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Implementation of Optimal Control for Thermal Regulation in Finite Volume Places Described by Second-Order Dynamics

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Abstract. Temperature control systems have various applications, from cooling to casting, and are crucial for ensuring quality in production. Although essential, their usage entails a significant energy consumption. This project focuses on implementing optimal control synthesized from the calculus of variations applied to the Hamilton-Jacobi-Bellman equation to regulate temperature within a finite volume place. The objective is to enhance thermal efficiency without compromising product quality. The approach not only aims to optimize energy consumption but also to ensure uniformity and quality in products and processes affected by temperature. This can be achieved by maintaining thermal stability at desired values and responsible resource management. In general, the article proposes improving efficiency and quality in temperature regulation, contributing to sustainable and effective industrial practices.
Keywords: Optimal Control, Performance Index, Temperature, Energy Consumption, Finite Volume Enclosure, Calculus of Variations.

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1 Introduction

Conducting a study on optimal control applied to processes involving temperature regulation in finite volume places is of utmost importance in the industrial context. It plays a crucial role in ensuring quality, safety, and efficiency in processes and products. For instance, in the food industry, temperature control is fundamental to reduce or prevent the risk of harmful bacteria proliferation, thus safeguarding health. In the pharmaceutical industry, temperature control helps prevent the spread of pathogens in medications or the deterioration of products before their expiration date. In the gas and oil industries, thermal control is vital to prevent workplace accidents and ensure optimal service (Jom, 2023).

In Braun (1990), the focus is on the application of thermal capacity in confined places with the purpose of mitigating operational costs through dynamic environmental control. Optimal temperature variations in different zones can reduce energy expenses and alleviate peak-hour demands. It's worth noting that the savings achieved are influenced by various factors, such as:

- The structure of public utility tariffs.
- Partial load characteristics of the cooling plant and air handling system.
- Climate conditions.
- Occupancy schedules.
- Building's thermal capacity.

Taking these factors into account, this research paves the way to explore the adaptability of a dynamic control system applied to buildings for thermal control, showing how through the application of optimization techniques, significant reductions in energy expenditure can be achieved.

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Additionally, in Gandur Adarme (2016), the significance of heat exchangers, their application in various contexts, and the pressing need for operational control are addressed. The proposal is to develop advanced control strategies tailored to specific heat exchangers. To achieve this, dynamic modeling of systems using white box and ARMAX models is examined. Limitations in control and optimization are also discussed. The primary aim of these proposals is to enhance control and reduce oscillations in heat exchanger operations.

In Madrigal, Cabello, Sagastume, & Balbis (2018), it is demonstrated that by implementing techniques such as thermography, computer-aided design, and finite element methods, it is possible to analyze air conditioning systems and propose improvements in refrigeration systems. However, the results of the study show that although thermography, simulation with software like Trnsys, and finite element methods (FEM) are valuable tools, they alone are insufficient for a comprehensive characterization of air conditioning systems. The research highlights the need for complementary approaches to achieve a more thorough evaluation.

In the study conducted by Herrera Segura (2021), the design of feedback control systems and heat flow estimation is addressed. Finite element methods, PID controllers, and linear state feedback control are employed. However, the study focuses on temperature control, primarily in greenhouses where heat flow measurement is not carried out. The proposal of an observer is introduced as a complementary tool.

On the other hand, in Gutiérrez & Arias (2017)., unlike previous researchers, the implementation of a control system that achieves optimal temperature regulation in a greenhouse is carried out. This is accomplished by using a comprehensive mathematical model specially tailored to the specific conditions of the environment. An optimal control is implemented in this specific regional model. When implemented, this control law not only increased production in the greenhouses but also achieved efficient energy consumption optimization. These results demonstrate a promising approach for future and diverse applications.

In non-ferrous hydrometallurgy research, Liu, Yang, Zhou, Li & Sun (2023) emphasizes that electrodeposition represents a critical process characterized by high energy consumption. Current efficiency and electrolyte temperature emerge as crucial factors for its operation. However, optimal control of the electrolyte temperature faces challenges due to process complexity and variable fluctuations. An approach using a temporal causal network and reinforcement learning (RL) is proposed to optimize the electrolyte temperature under various operating conditions. A case study on zinc electrodeposition confirms the effectiveness of this method in maintaining the electrolyte temperature in the optimal range without the need for complex models.

On the other hand, in Collado, Delgado, Bernal, Cárdenas, & Sáez (2023), the study aimed to improve the safety and performance of electronic circuits in hazardous areas with a high probability of death. The authors utilized the concept of nonlinear convex optimization to find the optimal operating point of transistors (BJT or MOSFET), considering the maximum surface temperature and nominal current. This ensures safe and efficient operation under various temperature and load conditions. The Karush-Kuhn-Tucker method is employed to solve the problem, and an algorithm is proposed to minimize the surface temperature while maintaining the necessary voltage and current levels.

The study by Del Angel, Solis, Villanueva & Huacuja (2019) present various combinatorial optimization methods, specifically heuristic methods used to find efficient solutions for tuning the parameters of a proportional-derivative controller applied to temperature control. In this work, a control system simulation is introduced in which an iterative algorithm finds controller gains that minimize the temperature error variable in a finite number of iterations.

This research aims to analyze and manage optimal energy performance in the regulation of temperature in finite volume places. To achieve this, the principles of thermal behavior in these environments are explored through the identification of their dynamics, using the Gauss-Newton algorithm, as evaluated in Section 2. In Section 3, the design of the optimal control law is presented, which is subject to a performance index obtained from the application of the first and second variation criteria of variational calculus to the Hamilton-Jacobi-Bellman equation. This equation is grounded in the theory of optimal control and game theory and describes the optimal value function for stochastic problems. Furthermore, in Section 4, the PID control is introduced in a general manner, outlining the contributions of each action of this controller, as well as its representation in the time domain for the continuous case and its pulse transfer function for the discrete case. Section 5 presents the results of the implementation of both controllers, along with the conditions for experimental validation. Additionally, a comparative performance study is presented, based on a series of performance indices related to the error variable. Finally, in Section 6, the conclusions drawn from this work are provided.

2 Experimental platform construction

This section addresses the construction of the experimental platform, describing the main components used, their dimensions, as well as the sensors and actuators along with their characteristics. Subsequently, it describes the process of model identification for the platform, which is obtained by applying the Gauss-Newton algorithm to a set of experimental data obtained from the response to a step input.

For the experimental validation of the control laws synthesized in this work, a platform with a cubic geometry was constructed using 9 mm thick MDF (Medium-Density Fiberboard) cut with a laser. This material and design provide strength and precision for analyzing temperature control in finite places. MDF was chosen for its thermal insulation properties and ease of manipulation, and laser cutting ensures precise dimensions. The platform has a total area of 96.4 cm^2 with openings to accommodate a fan, a 40-watt light bulb, and a DS18B20 temperature sensor. The incandescent bulb generates heat to simulate a heat source, creating a regulated thermal gradient controlled by the control system.

Regarding heat generation, a set of measurement and actuation devices of utmost importance for the platform's operation has been implemented. Specifically, a DS18B20 digital temperature sensor has been strategically placed on one of the faces of the platform. This location has been carefully selected, positioning it opposite to the thermal actuator. The DS18B20, being a high-precision component, plays a critical role by providing highly reliable measurements of the internal temperature of the experimental enclosure Koestoer, Saleh, Roihan & Harinaldi (2019). This thermometric information is of vital relevance as it serves as a fundamental basis for continuous monitoring and constant feedback of the control system, ensuring precise and efficient control of the temperature gradient generated by the heat source inside the enclosure.

Regarding heat generation, a 9-blade fan operating at 5V has been chosen. This component is placed on the face opposite to the temperature sensor inside the enclosure. Its main function is to introduce a stream of ambient temperature air into the interior of the experimental place. This process allows for the regulation and maintenance of the internal temperature at predefined levels according to the experiment's requirements.

For the simultaneous control of both actuators, the fan, and the light bulb, an H-bridge with the L298N model has been implemented to control the fan's rotation speed. The connection is made following these instructions: Initially, the common grounds are connected to both the Arduino and the power source, ensuring a common ground reference. Subsequently, the necessary current in the fan is ensured using the L298N H-bridge, which is connected between the Arduino output and the fan. Additionally, a light intensity regulator, known as DIMMER, is incorporated to manage the voltage supplied to the light bulb. Its main function is to adjust the amount of electrical energy supplied to the light source, which in turn controls the brightness or luminous intensity emitted. This configuration allows for precise control of the light intensity emitted by the light bulb. The combination of these elements, the L298N and the DIMMER, enables effective control of both actuators, allowing for precise and controlled temperature regulation within the enclosure according to the experiment's requirements.

Fig 1. Experimental platform.

For the identification of the mathematical model of the system, the Gauss-Newton algorithm is employed. This iterative method is widely used in the field of control engineering and numerical optimization. Its primary objective is to estimate the parameters of a mathematical model that best fit a set of experimental data, thereby minimizing the difference between the observations and the model predictions.

The iterative process of the Gauss-Newton algorithm begins with an initial estimate of the model parameters, represented by the vector θ. Through successive iterations, the parameters are adjusted to minimize the mean squared error between the experimental observations and the model predictions $f(x_i, \theta)$, This is achieved by updating the parameter vector in each iteration, as shown below:

$$\theta_{k+1} = \theta_k - (J_k^T J_k)^{-1} J_k^T r_k, \quad (1)$$

Where:

- θ_k is the parameter vector in the iteration k.
- J_k is the Jacobian matrix of the model evaluated at θ_k.
- r_k is the residual vector between the observations and the model predictions in iteration k

This algorithm plays a vital role in parameter identification from experimental data, and it is essential in the optimization and tuning of control systems and dynamic system modelling. It relies on initializing values, calculating residuals and the Jacobian, and updating parameters in each iteration until convergence.

In this study, parameters of a mathematical model for a wind turbine were adjusted using the Gauss-Newton algorithm. Experimental data were compared with predictions from the fitted model, and the fit was evaluated using the coefficient of correlation. Following the parameter identification and tuning process, the selected mathematical model exhibited a high correlation with the experimental data and a significant fit to the system's responses, achieving a correlation value of 94.45%.

By applying the Gauss-Newton algorithm to the experimental data representing the response to a step input, the mathematical model of the system is estimated. This model is identified as a second-order transfer function with underdamped poles in the following form:

$$G(s) = \frac{k}{T_{\omega}^2 s^2 + 2\zeta T_{\omega} s + 1} \quad (2)$$

Where:

- K is the numerator of the transfer function (indicating that the function has no zeros).
- T_{ω} is the natural undamped frequency.
- ζ is the damping coefficient.
- s represents the complex variable 's' in the Laplace domain.

with the parameters identified in the Gauss-Newton algorithm defined as K = 2.9394, T_{ω} = 95.067 and ζ = 0.3913.

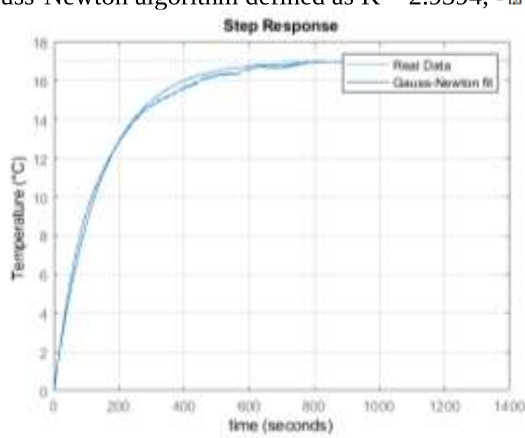

Fig 2. Second-order model.

The graphical representation clearly demonstrates that the correlation coefficient between the identified model and the experimental data is significantly high. This ensures that the identified model consistently reproduces the behaviour of the experimental platform when subjected to control laws that have been numerically validated.

3 Optimal Control Scheme

In this section, we present the synthesis of optimal control using the calculus of variations on the Hamilton-Jacobi-Bellman equation. It's important to mention that the Hamilton-Jacobi-Bellman (HJB) equation is a fundamental equation in the theory of optimal control and game theory. It describes the value function of a stochastic optimal control problem. This equation is crucial in dynamic optimization and is used to find the optimal control strategy in a dynamic system in the presence of uncertainty. The Hamilton-Jacobi-Bellman equation is typically presented in the context of a stochastic optimal control problem involving various components. Below, we provide a synthesis of optimal control using the criteria of the first and second variation in the calculus of variations. Consider a time-invariant linear system in its standard state-places form, as presented below:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \tag{3}$$

In which $x \in \mathbb{R}^2$ is the state vector, $y \in \mathbb{R}$ is the output vector, $u \in \mathbb{R}$ is the control signal vector, $A \in \mathbb{R}^{2\times 2}$, $B \in \mathbb{R}^2$, $C \in \mathbb{R}^{1\times 2}$ and $D \in \mathbb{R}$ are related to the system's dynamics. The objective is to find the control u(t) that minimizes energy consumption while subject to a performance index:

$$J\big(x(t), u(t)\big) = \int_0^\infty x^\intercal(t)\, Qx(t) + u^\intercal(t)Ru(t)dt. \tag{4}$$

Where Q>0 and R>0 are matrices of appropriate dimensions that penalize state convergence and energy consumption, respectively, and they satisfy an algebraic Riccati equation (Bellman, 1966). Given the Hamilton-Jacobi-Bellman equation, which is written as:

$$\frac{dV_B(x(t),u(t))}{dt}\bigg|_{\dot{x}(t)} + D\big(x(t), u(t)\big) = 0. \tag{5}$$

Where $D\big(x(t), u(t)\big)$ is the integrand of the performance index, and $V_B(x(t), u(t))$ is known as the Bellman function. Now, a quadratic form of the Bellman function is proposed as follows:

$$V_B(x(t), u(t)) = x^\intercal(t)Px(t). \tag{6}$$

Where its temporal derivative results as follows:

$$\frac{dV_B(x(t),u(t))}{dt} = \dot{x}^\intercal(t)Px(t) + x^\intercal(t)P\dot{x}(t). \tag{7}$$

And substituting this derivative into the Hamilton-Jacobi-Bellman equation, we have:

$$\dot{x}^\intercal(t)Px(t) + x^\intercal(t)P\dot{x}(t)|_{\dot{x}(t)} + D(x(t), u(t)) = 0. \tag{8}$$

Therefore, when evaluating this equation along the trajectories of the system (3), we have:

$$(x^\intercal(t)A^\intercal + u^\intercal(t)B^\intercal)Px(t) + x^\intercal(t)P(Ax(t) + Bu(t)) + D(x(t), u(t)) = 0, \tag{9}$$

which can be rewritten as:

$$x^\intercal(t)A^\intercal Px(t) + u^\intercal(t)B^\intercal Px(t) + x^\intercal(t)PAx(t) + x^\intercal(t)PBu(t) + D(x(t), u(t)) = 0. \tag{10}$$

From which it follows that $D\big(x(t), u(t)\big) = x^\intercal(t)Qx(t) + u^\intercal(t)Ru(t)$ and we have:

$$x^\intercal(t)A^\intercal Px(t) + u^\intercal(t)B^\intercal Px(t) + x^\intercal(t)PAx(t) + x^\intercal(t)PBu(t) + x^\intercal(t)Qx(t) + u^\intercal(t)Ru(t) = 0. \tag{11}$$

Upon closer examination of this equation, it's clear that it can be minimized with respect to the control $u(t)$, thus formulating the problem as:

$$\min_{u(t)}\{x^{\intercal}(t)A^{\intercal}Px(t)+u^{\intercal}(t)B^{\intercal}Px(t)+x^{\intercal}(t)PAx(t)+x^{\intercal}(t)PBu(t)+x^{\intercal}(t)Qx(t)+u^{\intercal}(t)Ru(t)=0\}. \quad (12)$$

The minimization process can be achieved by taking the partial derivative of this equation with respect to $u(t)$, leading to:

$$B^{\intercal}Px(t)+x^{\intercal}(t)PB+Ru(t)+u^{\intercal}(t)R=0. \quad (13)$$

This equation is dimensionally scalar and, therefore, simplifies to:

$$2B^{\intercal}Px(t)+2Ru(t)=0, \quad (14)$$

From this equation, it is possible to obtain the control $u(t)$, subject to the performance index (4), as:

$$u(t)=-R^{-1}B^{\intercal}Px(t), \quad (15)$$

And at the same time ensure that this control minimizes the performance index, as the second partial derivative of the Hamilton-Jacobi-Bellman equation yields:

$$2B^{\intercal}Px(t)>0. \quad (16)$$

It is evident that this control takes the form of a state feedback as follows:

$$u(t)=kx(t). \quad (17)$$

Where $k=-R^{-1}B^{\intercal}P$ and the matrix $P=P^{\intercal}>0$, satisfies the Lyapunov equation, thus ensuring the asymptotic stability of the closed-loop system with control *u(t)*.

4 Description of the Proportional-Integral-Derivative (PID) Controller

In this section, a succinct exposition of the PID (Proportional-Integral-Derivative) controller is presented. This implementation is undertaken to ease a comparative performance analysis in relation to optimal control methods specifically devised and penalized for temperature regulation, as proposed within the framework of this study.

The PID controller is a type of controller used in automatic control systems to keep a variable such as temperature, speed, position, among others, at a desired setpoint value by adjusting the control input. The acronym "PID" is related to the three fundamental control actions it performs (Astrom, 1995):

1. P (Proportional): The proportional action adjusts the controller's output in proportion to the current error, which is the difference between the measured value and the desired setpoint. The larger the error, the greater the proportional correction. This helps reduce the present error but may lead to a small steady-state error if used alone.
2. I (Integral): The integral action considers the accumulation of past errors over time and adjusts the output to reduce the accumulated error. This is especially useful for eliminating steady-state errors and improving long-term accuracy.
3. D (Derivative): The derivative action considers the rate of change of the error and adjusts the output to prevent oscillations or rapid changes in the controlled variable. Helps stabilize the system and reduce transient response.

Together, the PID controller calculates the control signal as the weighted sum of the proportional, integral, and derivative actions, as follows:

$$u(t)=K_pe(t)+K_i\int_0^1 e(t)\,dt+K_d\frac{de(t)}{dt} \quad (18)$$

- $u(t)$ is the control signal sent to the process or system.
- $e(t)$ is the current error (the difference between the desired value and the measured value)
- Kp, Ki, and Kd are the tuning coefficients for the proportional, integral, and derivative actions, respectively.

The proper tuning of these coefficients is essential for the optimal performance of the PID controller in a specific system. PID controllers are widely used in various industrial and automation applications due to their simplicity and effectiveness in controlling a wide range of dynamic systems. The equation describing the pulse transfer function of the digital PID controller is often referred to as the positional representation of the PID control scheme in the form Ogata (1996):

$$G_D(z) = \frac{U(z)}{E(z)} = k_P + \frac{k_I}{1-z^{-1}} + k_D(1-z^{-1}) \tag{19}$$

5 Results and Discussion

A The control laws analyzed in this work are implemented through the Arduino Integrated Development Environment (IDE). Within this environment, a PID controller has been created and configured, without the need to resort to pre-established libraries or those available in the Arduino IDE. Instead, the trapezoidal rule has been programmed for integral calculation, and the Euler's backward finite difference rule for derivative approximation.

This section presents the experimental conditions, the penalization for optimal control, the tuning of PID controller gains, and the results of implementing both control laws for temperature regulation within the experimental setup. For the experimental analysis, a point-to-point regulation task was performed, where the initial temperature was set according to the ambient temperature as $T(t_0) = 24°C$, while the desired temperature was defined as $T_{des} = 51°C$. For the control action, control actions (15) and (18) were implemented. For optimal control, the penalty pair (Q, R) is proposed as:

$$Q = \begin{bmatrix} 925.1 & 0 \\ 0 & 47.42 \end{bmatrix},$$

$R = 0.2$, and with the matrix P defined as:

$$P = \begin{bmatrix} 13.1437 & 3.0742 \\ 3.0742 & 210.2774 \end{bmatrix},$$

This set of matrices associated with the optimal controller satisfies a Riccati algebraic equation and minimizes the performance index (4). On the other hand, the PID control gains are determined as follows: $K_p=64.5$, $K_d=28.4$, and $K_i=22.7$ through a pole assignment process. Below, the temperature and error graphs associated with both experiments are presented:

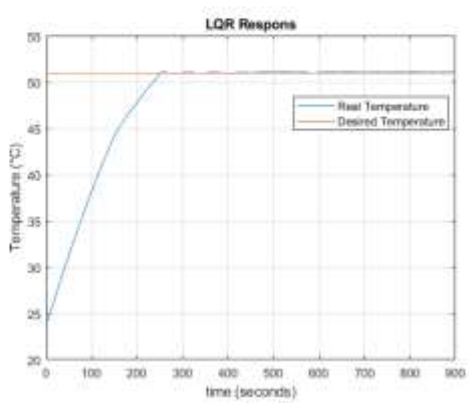

Fig 3. Measured temperature, optimal control.

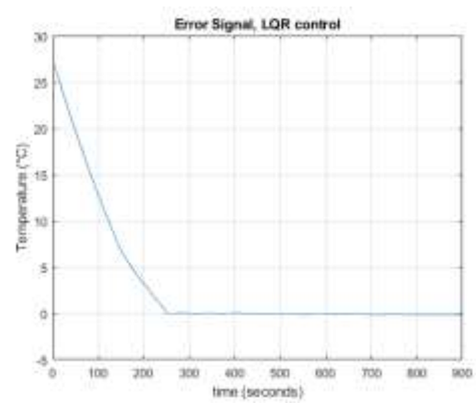

Fig 4. Error signal, optimal control.

As an example of these experiments, a video is presented through the following link: https://youtu.be/fj9UGhSuoY4.

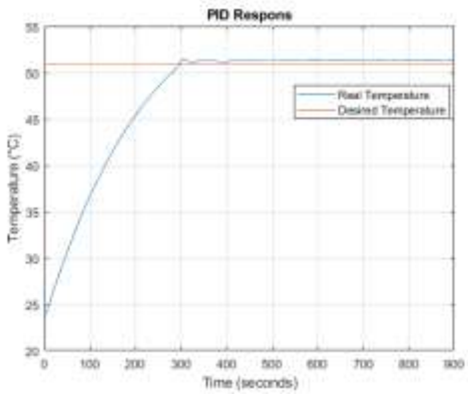

Fig 5. Measured temperature, PID control.

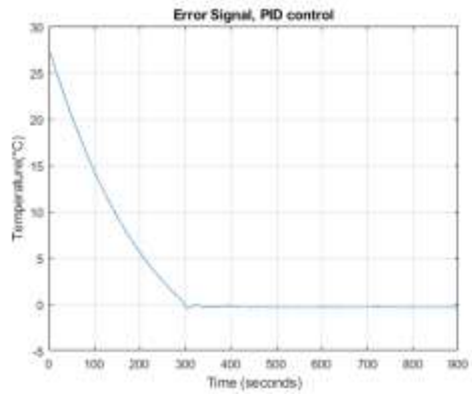

Fig 6. Error signal, PID control.

While stability is the most crucial criterion for any control law implemented in a physical system, the comparative study conducted here primarily focuses on the application of different error-based criteria. These criteria are often used as quantitative measures of system performance and help select controller settings for specific tasks or operations. Among the most important error-based criteria verified in this work are the following (O'dwyer, 2009):

A. Integral of the Absolute Error (IAE)

A system whose control parameters are penalized with this criterion exhibits optimal performance because the damping value is linked to energy consumption. This criterion can be easily implemented in discrete control systems through the following summation expression:

$$JIAE = \sum_{k=0}^{Nh} \left| k \frac{\bar{x}(k)+\bar{x}(k+1)}{2} \right| \quad (19)$$

Table 1. Integral of the Absolute Error

Controller	IAE
PID	3470.1348
LQR	2839.8538

B. Integral of time times absolute error (ITAE)

Systems penalized by this criterion exhibit significant initial errors in regulation tasks due to moderate penalization in the response. This leads to transient responses with low overshoot and appropriate damping. The evaluation of this criterion in systems is easily achieved through a specific operation.

$$J_{ITAE} = \sum_{k=0}^{N_h} K \left| h \frac{\bar{x}^2(k)+\bar{x}^2(k+1)}{2} \right| \quad (20)$$

Table 2. Integral of time times absolute error

Controller	ITAE
PID	407044.8797
LQR	239608.7398

B. Integral of the Squared Error (ISE)

Minimizing this criterion is said to make the system optimal and minimize energy consumption because the minimum value of the integral is obtained for a damping value that compromises between over-damped and critically damped values. Applying the trapezoidal rule to the absolute error results in the following summation expression for the discrete case:

$$J_{ISE} = \sum_{k=0}^{Nh} h \left[\frac{\overline{x^2}(k) + \overline{x^2}(k+1)}{2} \right] \tag{21}$$

Table 3. Integral of the Squared Error

Controller	ISE
PID	54568.3212
LQR	47199.7094

C. Integral of time multiplied square-error (ITSE)

This criterion is used for control parameters penalized based on the response to a step input, where the error starts at $x(t_0)$, can have a criterion that increases over time, penalizing the error more significantly. For discrete systems, the integral that defines this cost criterion is expressed in terms of the trapezoidal rule as:

$$J_{ITSE} = \sum_{k=0}^{Nh} Kh \left[\frac{\overline{x^2}(k) + \overline{x^2}(k+1)}{2} \right] \tag{22}$$

Table 4. Integral of time multiplied square-error

Controller	ITSE
PID	3322158.655
LQR	2397825.648

6 Conclusions

With a strong mathematical foundation and the application of mechatronics, both a PID controller and an optimal controller were successfully implemented. When comparing their responses, it is evident that the system controlled by the optimal controller reaches the desired temperature in a significantly shorter time and demonstrates more efficient energy consumption compared to the PID controller. This finding underscores the importance of using advanced control approaches with adjustments that penalize energy consumption to achieve optimal performance in temperature control systems, without sacrificing state convergence.

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