs to a membership function open on the left, a Sigmoidal function is selected, centered where the transition occurs, in this case where p_1 is. For the case of the other function f_2, a Sigmoidal function is also selected but open on the right, centered at **13**, which is the midpoint of the periods. Finally, the widths of these functions are proposed, as the width of the corresponding intervals. It only remains to define the slopes or boundaries of the membership functions, i.e., the value that allows the transition from one function to another, how smooth is the transition from one interval to another, are the parameters *a* and *b*, these parameters will be adjusted later to smooth the transitions between f_1 and f_2, the smaller these parameters are, the smoother the control curve will be. The semantic rules are expressed by the following membership functions and their parameters:

$$\mu(DB) = \mathrm{sigmf}_1(d; -a, 2) \tag{26}$$

$$\mu(DA) = \mathrm{sigmf}_2(d; -a, 13) \tag{27}$$

Step 5. Already having the Sugeno-Takagi-Kang model, having the control rules defined by equations (24 and 25), the semantics with the linguistic terms defined by the membership functions (26-27), through the Fuzzy Logic Designer tool of Matlab, to apply this model. The diagram of the fuzzy inference system is shown in **Figure 3a**, the membership function for the demand behavior in **Figure 3b**, membership function for the demand uncertainty level in **Figure 3c** and the membership function for the temporality behavior in **Figure 3d**.



3a 3b 3c 3d

Fig. 3. (a) Fuzzy inference system in Matlab R2023b. (b) the membership function for the demand behavior, (c) the membership function for the uncertainty level and (d) the membership function for the temporality behavior.

Step 6. Through Fuzzy Logic Designer of Matlab, different inferences about the level of demand forecasting can be obtained. The tool also allows developing control surfaces that allow visualizing the interaction between the different knowledge factors of the organization, based on the GM model (1,1), i.e., this model combines aspects of fuzzy logic and gray systems theory to estimate a higher level of forecasting than only aspects related to the time series could provide. An example of the resulting control surface is shown in **Figure 4**, where the level of forecasting on the Z-axis is obtained through the level of temporality and demand behavior.

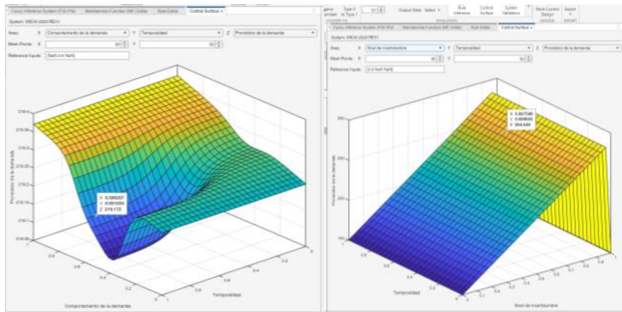


Fig. 4. Control surface of the fuzzy inference system in Matlab R2023b, obtained for the T3-F3 model combined with the GM (1,1) model.

4.1 Model performance evaluation

The model performance can be evaluated directly from the visualization of the rules and selecting the corresponding input. Thus, for example the demand behavior vs. temporality gives us a control surface as shown in **5a**, a temporality with a factor of 0.86 and demand behavior of 0.55, gives us a forecast of 219.152, while a demand behavior of 0.896 and a temporality of 0.310, gives us a forecast of 219. 355, while if we compare the demand behavior with the uncertainty level for a behavior of 0.862 and an uncertainty level of 0.206, we get a forecast of 178.771, but when that uncertainty level rises to 0.862, the forecast shoots up to 269.475, as shown in **Figure 5b**.
These results could not even be obtained through the GM (1,1) model alone since it does not consider factors such as user experience. The GM (1,1) model alone has been shown to obtain significantly better results than traditional methods.

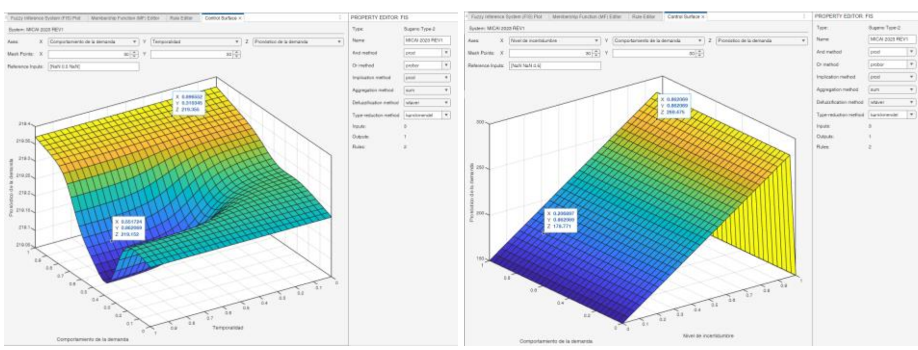


Fig. 5. (a) Control surface of the fuzzy inference system between demand behavior and seasonality and **(b)** the comparison between demand behavior and the level of uncertainty.

5 Conclusions and future research

The application of the combined Sugeno-Takagi-Kang type-3 (T3-FS) fuzzy model and the GM model (1,1) for demand forecasting in a 3PL warehousing company has the enormous advantage that, once the control curve, in this case the forecast, is known, through the GM model (1,1), it is easy to apply the Sugeno-Takagi-Kang model including the membership functions relevant to the user and the organization, It has the advantage that it is not necessary to defuzify the data, this saves time in calculations and allows different linear or constant functions to be established to make up the inference system. The mathematical requirement of these is minimal.

According to Bose and Mali [51] there are two important issues related to fuzzy time series design where more work needs to be done: (1) number of orders (past values) to be used (2) number of intervals to be considered. This paper provides an answer to the second question, establishing that as many intervals as linear functions must be created to form an inference system that softens the transitions between them.

To determine the number of intervals that represent these linear functions, they can be approximated through equation (20) and subsequent criteria, especially those that simply show us the graphical behavior of the data.

The quality of the results can be further improved, depending on a) smoothing between the overlaps of linear functions, b) membership functions relevant to the organization, as well as c) the fuzzy level of these. The important thing to note is that this combined model takes advantage of a highly accurate forecasting model (GM (1,1)), which is characterized by the small amount of data and the level of user experience, expressed through the fuzzy membership functions, which allow them to integrate their experience, all this without having to perform complex mathematical calculations. Finally, given the nature of the data generated over time it is broadly categorized as stationary, seasonal, trending, and random. The type of data considered in this paper proposes a forecasting method considering that these can be stationary, with seasonality or seasonal series and trends, but it places special emphasis on the vagueness or uncertainty of these, that is, there is many factors that are not known or of which there is no certainty or more information. to make a statistical inference. Just as most of the historical data generated over time in the real world is and although not statistically consistent, it is the data that is available for data analysis and forecasting purposes.

5.1 Future research in another Application Domains

In application domains, such as art, the two techniques can be complemented by allowing to identify by a GM Model what characteristics art collectors are looking for and with Fuzzy Logic Type-3 it is possible to identify the correct way of distribution of NFTs for a given piece and its commercial value and according to the trend in vogue of future buyers of a piece through this method of collective purchase, as is possible see in **Figure 6.**

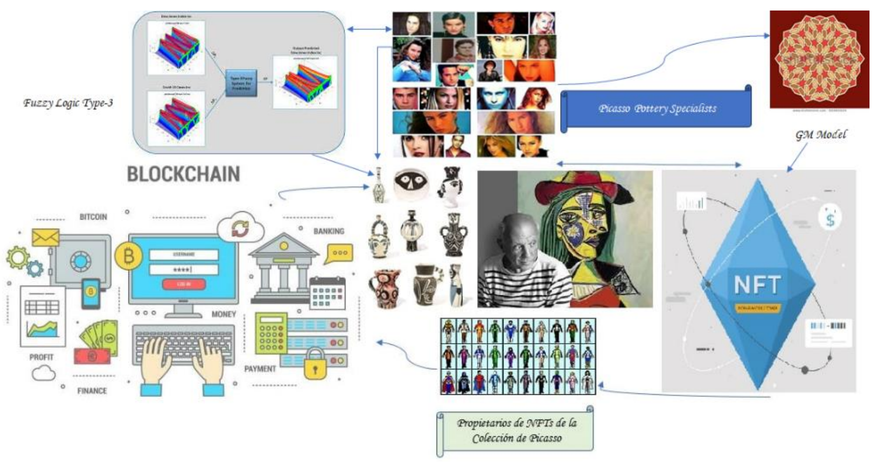


Fig. 6. A hybrid model associated with valuation, segmentation of NFTs and determination of collective purchase of high-end art in a Smart City.

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