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Multicriteria ordinal classification to improve strategic planning in the financial sector of the company

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Abstract. Globalization has changed the way companies do business in the world by trying to reach the objective of having a greater presence in more countries through strategic alliances, company acquisitions, investment projects, among others. For this purpose, companies need decision-making support to generate strategies that consider consumer demands in order to consolidate their presence. The aim of this study is to propose a novel methodology to facilitate the generation of efficient strategic planning in the financial area of the company. The proposed approach is based on the historical recognition of patterns and prediction of scenarios for multi-criteria ordinal classification using a recent classifier method. A Back-testing comparison with a benchmark is performed to show the effectiveness of the proposed methodology
Keywords: Multicriteria ordinal classification, stock selection, decision support system, INTERCLASS-nC.

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1 Introduction

Every day, companies look for where to invest their capital; therefore, efficient financial investment management is necessary. In fact, this area is one of the most investigated in financial decision-making [1]. There are many ways to invest the remaining capital of a company, and financial investments are one of the most employed methods for big companies. Typically, it consists of asset, purchase, and sale transactions (such as stocks, currencies, bonds, commodities, futures, contracts for difference (CFDs), or exchange-traded funds (ETFs)). Consequently, investments in the stock market are of special importance since they represent a relevant source of resources for organizations [2]. According to the World Bank, in 2019, the value of shares traded worldwide is estimated to be more than $60.35 trillion [3]. Since there are many ways to make financial investments, in this paper, we focus our analysis on stocks only. A stock is a share in which the ownership of a company is divided; therefore, owning a share represents the ownership of a fraction of that company. A portfolio of stocks can be viewed as a group of stocks, where an amount of money has been allocated to each one. Aspects such as investment objectives and risk aversion are crucial to building the "best" portfolio of stocks.

There are three basic stages in so-called stock portfolio management: price forecasting, stock selection, and portfolio optimization. Price forecasting provides a way to estimate what future equity returns will look like. Security selection allows the investor to determine which securities are "acceptable for investment." Portfolio optimization specifies the amounts (generally as a proportion of available resources) of investment to be allocated to selected stocks. In the literature, there are many approaches that focus on one or two of these stages of this process. For example, in [4], [5], and [6], only analysis on stock selection is performed. In [7], an approach where price forecasting and stock selection is presented. Other approaches only focus on portfolio optimization (e.g. [8], [9], [10] and [11]). Typically, if an approach focuses on two or more stages, one or more are performed in a simplistic manner. This is done to avoid complexity in the analysis, and therefore give more emphasis to the results. From these stages, stock selection is one of the most tedious; however, it is very important in order to reduce the options to build a portfolio. There are many approaches that focus on stock selection; some of them use fundamental analysis as a criterion to weigh the stocks, such as in [12]. However, other approaches combine technical and fundamental factors for this purpose, as in [13]. Commonly, these approaches use computational intelligence techniques with multicriteria decision aiding theory to select the “best” stocks for investment.

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Recently a novel method called INTERCLASS-nC was introduced in [14]. This approach gives the opportunity of defining parameters that can be represented by real or interval numbers, which gives flexibility and reduce the cognitive effort and time on the process for choosing the “best parameters”. In this paper, a novel use for the INTERCLASS-nC method is presented. For the first time, this method is used for the stock selection problem, and some interesting results are obtained. The working methodology can be defined as follows:

1. Define the set of decision alternatives. Here, we consider the set of stocks in the Standard and Poor’s 500 (S&P 500) 500 index (see Table 1).
2. Define the subset of factors to be considered. This step consists in finding the most outstanding factors (out of a considerably big universe of factors) that, according to the decision maker, would provide enough information as to perform a convenient decision process (ordinal classification). The factors used in this work are shown in Subsection 4.1.
3. Define reliable data about the factors. This step is oriented towards the collection and adaptation of the data. It is a crucial step since there are usually problems involved in the collection of reliable data (e.g., such data can only be found in databases that must be purchased). The data also can require adaptation (such as normalization) procedures. For this work, the data were obtained from capitaliq.com. The performances of the alternatives on these factors are shown in Table 2.
4. Define the parameters of INTERCLASS-nC. These parameters are (see Section 3): The credibility threshold to establish crisp preference relations, δ, the threshold λ used to define the strong majority in the outranking relation, the criteria weights, the indifference thresholds, the preference thresholds, the veto thresholds, and the characterizing profiles. All the values for these parameters are given in Subsection 4.2.
5. Perform the assignments of decision alternatives (stocks) to classes preferentially ordered.
6. Exploit the classification. In this work, we assume that the decision maker has established that only the stocks assigned to the best class will be supported. Furthermore, all the supported stocks will receive the same proportion of resources.

The structure of the paper is as follows. Section 2 shows a brief review of works related to the stock selection problem. In Section 3, a summary of the INTERCLASS-nC method is presented. The application of the INTERCLASS-nC for the stock selection problem and the obtained results are shown in Section 4. Finally, conclusions and future work are discussed in Section 5.

2 Literature review

Stock selection consists of defining, among a universe of stocks, the ones that are most convenient for investment. Evidently, a sufficiently small subset of stocks should be selected since there is an overwhelming number of available alternatives; so, the technique used to define such a subset is crucial in portfolio management. Many factors can be used to select stocks, from which fundamental analysis outstand [12],[15].

Fundamental analysis uses disclosures of the organizations underlying the stocks (cf. [16]) to build both qualitative and quantitative indicators that seek to shed light on the actual value of the organizations. Therefore, comparing the actual value to the current value, one can deduce if a given organization is undervalued or overvalued. In [17], the financial ratios Debt/Equity, Price/Earnings, Profit to earn are exploited for stock selection. Price/Earnings Ratio and New Loan/Market Capitalization Ratio are used in [18], and seventeen indicators of this kind are used in [19] for the same purpose. Common categories of fundamental indicators used during the stock selection stage are Profitability, Leverage, Liquidity, Efficiency, Growth, Solvency, Operational efficiency [7],[19]. Of course, combining fundamental indicators with other factors is a common practice for stock selection. Fundamental and technical indexes are combined in [20]. Also, in [7], twelve fundamental indicators are combined with the forecasted price. In [13], eight fundamental indicators in conjunction with eight technical indicators are used.

Regarding the diverse techniques used for stock selection, artificial neural networks, data envelopment analysis, evolutionary algorithms, sentiment analysis, and support vector machines are among the most common ones [1]. In [21], data envelopment analysis with multicriteria decision aiding theory are combined to perform fund selection. Similarly, in [22], a new three-stage network model in multiplier data envelopment analysis is proposed. Hybridization between a feed-forward neural network and an adaptive neural fuzzy inference system is presented in [23]. In [7], it is proposed to use differential evolution to exploit an objective function using historical prices to weigh a set of indicators from the fundamental analysis. Support vector machines are used in [24] and [25].

3 The INTERCLASS-nC method

INTERCLASS-nC [26] is a multi-criteria ordinal classification method that exploits the generalization of the outranking approach presented in [27], which was later improved in [14] to deal with interacting criteria structured as a hierarchy. The most recent version of INTERCLASS-nC allows considering criteria scores defined as real numbers but also as interval numbers. Next, a summary of the method is presented.

INTERCLASS-nC uses the main concept of the so-called interval theory, interval numbers. An interval number is a range of values that an uncertain quantity can attain; that is, given a quantity i whose precise value is unknown but whose highest and lowest attainable values can be defined by i^+ and i^-, respectively, then the interval number that represents such a quantity is defined by $\boldsymbol{i} = [i^-, i^+]$ (note that we use boldface font to denote an interval number).

In INTERCLASS-nC, the set of decision actions is denoted by $A = \{a_1, a_2, a_3, \ldots\}$. The method also uses a set of classes C_k, $k = 1, \ldots, M$, each $a_i \in A$ can be assigned to one class. To characterize each of these classes, INTERCLASS-nC uses a set $R_k = \{r_{k,j}; j = 1, \ldots, card(R_k)\}$ of characteristic actions $r_{k,j}$, where $\{R_0, R_1, \ldots, R_M, R_{M+1}\}$ is the set of all the characterizing decision alternatives (R_0, and $R_{M+1}$are composed of the anti-ideal and ideal actions, respectively).

Assume given $\delta>0.5$ and $\beta>0.5$. [14] provide steps to calculate the credibility of an action $x \in A$ dominating an action $y \in A$, this credibility is denoted by $xD(\alpha)y$. It is said that if $xD(\alpha)y \geq \delta$, then "x dominates y" is accepted. In [12] are also provided steps to calculate the credibility of x being at least as good as y, $\eta(x,y)$. It is said that "x is at least as good as y" if and only if $\eta(x,y) \geq \beta$, which is denoted by $xS(\beta)y$. INTERCLASS-nC exploits those calculations and the following conditions to assign actions from A to ordered classes.

Conditions

Each element in R_k must fulfill the following conditions:

i. For all k and for each action w in R_k, there is at least one action z in R_{k+1} such that $zD(\alpha)w$.
ii. For all k and for each action w in R_{k+1}, there is at least one action z in R_k such that $wD(\alpha)z$.
iii. For all k and for each action w in R_{k+1}, there is no action z in R_k such that $zS(0.5)w$.

The credibility index of the outranking relation of action x over the subset R_k is defined as follows:

$$\eta(\{x\}, R_k) = \max_{j=1,\cdots,card(R_k)}\{\eta(x, r_{k,j})\}.$$

While the credibility index of the outranking relation of subset R_k over an action x is defined as follows:

$$\eta(R_k, \{x\}) = \max_{j=1,\cdots,card(R_k)}\{\eta(r_{k,j} x)\}.$$

Such credibility indices allow to build interval crisp outranking relations between decision actions and sets of characteristic actions as follows:

$$xS(\delta, \boldsymbol{\lambda})R_k \Leftrightarrow \eta(\{x\}, R_k) \geq \delta;$$
$$R_k S(\delta, \boldsymbol{\lambda})x \Leftrightarrow \eta(R_k, \{x\}) \geq \delta.$$

The selection function is defined as $i(\{x\}, R_k) = \min\{\eta(\{x\}, R_k), \eta(R_k, \{x\})\}$.

The assignments of alternatives to classes are performed in INTERCLASS-nC using two joint rules, called the descending rule and the ascending rule, which should be used conjointly, as in both ELECTRE TRI-C and ELECTRE TRI-nC. These rules are defined as follows.

Descending assignment rule

i. Compare x to R_k for $k = M, \ldots, 0$, until the first value, k, such that $xS(\delta, \boldsymbol{\lambda})R_k$;
ii. For $k = M$, select C_M as a possible class to assign action x.
iii. For $0 < k < M$, if $i(\{x\}, R_k) \geq i(\{x\}, R_{k+1})$, then select C_k as a possible class to assign x; otherwise, select C_{k+1}.
iv. For $k = 0$, select C_1 as a possible class to assign x.

Ascending assignment rule

i. Compare x to R_k for $k = 1, \ldots, M+1$, until the first value, k, such that $R_k S(\delta, \boldsymbol{\lambda}) x$;
ii. For $k = 1$, select C_1 as a possible category to assign action x.
iii. For $1 < k < M+1$, if $i(\{x\}, R_k) \geq i(\{x\}, R_{k-1})$, then select C_k as a possible class to assign x; otherwise, select C_{k-1}.

4 Computational experiments

The assessment of the proposal is carried out by assigning stocks to classes through the so-called back-testing [28]. The idea of this kind of testing is to use historical performances of the stocks to discover the performance of the stock-selection approach in a given period; later, by using a sliding window, to use more recent historical performances to discover the performance of the immediately subsequent period, and so on. This way, the approach is assessed in a sufficiently high number of different scenarios.

4.1 Data

The dataset used in the experiments contains the historical performances of the stocks in the Standard and Poor's 500 (S&P 500) index, one of the most outstanding indexes in the world, listing five hundred companies. The dataset consists of daily historical performances of the stocks ranging from March 6th to July 14th, 2021. This time frame represents the last ninety working days in the United States (U. S.) market and contains both upward and downward trends. Table 1 shows a sample of the companies listed in the index. Some reliable databases to acquire these data are: capitaliq.com, koyfin.com, finviz.com, finance.yahoo.com, google.com/finance, sec.gov/edgar.shtml, bloomberg.com, iqoption.com.

From this time frame, the first sixty periods (March 6th to June 1st) were taken to "train" the model; that is, to estimate future returns and to obtain historical performances according to the fundamental indicators. Then, the investment is simulated to take place at the immediately subsequent period (June 2nd). Therefore, the return obtained by the proposed approach for this period is measured as (closing price June 2nd - closing price June 1st)/closing price June 1st.

Table 1. Sample of the S&P 500 index

Company (symbol)									
3M Company (MMM)	AT&T Inc. (T)	Church & Dwight Co. Inc. (CHD)	Duke Realty Corporation (DRE)	Generac Holdings Inc. (GNRC)	JPMorgan Chase & Co. (JPM)	Mohawk Industries Inc. (MHK)	Perrigo Company plc (PRGO)	State Street Corporation (STT)	Trane Technologies plc (TT)
A. O. Smith Corporation (AOS)	Atmos Energy Corporation (ATO)	Cigna Corporation (CI)	DuPont de Nemours Inc. (DD)	General Dynamics Corporation (GD)	Juniper Networks Inc. (JNPR)	Molson Coors Beverage Company (TAP)	Pfizer Inc. (PFE)	STERIS plc (STE)	TransDigm Group Incorporated (TDG)
Abbott Laboratories (ABT)	Autodesk Inc. (ADSK)	Cincinnati Financial Corporation (CINF)	DxC Technology Company (D0C)	General Electric Company (GE)	Kansas City Southern (KSU)	Mondelez International Inc. (MDLZ)	Philip Morris International Inc. (PM)	Stryker Corporation (SYK)	Trimble Inc. (TRMB)
AbbVie Inc. (ABBV)	Automatic Data Processing Inc. (ADP)	Cintas Corporation (CTAS)	Eastman Chemical Company (EMN)	General Mills Inc. (GIS)	Kellogg Company (K)	Monolithic Power Systems Inc. (MPWR)	Phillips 66 (PS0)	SVB Financial Group (SIVB)	Truist Financial Corporation (TFC)
Abiomed Inc. (ABMD)	AutoZone Inc. (AZO)	Cisco Systems Inc. (CSCO)	Eaton Corporation plc (ETN)	General Motors Company (GM)	KeyCorp (KEY)	Monster Beverage Corporation (MNST)	Pinnacle West Capital Corporation (PNW)	Synchrony Financial (SYF)	Twitter Inc. (TWTR)
Accenture plc (ACN)	AvalonBay Communities Inc. (AVB)	Citigroup Inc. (C)	eBay Inc. (EBAY)	Genuine Parts Company (GPC)	Keysight Technologies Inc. (KEYS)	Moody's Corporation (MCO)	Pioneer Natural Resources Company (P0D)	Synopsys Inc. (SNPS)	Tyler Technologies Inc. (TYL)
Activision Blizzard Inc. (ATVI)	Avery Dennison Corporation (AVY)	Citizens Financial Group Inc. (CFG)	Ecolab Inc. (ECL)	Gilead Sciences Inc. (GILD)	Kimberly-Clark Corporation (KMB)	Morgan Stanley (MS)	Pool Corporation (POOL)	Sysco Corporation (SYY)	Tyson Foods Inc. (TSN)
…	…	…	…	…	…	…	…	…	…

Six criteria are used to assess the stocks; the first five are taken from the so-called fundamental analysis, represented by a real number each. The last criterion, the forecasted return of the stock, is taken from the historical prices of the stock and is represented by an interval number:

1. Price to Earnings (PE), defined as the market value per share over earnings per share.
2. Price to Book (PB), defined as the market price per share over book value per share.
3. Price to Sales (PS), defined as the market price per share over revenue per share.
4. Return on equity (ROE), defined as the net income over average shareholder's equity.
5. Return on asset (ROA), defined as the net income over total assets.
6. Estimated future return, defined as the mean plus/minus three times de standard deviation of the stock's returns in the last sixty periods. Note that the forecasting of a quantity is very difficult to be precisely defined; this is the reason why this criterion has been represented by an interval number. INTERCLASS-nC gives one the possibility to define criteria scores defined by interval numbers without adding further complexities to the modeling procedure.

It is common for different companies to find themselves in different contexts. Therefore, the fundamental indicators can be considerably different when comparing companies, even when they present performances that indicate their buying convenience. Thus, we perform here a normalization of each stock's impacts on the fundamental indicators. This normalization considers the last sixty periods to transform the impacts to the range [0, 1]. A sample of the performances of the alternatives is shown in Table 2.

Table 2. Performances of the alternatives on the six criteria

Company	Price to Earnings	Price to Book	Price to Sales	Return on equity	Return on asset	Estimated future return
3M Company	0.5311	0.4851	0.8129	0.1275	0.0152	[-0.0293,0.0333]
A. O. Smith Corporation	0.4345	0.9146	0.8256	0.7421	0.6675	[-0.0445,0.0492]
Abbott Laboratories	0.0000	0.0000	0.0000	0.8195	0.7874	[-0.0506,0.0475]
AbbVie Inc.	0.1940	0.3067	0.4091	0.0000	0.0000	[-0.0374,0.0392]
Abiomed Inc.	0.1280	0.1587	0.1650	0.0545	0.0129	[-0.0673,0.0654]
Accenture plc	0.5734	0.6236	0.5849	0.1860	0.1697	[-0.0313,0.0354]
Activision Blizzard Inc.	0.5529	0.7218	0.5218	0.0280	0.1016	[-0.0463,0.048]
Adobe Inc.	0.3087	0.4041	0.3657	0.2944	0.0192	[-0.0481,0.0523]
Advance Auto Parts Inc.	0.7751	0.5280	0.6078	0.9358	0.8638	[-0.042,0.0464]
Advanced Micro Devices Inc.	0.3030	0.3572	0.2977	0.2364	0.7632	[-0.0742,0.0758]
Aflac Incorporated	0.4932	0.9710	0.9531	0.7974	0.7300	[-0.0301,0.0348]
Agilent Technologies Inc.	0.1597	0.6139	0.5305	0.9020	0.5347	[-0.0327,0.0384]
Air Products and Chemicals Inc.	0.9250	0.9177	0.8976	0.0193	0.0242	[-0.0318,0.0365]
Akamai Technologies Inc.	0.6980	0.7562	0.7451	0.0403	0.0361	[-0.0325,0.038]
Alaska Air Group Inc.	0.0000	0.8491	0.9022	0.0000	0.1966	[-0.0663,0.0691]
Albemarle Corporation	0.7002	0.3239	0.6420	0.0178	0.1296	[-0.083,0.0897]
Alexandria Real Estate Equities Inc.	0.6793	0.5179	0.6220	0.0306	0.1190	[-0.0305,0.035]
Alexion Pharmaceuticals Inc.	0.3304	0.6120	0.6592	0.0320	0.0100	[-0.0268,0.0322]
Align Technology Inc.	0.9042	0.5685	0.2826	0.0008	0.6187	[-0.0747,0.0787]
Allegion plc	0.1171	0.9219	0.8803	0.9939	0.2622	[-0.0368,0.0439]
…	…	…	…	…	…	…

4.2 Parameter settings

A convenient feature of the integrated outranking approach presented in [14] is its flexibility. This approach gives the DM-Analyst pair the opportunity of defining parameters that can be represented by real or interval numbers. Among the several advantages derived from this flexibility, reducing the DM's cognitive effort during a direct elicitation is outstanding. Below, we assume that the DM is engaged in a direct elicitation of the proposed approach's parameters.

Preference parameters
The credibility threshold to establish crisp preference relations, δ, has been defined to be 0.51. The threshold **λ** used to define the strong majority in the outranking relation is set to [0.51, 0.66]. All the criteria are assumed to have the same weight.
The definition of the threshold values assumes that the impacts on the criteria are normalized in [0, 1]. The indifference thresholds are defined as [0, 0.1], the preference thresholds as [0.1, 0.2], and the veto thresholds as [0.5, 0.7].

Profiles
Two classes were considered to select stocks, the Very Convenient Stocks class (C_1), and the Dismissed Stocks class (C_2). The profiles used to characterize these classes are shown in Table 3.

Table 3. Profiles used to characterize the classes

	Price to Earnings	Price to Book	Price to Sales	Return on equity	Return on asset	Estimated future return
C_1	0.7	0.7	0.7	0.7	0.7	[0.05, 0.10]
C_2	0.2	0.2	0.2	0.2	0.2	[-0.05, 0.05]

4.3 Results

The results presented below show the class to which each stock was assigned in each period. Once the stocks belonging undoubtedly to the Very Convenient Stocks class (C_1) in each period were identified, we obtained these stocks' returns in the corresponding periods to assess the performance of the proposed approach.

Table 4. Stocks assigned to classes

	2/6/21	3/6/21	4/6/21	7/6/21	8/6/21	9/6/21	10/6/21	11/6/21	…
3M Company	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	…
A. O. Smith Corporation	[C1, C2]	[C1, C2]	C2	C2	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	…
Abbott Laboratories	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	…
AbbVie Inc.	C1	C1	C1	C1	C1	C1	C1	[C1, C2]	…
Abiomed Inc.	C1	C1	C1	C1	C1	C1	C1	C1	…
Accenture plc	[C1, C2]	[C1, C2]	C1	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	…
Activision Blizzard Inc.	[C1, C2]	C1	C1	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	…
Adobe Inc.	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	…
Advance Auto Parts Inc.	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	C2	[C2, C2]	C2	…
Advanced Micro Devices Inc.	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	…
Aflac Incorporated	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	…
Agilent Technologies Inc.	[C1, C2]	[C1, C2]	[C1, C2]	C2	[C1, C2]	C2	C2	C2	…
Air Products and Chemicals Inc.	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	…
Akamai Technologies Inc.	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	…
Alaska Air Group Inc.	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	…
Albemarle Corporation	[C1, C2]	C1	C1	[C1, C2]	C1	[C1, C2]	[C1, C2]	C1	…
Alexandria Real Estate Equities Inc.	C1	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	…
Ale0ion Pharmaceuticals Inc.	C1	C1	C1	C1	C1	C1	[C1, C2]	[C1, C2]	…
Align Technology Inc.	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	[C1, C2]	…
Allegion plc	C2	C2	C2	C2	C2	C2	C2	C2	…
…	…	…	…	…	…	…	…	…	…

Assignments of stocks to ordered classes

Table 4 shows how the approach assigned the stocks to classes. In this table, we can see, for example, how the approach could not precisely assign 3M Company to a class in any of the first eight periods; it only could precisely assign A. O. Smith in 4/6/21and 7/6/21; stocks like Abiomed Inc. and Allegion plc were precisely assigned in all the first eight periods. Evidently, an imprecise assignment of a stock means that the approach did not find enough arguments to classify the stock as very convenient to buy.

Assessing the stock returns produced by the approach

Since the main purpose of this work is to provide decision support to investors regarding the maximization of their investments' returns, we now proceed to assess the effectiveness of the approach by determining the return produced by the stocks that were better classified.

From Table 4, we can see that only Allegion plc was assigned to the best class in 3/6/21; thus, from the information shown in this table, the approach found that there are no arguments to invest in any other stock in that period. Table 5 shows the number of stocks that were assigned to C_2 per period. Note that the number of stocks significantly decreased regarding the original set considered; this is important in the context of the problem since, usually, investors are not willing to support many stocks.

Table 5. Number of stocks selected per period

Period	# stocks selected	Period	# stocks selected
2/6/2021	48	23/6/2021	36
3/6/2021	40	24/6/2021	33
4/6/2021	38	25/6/2021	34
7/6/2021	44	28/6/2021	37
8/6/2021	38	29/6/2021	32
9/6/2021	41	30/6/2021	31
10/6/2021	34	1/7/2021	25
11/6/2021	37	2/7/2021	26
14/6/2021	40	6/7/2021	22
15/6/2021	33	7/7/2021	21
16/6/2021	39	8/7/2021	21
17/6/2021	35	9/7/2021	14
18/6/2021	35	12/7/2021	24
21/6/2021	24	13/7/2021	23
22/6/2021	29	14/7/2021	14

The summation of Allegion plc's actual return and the actual returns of all the other stocks precisely assigned to C_2 in 3/6/21 is 1.4%, while the summation of actual returns of all the stocks in the S&P 500 index (hereafter, the return of the market) was -98%. Similar assessments were performed for the thirty testing periods considered (3/6/21 – 14/7/21) according to Table 6.

Several conclusions can be obtained from Table 6. First, we can see that there is not a perfect correlation between the direction of the market's returns and our approach. This means that the subset of stocks selected by our approach according to the recommendations of INTERCLASS-nC is not necessarily representative of the whole set of stocks in the index. This can be beneficial in the presence of strong market declines as the one seen in 16/06/21-18/06/21. Another conclusion drawn from the table is that the market is much more volatile than our approach (see Figs. 1 and 2). This can be determinant in this kind of decision problem since investors will always prefer lower levels of volatilities given that the rest of the important factors are the same.

Fig. 1 graphically shows the returns provided in Table 6; this visualization provides a clear insight on the difference of the volatilities described above. Fig. 2 also shows these returns but according to the proportions of the total returns obtained by both the market and our approach. Fig. 2 also shows that two (resp. three) periods out of the thirty periods, the market (resp. our approach) provided positive returns when our approach (resp. the market) provided negative returns.

Finally, Table 6 shows that our approach beat the market overall. This can be shown not only in terms of average return, but also in terms of the so-called cumulative return. Cumulative return is a way to aggregate return over time since negative returns usually make worse impacts if obtained at the first periods. It is calculated as $CumulativeReturn_t = (1 + return_t) \cdot (1 CumulativeReturn_t\text{-}1) - 1$; where $return_t$ and $CumulativeReturn_t$ are the return and cumulative return at period t, respectively, and $CumulativeReturn_0 = return_0$.

Table 6. Summation of the stocks' returns in the S&P 500 (Market) and the returns of the stocks selected by our approach

Period	Market	Our approach
02/06/21	53%	-32%
03/06/21	-98%	1%
04/06/21	251%	9%
07/06/21	-93%	-23%
08/06/21	87%	4%
09/06/21	-230%	-48%
10/06/21	49%	-19%
11/06/21	179%	27%
14/06/21	-208%	-25%
15/06/21	17%	10%
16/06/21	-340%	-19%
17/06/21	-508%	-72%
18/06/21	-833%	-61%
21/06/21	972%	39%
22/06/21	69%	12%
23/06/21	-69%	7%
24/06/21	322%	15%
25/06/21	364%	27%
28/06/21	-220%	-22%
29/06/21	-56%	3%
30/06/21	125%	15%
01/07/21	330%	19%
02/07/21	114%	13%
06/07/21	-437%	-13%
07/07/21	103%	5%
08/07/21	-530%	-34%
09/07/21	812%	25%
12/07/21	135%	4%
13/07/21	-467%	-25%
14/07/21	-78%	-5%
Mean	**-6.21%**	**-5.52%**
Std. Dev.	**374%**	**27%**

Fig. 1. Summation of returns of the stocks in the S&P 500 index (market) and our approach

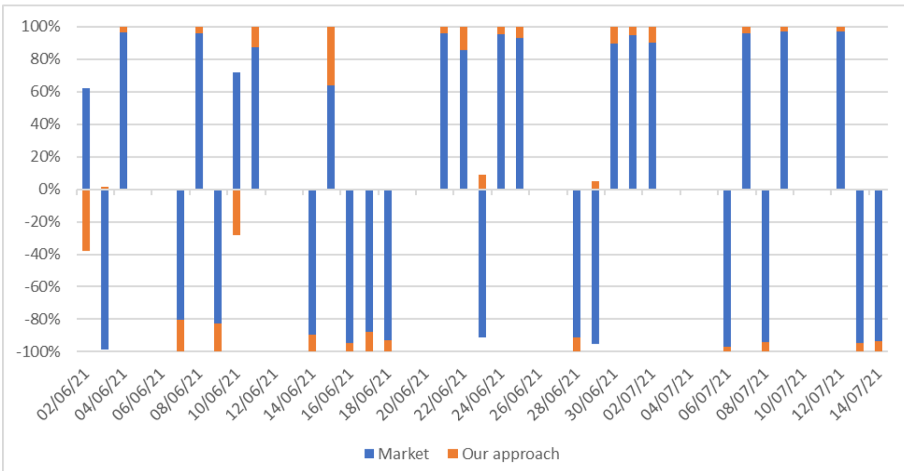

Fig. 2. Composition of returns of the stocks in the S&P 500 index (market) and our approach

5 Conclusions

The problem of selecting the best stocks was addressed here in a novel way. We present the proposal of assigning stocks to ordered classes by exploiting an extension of the outranking approach. This extension allows us to flexibly consider many realistic situations that decision makers usually face when addressing the problem. One of these situations is using many criteria to assess the stocks. Practitioners often use factors from the fundamental analysis that allow them to take into consideration the financial status of the stocks. The financial status of a stock would indicate if the stock were under- or over-valued, thus providing evidence to support (or not support) the decision of investing in the stock. This, however, turns difficult when the decision maker wants to consider many factors. Another important realistic consideration is assessing the stocks in the presence of uncertainty, which is derived from vagueness, imprecision, errors in measurements, or even missing scores on the criteria.
The generalization of the outranking approach presented in [26] and improved in [14] allows building multi-criteria ordinal classification methods able to consider these real situations. In this work, we have exploited such improvement of the outranking approach to assign stocks to ordered classes in such a way that we can determine, according to criteria often used by experts, which are the most convenient stocks.

The experiments consisted of assigning the 500 stocks in the Standard and Poor's 500 index to ordered classes based on their historical performances. Later, we simulated investing on the stocks undoubtedly assigned to the best class for a given period, calculating the final return of supported stocks, and moving to the next period. We considered thirty periods in order to cover a wide time frame where several market trends are involved.

Results of Tables 4 and 5 indicate that the proposed approach considerably reduced the number of supported stocks, decreasing from 500 originally considered stocks to less than 50 selected stocks. This is an important result because the stock selection stage (second stage of the so-called portfolio management) pursuits to create portfolios with few stocks; this is because of transaction costs and other difficulties related to handling many stocks.

Table 6 and Figs. 1 and 2 clearly show that the results obtained by the proposed approach beat the market overall in terms of three main contexts: i) volatility, ii) average return, and iii) cumulative return. The market presented much more volatility than the returns obtained by the selected stocks, which is always convenient since investments tend to be more stable and reliable. On the other hand, the average return of the selected stocks (-5.52%) was greater than that of the market (-6.21%), which indicates the potentiality of the proposed approach. This was confirmed by the cumulative return (that takes into consideration the return over time). Thus, we can conclude that the proposed approach can be seriously considered as an interesting alternative to address the problem of stock selection.

Future research lines include assessing the proposed approach i) conjointly with other methods addressing more stages of portfolio management (such as price forecasting, portfolio optimization, and portfolio rebalancing), ii) when different characterization of the problem is considered (such as more classes, more profiles per class, different numbers of criteria), iii) to consider different attitudes of the decision maker in the presence of risk, iv) in different markets.

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