Effects of Globalization under Consistent Conjectures

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Abstract. We study the effects of merging two separate markets each originally monopolized by a producer into a globalized duopoly market. We consider a linear inverse demand with cap price and quadratic cost functions. After globalization, we find the consistent conjectural variations equilibrium (CCVE) of the duopoly game. Unlike in the Cournot equilibrium, a complete symmetry (identical cost functions parameters of both firms) does not imply the strongest coincident profit degradation. For the situation where both agents are low-marginal cost firms, we find that the company with a technical advantage over her rival has a better ratio of the current and previous profits. Moreover, as the rival becomes ever weaker, that is, as the slope of the rival's marginal cost function increases, the profit ratio improves.

Keywords: Duopoly Game, Conjectural Variations Equilibrium, Cap Price, Globalization.

1. Introduction

The purpose of the present paper is to investigate a market with two competing producers of an identical commodity. We consider two stages: before globalization (separate markets) and after globalization (united market). Before globalization, each producer satisfies the separate demand of the market that it monopolizes. After globalization, both firms compete in a united market. This model is often said to have the structure of a pure (classic) duopoly where both companies satisfy the complete market demand.

One can find numerous studies on the effects of combining two or more markets in the literature. According to [1], there are two types of global markets: a) the free trade market which allows the existence of *n* different markets with a separate supplier; and b) a single integrated market in which all producers compete.

Since the 1980s, there has been a lot of research on the role of imperfect competition. This was pointed out in [2], which deals with global markets of type a). In fact, there are several works which models correspond to these type of markets. Some examples are found in [3]–[5], too. On the other hand, [1] analyzes a globalized market of type b), through a Nash-Cournot equilibrium model, whereas in [1], the authors examine cases where all producers' profits are degraded in the same manner. For each producer, they use the ratio of the profit obtained after globalization to the profit before globalization to represent the degree of the profit degradation, and the largest of the ratios among the producers is a measure of coincident degradation. They found that under a complete symmetry, i.e., when the values of parameters of cost and demand functions are equal, all producers have profit degradation coincidently. For the model they use which boasts linear demand functions for the separated markets and the globalized market, as well as linear cost functions, under Nash-Cournot conjectures, the value of the measure of coincident degradation is the lowest (the worst) when the firms are identical.

The present paper also discusses a situation of type b). As in [1], we use the ratio of the profit obtained after globalization to the profit before globalization to represent the degree of the profit degradation or improvement. However, our purpose is to analyze the effects of globalization considering the diverse values that can take the parameters of the cost functions of the companies, which in our case are quadratic. We reveal that is possible that one producer loses while the other one gains; or both lose.

As well, at the stage of globalization, when competition takes place, we raise a kind of equilibrium with consistent conjectural variations (CCVE). Conjectural Variations Equilibria (CVE) were introduced by Bowley in 1924 [6] and Frisch in 1933 [7] as another possible solution concept in static games. According to this concept, agents behave as follows as was stated in [8]: each agent chooses her most favorable action taking into account that every rival's strategy is a conjectured function of her own strategy. In [8] and [9], we studied mixed oligopoly models with consistent conjectural variations (CCV), which correspond to the market price variations due to the change in the output level of a producer. Concepts such as exterior and interior equilibrium were introduced, and proofs of existence and uniqueness of equilibrium were presented in the above-mentioned papers. We apply these concepts in our present paper, too.

The paper is organized as follows. In Section 2, we describe the mathematical model and specify the assumptions to accept for each stage. This section shows the optimal output levels produced by each firm before globalization as well and finds the consistent conjectural variations equilibrium price and production volumes. In Section 3, we define two types of agents: *low-marginal* and *high-marginal cost* firms. As we study a market with 2 agents, we have four feasible situations corresponding to the possible combination of types of firms. We define the profit ratio and compute it for each situation in terms of the parameters in order to analyze the effect of the cost parameters on this ratio. To do so, we use the concept of technical advantage introduced by [10]. In this section, we also display an example which shows that, unlike the Nash-Cournot case [1], a complete symmetry does not necessarily render the worst-case ratio under consistent conjectures. Finally, in Section 4, we present our conclusions and outline our future work.

2. Model Specification

Before globalization, consider two monopolistic markets. Each monopoly faces an active demand $D_i, i \in \{0,1\}$ which does not depend on market price and a passive demand $G_i(p_i)$ whose argument p_i is the market clearing price. We will assume that in every market the price value $p_i = \bar{P}$ is the *cap price*. This means that the demand functions have a discontinuity point (a break) and for prices higher than $\bar{P}$ the demand is zero. Therefore, the company i output volume, $q_i \geq 0$, will satisfy the following inequality if the market is "balanced":

$$g_i(p_i) + D_i \leq q_i \leq G_i(p_i) + D_i. \tag{1}$$

Here, $g_i(p_i)$, is the right limit of the function G_i at any point while the left limit of this function at each point is assumed to coincide with its proper value.

After globalization, both firms compete in a globalized market. The consumers' (passive) demand is described by a demand function $G(p_w)$, whose argument p_w is the market clearing price. An active demand value D is non-negative and does not depend on the market price. Here we take for granted that after globalization, the cap price will be the same as before globalization. Since the demand function has a point of discontinuity (a break at the cap price $\bar{P}$), the balance between the demand and supply for a given price p_w is described by the following ("balance") inequality:

$$g(p_w) + D \leq Q \leq G(p_w) + D. \tag{2}$$

Here, $g(p_w)$ is the right limit of the function G at any point p_w and $Q = q_0 + q_1$.

2.1 Assumptions

Accept the following assumptions about the demand and cost functions in order to study the effects of globalization.

Before Globalization
A1.1. The inverse demand function for each firm $i, i \in \{0, 1\}$, is defined as follows:

$$p_i(\theta_i) = \begin{cases} \bar{P} & if\ 0 \leq \theta_i \leq \bar{Q}/2 ; \\ c - d\theta_i & if\ \bar{Q}/2 < \theta_i \leq c/d. \end{cases} \tag{3}$$

Here c and d are positive values, and $\bar{P} = c - d\bar{Q}/2$. The total quantity demanded in the market i at the price p_i is θ_i, which includes the passive and the active quantities demanded.
A1.2. For each $i \in \{0, 1\}$, the cost function $f_i(q_i)$ is quadratic, i.e., $f_i(q_i) = \frac{1}{2}a_i q_i^2 + b q_i$ where $a_i > 0$ and $0 < b < c$.
A1.3. $\bar{P} > \max_{i \in \{0,1\}} \{a_i \bar{Q}/2 + b\}$.

After Globalization
A2.1. The market inverse demand function is defined as follows:

$$p_w(\theta) = \begin{cases} \overline{P} & if\ 0 \le \theta \le \overline{Q}; \\ c - d\theta/2 & if\ \overline{Q} < \theta \le 2c/d. \end{cases} \tag{4}$$

Here c, d, and $\overline{P}$ are defined as in **A1.1**. The variable θ is the total quantity demanded (including the passive and the active demand).

Assumption **A1.2** about the cost function is also made; the cost structure won't change after globalization. As a consequence of **A1.3**, if $q_0 + q_1 < \overline{Q}$ then $\overline{P} > a_i q_i + b$, for at least one $i, i \in \{0,1\}$.

2.2 Objective functions of the companies

2.2.1 Before Globalization

Recall that before globalization exists a single company in each market commercializing the commodity. Company i, ($i \in \{0,1\}$) chooses its output volume so as to maximize its net profit function: $\pi_i(q_i) = p_i(q_i)q_i - f_i(q_i)$.

Note that assumption **A1.3** implies that the output value that maximizes the benefits cannot be less than $\overline{Q}/2$. Because of that, we can rewrite the maximization problem of any firm with sub-index i as follows:

$$\begin{aligned} \underset{q_i}{\text{maximize}} &\qquad \pi_i(q_i) = p_i(q_i)q_i - f_i(q_i), \\ \text{subject to} &\qquad q_i \ge \overline{Q}/2, \end{aligned} \tag{5}$$

which can be easily solved by Karush-Kuhn-Tucker (KKT) equations. The optimal output value $\bar{q}_i$ for private firm $i, i \in \{0,1\}$ is found as:

$$\bar{q}_i = \begin{cases} \dfrac{c - b}{2d + a_i} & if\ \dfrac{c - b}{2d + a_i} > \dfrac{\overline{Q}}{2}; \\[3mm] \dfrac{\overline{Q}}{2} & if\ \dfrac{c - b}{2d + a_i} \le \dfrac{\overline{Q}}{2}. \end{cases} \tag{6}$$

2.2.2 After Globalization

After globalization, there is an integrated market where both companies compete in a classic duopoly. The price at this stage is determined in the global market, so it obeys the inverse demand function (**4**) cited in assumption **A2.1**.

The problem of each private company i is to maximize its net profit

$$\xi_i(q_i) = p_w(Q)q_i - f_i(q_i), i = 0,1 . \tag{7}$$

The output level of each company under the assumptions made is found using the theory of [11]. As in [11], we also claim that the output volume chosen by a producer influences the market price. This can be described by a conjectured function of the variations of the price upon variations of the production volume. Then, the first order maximum condition to define the equilibrium would have the form $i, i \in \{0,1\}$:

$$\frac{\partial \xi_i}{\partial q_i} = p_w(Q) + \frac{\partial p_w(Q)}{\partial q_i} \cdot q_i - a_i q_i - b \begin{cases} = 0 & if\ q_i > 0; \\ \le 0 & if\ q_i = 0. \end{cases} \tag{8}$$

As in [11], denote $v_i = -\partial p_w(Q)/\partial q_i$. In order to describe each agent's behavior, we need to evaluate v_i. The conjectured dependence of p_w on q_i must account for the (local) concavity of the i –th agent's objective function; otherwise one cannot guarantee that the output volumes found via the *first order optimality conditions* (**8**) *maximize* (but not minimize) the profit functions. For instance, it suffices to assume that the coefficient v_i (from now on referred to as the i –th agent's *influence coefficient*) is nonnegative and constant, for $i, i \in \{0,1\}$.

In [11] and [12], we defined the concept of *exterior equilibrium*, i.e., *conjectural variations equilibrium* (CVE) with the influence coefficients fixed in an exogenous mode. As the competition after globalization has been represented by the model presented in [11], the equilibrium would be found as in the mentioned publication. Theorem 1 in [11] establishes the existence and unicity of the exterior equilibrium $(p_w ; \tilde{q}_0, \tilde{q}_1)$ under assumptions **A1.2** and **A2.1**, and also provides the left and right derivatives of the equilibrium price $p_w = p_w(D, v_0, v_1)$ with respect to D. This theorem serves as a base for the concept of *interior equilibrium*, which was defined in [11] as the exterior equilibrium with *consistent* conjectures (influence coefficients). Under the above assumptions, according to Theorem 2 in [11], there exists *interior equilibrium* after globalization.

Consistent Conjectural Variations Equilibrium:

Let

$$\tau = \begin{cases} -\infty & if \ p_w = \bar{P}, \\ -2/d & otherwise. \end{cases}$$

Given the previous results obtained in [11] the influence coefficient of the agent $i, i \in \{0,1\}$, after globalization is:

$$v_i = \cfrac{1}{\cfrac{1}{v_{-i}+a_{-i}} - \tau} \ , i = 0,1,$$ (9)

where the symbol $-i$ represents the competitor's sub-index.

In (**9**), $\tau \in [-\infty, 0]$. When $\tau = -\infty$, system (**9**) has the unique solution $v_i = 0, i \in \{0, 1\}$. The latter result corresponds to the perfect competition equilibrium (*cf.*, [11]).

The following result was already derived and published as Theorem 3 in [11] and Theorem 4.3 in [12], including for the case of a mixed oligopoly (competition among a public firm and several private companies).

Theorem 2.1([11]-[12]). *Under assumptions A1.2 and A2.1, for any $\tau \geq 0$, Eq. (**9**) has a unique solution $v_i = v_i(\tau), i \in \{0, 1\}$, continuously depending upon τ. Furthermore, $v_i(\tau) \to 0$ when $\tau \to -\infty$, and strictly increases and tends up to $v_i(\tau)$ as $\tau \to 0, i = 0,1$.*

Therefore, the solution of the system formed by equations (**9**) for the firm i's influence coefficient is:

$$v_i = \begin{cases} \dfrac{-a_i}{2} + \sqrt{\dfrac{a_i{}^2}{4} + \dfrac{\Gamma}{K_{-i}}}, & \tau = -\dfrac{2}{d}; \\ \\ 0, & \tau = -\infty, \end{cases}$$ (10)

$i = 0,1$, where $\Gamma = a_i + a_{-i} + 2a_i a_{-i}/d$ and $K_{-i} = 2\left(2 + \frac{2}{d}a_{-i}\right)/d$.

For the interior equilibrium price $p_w > b$, Theorems 1 and 2 in [11] imply that relationship (**8**) defines uniquely the equilibrium production volumes $\tilde{q}_i, i \in \{0, 1\}$, taking into account that $p_w = \bar{P}$ implies $v_i = 0$:

$$\tilde{q}_i = \begin{cases} \dfrac{p_w - b_i}{v_i + a_i}, & p_w < \bar{P}; \\ \\ \dfrac{p_w - b_i}{a_i}, & p_w = \bar{P}, \end{cases}$$ (11)

$i = 0,1$. However, assumption **A1.3** entails that the total output level given by (**11**) at $p_w = \bar{P}$, is greater than $\bar{Q}$, but at this price the quantity demanded is at most $\bar{Q}$, which means that the market is *not* balanced. Hence, the equilibrium can be reached only when $p_w < \bar{P}$.

In the equilibrium when $p_w < \bar{P}$, the total supply output equals the demand in the market. Then, from **A2.1**, $p_w = c - dQ/2$, where $Q = \tilde{q}_0 + \tilde{q}_1$. Plug in this in the equilibrium outputs (**11**) to obtain the total output and the equilibrium price p_w . These values can be found in **Appendix A**.

3. Effects of Globalization on Profits

To find the effects of globalization on profits we look for the ratio of benefits. We determine conditions involving the parameters under which these ratios are greater or smaller than 1. If the profit ratio is greater than 1 for company $i, i \in \{0, 1\}$ we say that globalization is beneficial for this firm, and it is *not* otherwise, that it, if the profit ratio is less than 1. In order to do that, we first introduce the properties of companies being *low-marginal*, or vice versa, *high-marginal* cost firms.

Definition 3.1. We say that agent i is a *low-marginal cost firm* (*LMCF*) if the marginal cost $f'_i(q_i)$ evaluated at $\bar{Q}/2$ is less than the cap price minus a proportion d of the quantity $\bar{Q}/2$, that is, $f'_i(\bar{Q}/2) < \bar{P} - d\,\bar{Q}/2$. Conversely, agent i is a *high-marginal cost firm* (*HMCF*) if $f'_i(\bar{Q}/2) \geq \bar{P} - d\,\bar{Q}/2$.

Before globalization, the output level produced by firm i to supply to a separate market depends on the value of the corresponding parameters. On the one hand, if firm i is an ***LMCF***, it produces $\bar{q}_i = \frac{c-b}{2d+a_i}$. On the other hand, if it is an ***HMCF*** it supplies $\bar{q}_i = \frac{\bar{Q}}{2}$. Because of that, before globalization four situations in total are feasible depending on the characteristics of the firms of both markets. These situations are described in Table 1, which shows the optimal outputs and the profits for both firms.

Table 1. Possible Situations.

		Before Globalization	
Sit.	Type of firms	Outputs: $\bar{q}_i, i \in \{0,1\}$	Profits: $\pi_i(\bar{q}_i), i \in \{0,1\}$
1	Both are LMCF	$\bar{q}_0 = \frac{c-b}{2d+a_0}, \bar{q}_1 = \frac{c-b}{2d+a_1}$	$\pi_0(\bar{q}_0) = \frac{(c-b)^2}{2(2d+a_0)}, \pi_1(\bar{q}_1) = \frac{(c-b)^2}{2(2d+a_1)}$
2	Agent 0 is an LMCF and agent 1 is an HMCF	$\bar{q}_0 = \frac{c-b}{2d+a_0}, \bar{q}_1 = \frac{\bar{Q}}{2}$	$\pi_0(\bar{q}_0) = \frac{(c-b)^2}{2(2d+a_0)}, \pi_1(\bar{q}_1) = \frac{1}{8}\bar{Q}\big(4(c-b) - \bar{Q}(2d+a_1)\big)$
3	Agent 0 is an HMCF and agent 1 is an LMCF	$\bar{q}_0 = \frac{\bar{Q}}{2}, \bar{q}_1 = \frac{c-b}{2d+a_1}$	$\pi_0(\bar{q}_0) = \frac{1}{8}\bar{Q}\big(4(c-b) - \bar{Q}(2d+a_0)\big), \pi_1(\bar{q}_1) = \frac{(c-b)^2}{2(2d+a_1)}$
4	Both are HMCF	$\bar{q}_0 = \frac{\bar{Q}}{2}, \bar{q}_1 = \frac{\bar{Q}}{2}$	$\pi_i(\bar{q}_i) = \frac{1}{8}\bar{Q}\big(4(c-b) - \bar{Q}(2d+a_i)\big), i \in \{0,1\}$

Let R_i denote the profit ratio of company $i, i \in \{0,1\}$ and be given by:

$$R_i = \frac{\xi_i(\tilde{q}_i)}{\pi_i(\bar{q}_i)}.$$ (12)

Formula (**12**) would take different values according to the faced situation.

3.1 Measure of coincident profit degradation

Globalization may improve or degrade the profits of the companies. However, [1] study the cases when coincident profit degradation occurs, that is, both firms have smaller profits after globalization than before. In the above-mentioned work, the profit ratio of a producer after globalization to that before globalization is proposed as the degree of profit degradation for the producer due to globalization. They utilize the largest of the ratios of profit degradation among producers as a measure of *coincident degradation*. According to [1], the reason is: a smaller value of the measure is supposed to indicate stronger coincident degradation. The situation where only one of the producers suffers profit degradation cannot be considered overall coincident producer profit degradation as far as the other producer enjoys profit improvement. The measure of coincident profit degradation used in [1] for a duopoly is defined by the following equality:

$$k^R = max\{R_0, R_1\}.$$ (13)

The main result obtained in [1] is that the worst-case ratio of coincident profit degradation for all producers due to globalization

is reached by a market system if, and only if the system is in complete *symmetry*. In the next subsection, we show how this result is not necessarily true in the case of the equilibrium with consistent conjectural variations, at least for a system of two firms with quadratic cost functions. We also describe the effect of the cost parameters values, a_i and a_{-i} on the profit ratios. In order to do so, we use the concept of *technical advantage*. According to the definition of technical advantage introduced in [10], a firm has technical advantage over its rival, if it can produce the same output that its rival produces at lower marginal and total costs than its rival. We say that firm i has technical advantage over firm $(-i)$ if $a_i < a_{-i}$.

The proofs of the propositions are exported to ***Appendix*** due to the space issues.

3.2 Situation 1

Situation 1 stated in Table 1 refers to the case when both agents are low-marginal cost firms. Substitute the profits at the equilibrium (shown in ***Appendix A***) and the optimal profits (Table 1) into formula (**12**), and after some algebraic manipulations obtain:

$$R_i = \frac{2d + a_i}{2v_i + a_i}\left(\frac{2}{d}v_i\right)^2, \tag{14}$$

where $v_i = \dfrac{-a_i}{2} + \sqrt{\dfrac{a_i{}^2}{4} + \dfrac{\Gamma}{K_{-i}}}$ according to equation (**10**).

Proposition 3.1. *There is degradation of the profits of private firm i if, and only if*

$$\lambda_1(a_i, a_{-i}, d) + \lambda_2(a_i, a_{-i}, d) > 1, \tag{15}$$

where $\lambda_1(a_i, a_{-i}, d), \lambda_2(a_i, a_{-i}, d) \in (0,1)$ *are continuous functions specified in Appendix A.*
Proof. See ***Appendix B***.

Coincident degradation of the benefits occurs when inequality (**15**) is satisfied for both $i = 0, 1$. The degradation or increase of company i's profit depends not only on the cost parameters of the same company but also on the cost parameters of the other agent.

Proposition 3.2. *The profit ratio of competitor i increases if* a_{-i} *grows.*
Proof. See ***Appendix B***.

Proposition 3.2 states that the larger the coefficient of the quadratic term of the rival's cost function, the lower the profit degradation for producer i (or, its profits may even increase). The proof of Proposition 3.2 shows that the increase of the parameter a_{-i} of the cost function of the rival affects positively the profit ratio of player i, as expected. However, the parameter a_i of its own cost function has an ambiguous effect on this ratio. In order to show that, calculate the derivative of R_i with respect to a_i

$$\frac{\partial R_i}{\partial a_i} = \frac{v_i}{d^2(d + a_{-i})(2v_i + a_i)^3}\chi_i(a_i, a_{-i}, d), \tag{16}$$

where $\chi_i(a_i, a_{-i}, d)$ is a continuous function specified in ***Appendix A*** . The sign of this derivative is the same as the sign of $\chi_i(a_i, a_{-i}, d)$. In order to describe the behaviour of $\chi_i(a_i, a_{-i}, d)$, let $a_i = \rho a_{-i}$ and $d = \sigma a_{-i}$; $\rho, \sigma > 0$. By substituting them in χ_i, we have: $\chi_i(\rho, \sigma, a_{-i}) = a_{-i}^4\tilde{\chi}_i(\rho, \sigma)$, where $\tilde{\chi}_i(\rho, \sigma)$ is a continuous function of two variables specified in ***Appendix A***.

In general, the sign of $\tilde{\chi}_i$ depends on both values ρ and σ. However, in particular, we can say that $\tilde{\chi}_i$ shows various types of behavior when ρ takes values around zero, however $\tilde{\chi}_i \to 0$ when $\rho \to 0$. While ρ is cut off from zero, that is, if $\rho \geq \bar{\rho}_1 > 0$, then $\tilde{\chi}_i$ takes negative values, for any value of $\sigma > 0$. That is, a little increments in a_i affect negatively the profit ratio R_i, regardless of whether firm i has technical advantage over her rival or not, while the value of d doesn't exceed $\bar{\rho}_1$ times a_{-i} (in this case, d may also be less than a_{-i}, but remember that the behavior of χ is ambiguous when ρ is very close to zero). If $s \leq 1$, which means that the rival firm $(-i)$ has no technical advantage over firm i, the effect of little increments in a_i is positive when $\rho < \bar{\rho}_2$. This happens if d is large enough as compared to a_{-i}.

For the current situation, the profits of the weaker firm $(-i)$ are degraded after globalization. Another important fact is that if a firm has a technical advantage over the other, the degradation of her own profit due to globalization (if the latter happens at all) is lower than the profit degradation of the other firm. Even more, the profits of firm i can increase. These results are summarized in Proposition 3.3.

Proposition 3.3. *If $a_i < a_{-i}$, i.e., competitor i has technical advantage over $(-i)$, then*
   a) $R_{-i} < 1$;
   b) $R_{-i} < R_i$.
Proof. See ***Appendix B***.

Note that if we consider the case where there is coincident degradation of the profits, the measure of the latter in this case would equal $K^R = R_i$, i.e., the profit degradation of the firm with the technical advantage.

Under the complete symmetry, both producers suffer coincident profit degradation. This result is the same as in [2] and is stated in Proposition 3.4:

Proposition 3.4. *If the firms are symmetric $(a_i = a_{-i} = a)$ the ratio of both firms is given by*

$$R = \frac{2d + a}{2\sqrt{\frac{a^2}{4} + \frac{ad}{2}}} \left(\frac{a}{\frac{a}{2} + \sqrt{\frac{a^2}{4} + \frac{ad}{2}}} \right)^2 .$$ (17)

This resulting value is less than 1 for any positive values of a and d, which means that globalization degrades profits for each company when both firms face the same costs.
Proof. See ***Appendix B***.

In contrast to [1], the latter is *not* necessarily the worst case under consistent conjectures. We introduce a numerical example to show it. In the following examples, we compute, together with the consistent conjectural variations equilibrium (CCVE), the equilibrium under Nash-Cournot conjectures considering the quadratic cost functions. In [1], the cost function is linear. Nevertheless, our examples with quadratic costs show that a complete symmetry implies the worst-case ratio under Nash-Cournot conjectures, too.

Example 1. Consider a duopoly with $c = 50, d = 10, \bar{P} = 30, \bar{Q} = 4, b = 1$, and $a_0 = 0.1$. Table 2 (which has been exported to ***Appendix A*** due to the space issues), simulates Situation 1 for the above-given values of the parameters and different values of the parameter a_1, starting with $a_1 = 0.06$ and increasing with a mesh of 0.02. The above-mentioned table shows the influence coefficients in the case of CCVE, while the values of the influence coefficients at the Nash-Cournot equilibrium are always $v_0 = v_1 = -d/2$. The minimal value of the measure k^R among the values presented in Table 2 is achieved when $a_1 = 0.06$, $k_{CV}^R = 0.23021$, which means that the worst case is not the one where the firms are symmetric, unlike the Nash-Cournot case in which the worst case ratio is obtained when $a_1 = a_0 = 0.1$. In the CCVE, as long as firm 1 has technical advantage over firm 0, $k_{CV}^R = 0.23021$ (when $a_1 = a_0, k_{CV}^R = R_1 = R_0$). With regard to R_1, notice that $\rho = 10 = 100 > \bar{\rho}_2$. Therefore, at least when firm 0 has no technical advantage over firm 1 ($\sigma \leq 1$), R_1 will increase along with a_1. This explains why a complete symmetry does not necessarily entail the worst case ratio in for conjectural CVEs.

However, in **Example 2** where $c = 50, d = 0.1, \bar{P} = 49.8, \bar{Q} = 4, b = 1$, and $a_0 = 0.1$, the profit ratio of firm 1 decreases as it loses the technical advantage and becomes weaker ($\rho = 0.1 = 1$), whereas the profit ratio of firm 0 increases as a_1 grows. In this case, the worst measure occurs when the firms are symmetric. These results are found in Table 3 also exported to ***Appendix***.

3.2 Situations 2 and 3

In Situation 2 from Table 1, there is one *low-marginal cost* firm and the rival is a *high-marginal cost*. Plug in the equilibrium and optimal values into formula (**12**), and simple algebraic manipulations yield (in Situation 3, the results are similar):

$$R_0 = \frac{2d + a_0}{2v_0 + a_0}\left(\frac{2}{d}v_0\right)^2, \qquad R_1 = \frac{\frac{1}{2(2v_1 + a_1)}((c - b)\frac{2}{d}v_1)^2}{\frac{\overline{Q}}{2}(2d + a_1)\left(\frac{c - b}{2d + a_1} - \frac{1}{2}\frac{\overline{Q}}{2}\right)}. \tag{18}$$

Here, propositions 3.1 and 3.2 are still valid for firm 0 because the formulas of the profit ratio for firm 0 from equations (18) are identical to formulas (14). Firm 0 would face degradations of her profits if, and only if (15) holds. The profit ratio of firm 0 increases with respect to a_1. The larger the value of a_1 the higher is the profit ratio for firm 0. The latter means that if globalization damages the profits of firm 0, the degradation would not be too strong as it would be with a smaller value of a_1. For firm 1, the higher values of the slope of the rival's marginal cost would result in a better profit ratio as stated in Proposition 3.5.

Proposition 3.5. *Profit ratio of competitor* 1 *increases on* a_0.
Proof. See ***Appendix C***.

3.3 Situation 4

In Situation 4 from Table 1, both producers are high-marginal cost firms. Substituting the equilibrium and optimal values in formula (12), after simple algebraic manipulations one obtains for $i = 0, 1$:

$$R_i = \frac{\frac{1}{2(2v_i + a_i)}((c - b)\frac{2}{d}v_i)^2}{\frac{\overline{Q}}{2}(2d + a_i)(\frac{c - b}{2d + a_i} - \frac{1}{2}\frac{\overline{Q}}{2})}. \tag{19}$$

Proposition 3.6. *The profit of competitor i increases together with* a_{-i}.
Proof. See ***Appendix D***.

Therefore, the effect of increase of the quadratic cost coefficient a_{-i} on the rival's profit (player i) is positive.

4. Conclusions and Future Work

In this paper, we examine consistent conjectural variations equilibrium (CCVE) for a duopoly in a market of a homogeneous product. We study the effects of uniting two separate markets each monopolized by a producer: after globalization, both firms compete in one common market. Our model assumes an inverse demand function with a cap price and quadratic cost functions of both agents. Similar to previous studies, we investigate if the companies lose or gain due to globalization by evaluating their profit ratios, i.e., the ratios of their net profits after and before entering the common market. For the situations where both agents are low-marginal cost firms, we find that the company with a technical advantage over her rival has a better profit ratio. In addition, as the rival becomes weaker, this is, as the slope of the rival's marginal cost function increases, the agent's profit ratio enhances, too. Moreover, when both agents are low-marginal cost firms, at least the weaker company suffers degradation of her profits due to the globalization.

Unlike the previous study [1] which considers Nash-Cournot equilibrium, we show with an example that the complete symmetry of the agents does *not* always provide the worst case (the lowest profit ratio) in the case of CCVE. As a consequence, we demonstrate that under consistent conjectures it is important to analyze not only the case where firms are symmetrical, although this leads us to deal with more complicated or even intractable problems.

In our forthcoming papers, we are going to analyze a system with a public firm whose maximized objective is distinct from its net profit.

Acknowledgments

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5. Appendices

5.1 Appendix A

Profits before globalization

$$\pi_i = \begin{cases} \dfrac{(c-b)^2}{2(2d+a_i)}, & if \ \dfrac{c-b}{2d+a_i} > \dfrac{\bar{Q}}{2}; \\ \dfrac{1}{8}\bar{Q}\big(4(c-b) - \bar{Q}(2d+a_i)\big), & if \ \dfrac{c-b}{2d+a_i} \le \dfrac{\bar{Q}}{2}. \end{cases} \tag{20}$$

Equilibrium price, outputs, and profits. We know from (**9**) that $\sum_{j=0}^{1} \frac{1}{v_j+a_j} = \frac{1}{v_i+a_i} + \frac{1}{v_i} - \frac{2}{d}$, $i \in \{0,1\}$.   After some algebraic manipulations one yields:

$$Q = \frac{\sum_{j=0}^{1}\dfrac{c-b}{v_j+a_j}}{\dfrac{d}{2}\left(\dfrac{1}{v_i+a_i} + \dfrac{1}{v_i}\right)}, \qquad\qquad p_w = c - \frac{\sum_{j=0}^{1}\dfrac{c-b}{v_j+a_j}}{\dfrac{1}{v_i+a_i} + \dfrac{1}{v_i}}, \tag{21}$$

$$\xi_i(\widetilde{q_i}) = \frac{(2v_i(c-b))^2}{2d^2(2v_i+a_i)}. \tag{22}$$

Profit Ratios in Situation 1

From (**8**), $p_w = (v_i+a_i)\widetilde{q_i} + b$, and from **A1.1**, $p_i(\bar{q}_i) = c - d\bar{q}_i$.   Plug   in   these   values   in   (**12**)   and   note   that $(c - d\bar{q}_i)\bar{q}_i - \frac{1}{2}a_i\bar{q}_i^2 - b\bar{q}_i = \frac{1}{2}\frac{(c-b)^2}{(2d+a_i)} = \frac{1}{2}(2d+a_i)\bar{q}_i^2$, then (**12**) can be rewritten as $R_i = \frac{2v_i+a_i}{2d+a_i}\left(\frac{\widetilde{q_i}}{\bar{q}_i}\right)^2$.

In the equilibrium, when $p_w < \bar{P}$, the total supply output equals the demand in the market. Then, from **A2.1**, $p_w = c - \frac{d}{2}Q$, where $Q = \tilde{q}_0 + \tilde{q}_1$.

Insert the equilibrium outputs in (11) to obtain the total output and the equilibrium price p_w: $Q = p_w \sum_{j=0}^{1} \frac{1}{v_j+a_j} -$

$\sum_{j=0}^{1} \frac{b}{v_j+a_j}$. Then $Q = \dfrac{\sum_{j=0}^{1} \frac{c-b}{v_j+a_j}}{1+\frac{d}{2}\sum_{j=0}^{1} \frac{1}{v_j+a_j}}$. We know from (9) that $\sum_{j=0}^{1} \frac{1}{v_j+a_j} = \frac{1}{v_i+a_i} + \frac{1}{v_i} - \frac{2}{d}$, $i \in \{0,1\}$, hence $Q =$

$\dfrac{\sum_{j=0}^{1} \frac{c-b}{v_j+a_j}}{\frac{d}{2v_i+a_i}+\frac{1}{v_i}}$. The equilibrium price $p_w = c - dQ/2$ can be plugged in (11), hence the output values ratio of firm i is:

$$\frac{\tilde{q}_i}{\bar{q}_i} = \frac{2d+a_i}{v_i+a_i} - \frac{(2d+a_i)(\sum_{j=0}^{1}\frac{c-b}{v_j+a_j})}{(c-b)(2+\frac{a_i}{v_i})}.$$

Substitute this ratio in $R_i = \frac{2v_i+a_i}{2d+a_i}\left(\frac{\tilde{q}_i}{\bar{q}_i}\right)^2$ and after a little of algebra obtain $R_i = \frac{2d+a_i}{2v_i+a_i}\left(1 - \frac{v_i}{v_{-i}+a_{-i}}\right)^2$.

Note from (9) that $\frac{v_i}{v_{-i}+a_{-i}} = 1 - \frac{2}{d}v_i$. Then $R_i = \frac{2d+a_i}{2v_i+a_i}\left(\frac{2}{d}v_i\right)^2$

The desired result is obtained by plugging in the influence coefficients (10) in the last formula for R_i.

Functions omitted in text for space issues

$$\lambda_1(a_i, a_{-i}, d) = \left(\frac{2\sqrt{\frac{a_i^2}{4}+\frac{\Gamma}{K_{-i}}}}{2d+a_i}\right)^{\frac{1}{2}}$$

$$\lambda_2(a_i, a_{-i}, d) = 1 - \frac{2}{d}\left(-\frac{a_i}{2}+\sqrt{\frac{a_i^2}{4}+\frac{\Gamma}{K_{-i}}}\right)$$

$$\chi_i(a_i, a_{-i}, d) = -(8a_2 d^3 + 6a_i d^2(3a_2+d) + a_i^2 d(14a_2+9d) + 4a_i^3(a_2+d)) + (2d^2(3a_2+d)$$
$$+ a_i d(10a_2+7d) + 4a_i^2(a_2+d))\sqrt{\frac{d+a_i}{d+a_2}}(da_i + da_2 + a_1 a_{-i})$$

$$\tilde{\chi}_i(\rho, \sigma) = -(8\rho^3 + 4(\rho+3)\rho^2\sigma + 2(\rho+2)\rho\sigma^2)$$
$$- (2(\rho+3)\rho^2 + 4(\rho+1)\sigma^2 + (7\rho+10)\rho\sigma)\left(\sigma - \sqrt{(\rho+\sigma)\left(\frac{\rho}{1+\rho}+\sigma\right)}\right)$$

Table 2. Example 1

Table 1. Example 1: $c = 50, d = 10, \bar{P} = 30, \bar{Q} = 4, b = 1$ and $a_0 = 0.1$								
a_1	v_0	v_1	R_0^{CV}	R_1^{CV}	k_{CV}^R	R_0^C	R_1^C	k_C^R
0.06	0.58490	0.60238	0.21661	0.23021	**0.23021**	0.88571	0.89442	0.89442
0.08	0.62298	0.63164	0.23183	0.23856	0.23856	0.88804	0.89238	0.89238
0.10	0.65887	0.65887	0.24618	0.24618	0.24618	0.89036	0.89036	**0.89036**
0.12	0.69290	0.68438	0.25980	0.25319	0.25980	0.89266	0.88834	0.89266
0.14	0.72532	0.70839	0.27277	0.25968	0.27277	0.89496	0.88634	0.89496

Table 3. Example 2

Table 1. Example 2: $c = 50, d = 0.1, \bar{P} = 49.8, \bar{Q} = 4, b = 1$ and $a_0 = 0.1$

a_1	v_0	v_1	R_0^{CV}	R_1^{CV}	k_{CV}^R	R_0^C	R_1^C	k_C^R
0.06	0.03292	0.03633	0.78401	1.03483	1.03483	0.83424	1.07555	1.07555
0.08	0.03498	0.03649	0.86406	0.97464	0.97464	0.90354	1.01047	1.01047
0.10	0.03660	0.03660	0.92820	0.92820	**0.92820**	0.96000	0.96000	**0.96000**
0.12	0.03790	0.03670	0.98068	0.89125	0.98068	1.00682	0.91973	1.00682
0.14	0.03898	0.03677	1.02439	0.86112	1.02439	1.04625	0.88685	1.04625

5.1 Appendix B. Proofs for Situation 1

Proof of Proposition 3.1. We say that profit degrades when the benefits earned after globalization are less than before, i.e. $R_i < 1$. As all parameters are positive formula $R_i < 1$ is equivalent to $\left(\frac{2}{d}\left(-\frac{a_i}{2} + \sqrt{\frac{a_i^2}{4} + \frac{\Gamma}{K_{-i}}}\right)\right)^2 < \frac{2\left(\sqrt{\frac{a_i^2}{4} + \frac{\Gamma}{K_{-i}}}\right)}{2d + a_i}$ which leads to (15).

From (9), $\frac{v_i}{v_{-i} + a_{-i}} = 1 - \frac{2}{d}v_i$. Recall that $v_i > 0, i = 0,1$, if $\tau = -\frac{2}{d}$, and the demand and cost functions' parameters are also positive, then $0 < v_i < \frac{d}{2}$. Hence, given (10), we have $0 < 2\sqrt{\frac{a_i^2}{4} + \frac{\Gamma}{K_{-i}}} < d + a_i$, therefore $0 < \left(\frac{2\left(\sqrt{\frac{a_i^2}{4} + \frac{\Gamma}{K_{-i}}}\right)}{2d + a_i}\right)^{1/2} < 1$.

Assumptions **A1.2** and **A2.2** ensure that both firms actually produce maintain nonzero output levels before and after globalization, then $\frac{\tilde{q}_i}{\bar{q}_i} > 0$. Since d, a_i, v_i are positive, the product $\frac{2v_i + a_i}{2d + a_i}\left(\frac{\tilde{q}_i}{\bar{q}_i}\right)$ is positive, too. Similar to the proofs in ***Appendix A***, substitute the value of $\frac{\tilde{q}_i}{\bar{q}_i}$ in the formulas for the ratios and find $\frac{2v_i + a_i}{2d + a_i}\left(\frac{\tilde{q}_i}{\bar{q}_i}\right) = \frac{2}{d}v_i > 0$. Therefore, the second element of the sum in (**15**) is also smaller than 1.

■

Proof of Proposition 3.2. Differentiate (**14**) by a_{-i}, and a simplification yields $\frac{\partial R_i}{\partial a_{-i}} = \frac{2d + a_i}{4\left(\sqrt{\frac{a_i^2}{4} + \frac{\Gamma}{K_{-i}}}\right)^3}\left(\frac{2}{d}\right)^2\left(\frac{\Gamma}{K_{-i}}\right) \cdot \frac{K_i}{K_{-i}^2} > 0$

■

Proof of Proposition 3.3.

a)   We want to show that, if $a_i < a_{-i}$ then

$$R_{-i} = \frac{2d + a_{-i}}{2v_{-i} + a_{-i}}\left(\frac{2}{d}v_{-i}\right)^2 < 1. \tag{23}$$

Expression (**23**) is equivalent to

$$(2d + a_{-i})(v_{-i})^2 < (2v_{-i} + a_{-i})\left(\frac{d}{2}\right)^2. \tag{24}$$

By substituting v_{-i}, rewrite formula (**24**) as follows:

$$\frac{(2d + a_{-i})(a_i(2a_{-i}(d + a_{-i}) + d^2) + a_{-i}d(2a_{-i} + d))}{4(d + a_i)} - (2da_{-i} + a_{-i}^2)\sqrt{\frac{a_{-i}^2}{4} + \frac{d\left(a_{-i} + a_i + \frac{2}{d}a_i a_{-i}\right)}{2\left(2 + \frac{2}{d}a_i\right)}}$$

$$< \frac{1}{2}d^2\sqrt{\frac{a_{-i}^2}{4} + \frac{d\left(a_{-i} + a_i + \frac{2}{d}a_i a_{-i}\right)}{2\left(2 + \frac{2}{d}a_i\right)}},$$

(25)

which is tantamount to

$$\frac{(2d + a_{-i})(a_i(2a_{-i}(d + a_{-i}) + d^2) + a_{-i}d(2a_{-i} + d))}{4(d + a_i)}$$

(26)

$$< \left((2da_{-i} + a_{-i}^2) + \frac{1}{2}d^2\right)\sqrt{\frac{a_{-i}^2}{4} + \frac{d\left(a_{-i} + a_i + \frac{2}{d}a_i a_{-i}\right)}{2\left(2 + \frac{2}{d}a_i\right)}}.$$

Note that both sides of the inequality are positive, so we can square inequality (26) and after some algebraic manipulations obtain the following equivalent relationship:

$$(a_{-i} + d)(2a_{-i}(a_{-i} + 2d) + d^2)^2(a_i + d)(a_i d + a_{-i}(a_i + d)) > (a_{-i} + 2d)^2(a_i(2a_{-i}(a_{-i} + d) + d^2) + a_{-i}d(2a_{-i} + d))^2,$$

(27)

or, which is the same,

$$(a_{-i} + a_i)d^7 + (5a_{-i}^2 + 3(a_{-i} - a_i)a_i)d^6 + 2a_{-i}(2a_{-i}^2 + (a_{-i} - a_i)(3a_i + 2(a_{-i} + a_i)))d^5 + a_{-i}^2(a_{-i}^2 + (a_{-i} - a_i)(10a_i + 2(a_{-i} + a_i)))d^4 + 4a_{-i}^3(a_{-i} - a_i)a_i d^3 > 0.$$

(28)

Inequality (28) is true since $a_i < a_{-i}$.

∎

b)   Let $a_{-i} = a$. Then, after some manipulations:

$$R_i = \frac{(2d + a_i)\left(a_i - \sqrt{\frac{d + a_i}{d + a}}\sqrt{da + da_i + aa_i}\right)^2}{d^2\sqrt{\frac{d + a_i}{d + a}}\sqrt{da + da_i + aa_i}},$$

$$R_{-i} = \frac{(2d + a)\left(a - \sqrt{\frac{d + a}{d + a_i}}\sqrt{da + da_i + aa_i}\right)^2}{d^2\sqrt{\frac{d + a}{d + a_i}}\sqrt{da + da_i + aa_i}}.$$

In this situation, $R_{-i} < R_i$ if

$$\frac{d + a}{2d + a}\left(\sqrt{\frac{d + a_i}{d + a}}\sqrt{ad + a_i d + aa_i} - a_i\right)^2 > \frac{d + a_i}{2d + a_i}\left(\sqrt{\frac{d + a}{d + a_i}}\sqrt{ad + a_i d + aa_i} - a\right)^2.$$

(29)

The last inequality is obtained by simplifying the terms in the original $R_{-i} < R_i$. Note that if $a_i < a$, then

$$\frac{d + a_i}{2d + a_i} < \frac{d + a}{2d + a}.$$

(30)

Also, if $a_i < a$, then $\frac{a_i}{a_i+d} < \frac{a}{a+d}$, because $g(x) = \frac{x}{x+d}$ is an increasing function by x, with $d > 0$. Multiply both sides by $a - a_i > 0$ to get $\frac{a_i}{a_i+d}(a - a_i) < \frac{a}{a+d}(a - a_i)$, which is equivalent to $a_i\left(\frac{a+d}{a_i+d} - 1\right) < a\left(1 - \frac{a_i+d}{a+d}\right)$. This inequality is tantamount, after some algebra, to:

$$(a - a_i)^2 > \left(\left(\sqrt{\frac{d+a}{d+a_i}} - \sqrt{\frac{d+a_i}{d+a}}\right)\sqrt{da + da_i + aa_i}\right)^2.$$

Because $a - a_i > 0$, which implies that $\sqrt{\frac{d+a}{d+a_i}} - \sqrt{\frac{d+a_i}{d+a}} > 0$, we have $a - a_i > \left(\sqrt{\frac{d+a}{d+a_i}} - \sqrt{\frac{d+a_i}{d+a}}\right)\sqrt{da + da_i + aa_i}$

. Hence, $\left(\sqrt{\frac{d+a_i}{d+a}}\right)\sqrt{da + da_i + aa_i} - a_i > \left(\sqrt{\frac{d+a}{d+a_i}}\right)\sqrt{da + da_i + aa_i} - a$. Note that both sides are positive,

therefore:

$$\left(\sqrt{\frac{d+a_i}{d+a}}\sqrt{ad + a_id + aa_i} - a_i\right)^2 > \left(\sqrt{\frac{d+a}{d+a_i}}\sqrt{ad + a_id + aa_i} - a\right)^2. \tag{31}$$

Inequalities (**30**) and (**31**) combined together imply the desired relationship (**29**).

■

Proof of Proposition 3.4.
As $\quad 4a^6 + 16a^5d + 20a^4d^2 + 8a^3d^3 < 4a^6 + 16a^5d + 20a^4d^2 + 8a^3d^3 + a^2d^4 \quad$ then $\quad \left((2a^2 + 2ad)\sqrt{a(a + 2d)}\right)^2 < (ad^2 + 2a^3 + 4a^2d)^2$. Therefore, $(2a^2 + 2ad)\sqrt{a(a + 2d)} < (ad^2 + 2a^3 + 4a^2d)$, given that both sides of the inequality are

positive. Hence, $\sqrt{a(a + 2d)}\left(a - \sqrt{a(a + 2d)}\right)^2 < ad^2$, which entails that $\frac{\sqrt{a(a+2d)}\cdot\left(a-\sqrt{a(a+2d)}\right)^2}{ad^2} < 1$, and thus,

$$R = \frac{2d + a}{2\left(\sqrt{\frac{a^2}{4} + \frac{ad}{2}}\right)}\left(\frac{2}{d}\left(\frac{-a}{2} + \sqrt{\frac{a^2}{4} + \frac{ad}{2}}\right)\right) = \frac{\sqrt{a(a + 2d)}\left(a - \sqrt{a(a + 2d)}\right)^2}{ad^2}. \tag{32}$$

On account of that, $R < 1$.

■

5.3 Appendix C. Proofs in Situations 2 and 3

Proof of Proposition 3.5. Differentiate R_1 by a_0 to get

$$\frac{\partial R_1}{\partial a_0} = \frac{2d^2(d + a_1)(c - b)^2\Gamma}{\overline{Q}((d + a_0)(d + a_1)(a_0d + a_1d + a_0a_1))^{\frac{3}{2}}(4(c - b) - \overline{Q}(2d + a_1))}. \tag{33}$$

By assumption **A1.2**, $c - b > 0, i = 0,1$; Γ is positive by definition. The expression $4(c - b) - \overline{Q}(2d + a_1)$ is positive. Assumption **A1.3** states $\overline{P} > \max_{i\in\{0,1\}}\{a_i\overline{Q}/2 + b\}$, which is equivalent to $\frac{c-b}{2d+2a_i} > \frac{\overline{Q}}{4}$. Therefore, clearly,

$$\frac{\partial R_1}{\partial a_0} > 0.$$

The proof is complete.

■

5.4 Appendix D. Proof in Situation 4

Proof of Proposition 3.6. Observe that R_i in (19) for $i = 1$ is the same R_1 of (18). *See Proof of Proposition 3.5*

■