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Future-Value Assessment of a Multi-Facility Allocation Problem with Inventory Return Costs

Jonathan David Aguirre Bello, Oscar Flores Carreto,
Ninfa Alejandra Guzmán Velasco, Ricardo Ernesto Morales Guerrero,
Santiago Omar Caballero Morales

[1] Universidad Popular Autónoma del Estado de Puebla A.C., 17 Sur 901, Barrio de Santiago, 72410, Puebla, México

jonathandavid.aguirre@upaep.edu.mx, oscar.flores2@upaep.edu.mx, ninfaalejandra.guzman@upaep.edu.mx, ricardoernesto.morales@upaep.edu.mx, santiagoomar.caballero@upaep.mx

Abstract. Decision models within the logistic field are crucial for the optimization of economic resources throughout the supply chain. In this context, facility allocation is important to achieve an efficient distribution network. The present work extends on the multi-facility allocation problem by integrating the time-dependent costs associated with the dynamic behaviour of inventory management and transportation through a planning horizon. These costs are evaluated by considering their future value in different periods and an integrated cost metric, including distance, supply frequency, and fuel performance. The contribution of this approach leads to support the decision process regarding the viability of the investment required to establish a set of facilities in terms of future costs, how these costs may increase if the decision is delayed, and when the investment is to be compensated by the savings obtained by the solution of the multi-facility allocation problem.

Keywords: multi-facility allocation problem, inventory control, future value, logistics

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1 Introduction

Various efforts have been made to improve supply chain operations and to develop practical tools and models to improve competitiveness. This has implied the use of mathematical models for decision making that directly involves the optimization of resources. Among these models, some contribute to the design of distribution networks by integrating variables such as demographic data, times, capacities and restrictions. Among the most important aspects in solving problems related to distribution networks is that all services and products require efficient delivery to customers and industries [1].

An important aspect of achieving efficient delivery of goods is through the effective facility location-allocation of customers to distribution centers. In general, goods and resources should be allocated to close facilities, distribution centers, and/or customers to avoid setbacks, resulting in high-cost charges and ensuring that raw materials arrive on time [2]. Thus, facilities must be located at the most strategic places to ensure that the allocated companies have the appropriate infrastructure and means of transportation for the delivery of goods and/or services [3, 4]. The strategy must be supported by formal decision models as uncertainty and vagueness directly impact the economic assets of the company [5].

The facility location-allocation problem is frequently solved considering just distances from customers to distribution centers and opening costs. However, the design of a comprehensive supply chain network involves more elements which, if unified, can lead to more significant savings [6, 7]. Particularly, it is important to consider inventory management because it involves transportation and inventory supply costs which are frequently absent from standard facility location-allocation problems.

The issue of managing investment for inventories may have more relevance compared to the decision to invest in the business itself. Good economic management, well-thought-out decision-making and inventory planning are vitally important to avoid financial problems after opening or starting a company's operations. In this context, considering the time-dependent aspect of costs can support the viability of this decision process and evaluate the additional costs of postponing the opening or starting operations.

Thus, the present work extends on the multi-facility allocation problem by integrating the time-dependent costs associated to the dynamic behavior of inventory management and transportation through a planning horizon. The allocation strategy of six distribution centers is performed by considering an integrated cost metric which includes distance from the distribution centers to customers, fuel consumption, appropriate inventory levels required by the allocated customers, the optimal number of transfers, and the investment required to operate the distribution centers. The integrated costs are evaluated by considering their future value in different periods to determine how these costs may increase if the decision is delayed, and when the investment is to be compensated by the savings obtained by the solution of the multi-facility allocation problem.

This work is structured as follows: Section 2 presents the development of the integrated cost metric and the multi-facility allocation model. Then, Section 3 presents its application for a case study. Results are presented in Section 4 with a discussion of future-value assessment. Finally, in Section 5 our conclusions and future work are presented.

2 Integrated Cost Metric

Distribution frequency is determined by the supply requirements of the allocated customers. An inventory control policy frequently determines this. For example, consider the Economic Order Quantity (EOQ) model, which determines the purchase order quantity for replenishment. The main objective is to minimize total inventory costs [8]. The EOQ model considers the variables presented in Table 1.

Table 1. Variables and costs considered by the EOQ model

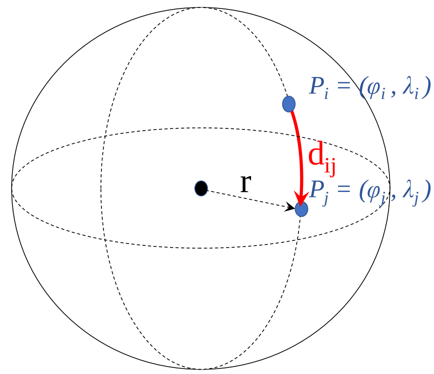
Variable	Description
D	Cumulative demand (units of product) through a planning horizon
S	Order or enlist cost ($ per lot of products)
C	Cost of the unit of product ($ per unit of product)
i	Maintenance rate (% per unit of product)
$H = i \times C$	Maintenance cost ($ per average units of products)
Q	EOQ = Q = sqrt((2×D×S)/H) size of economic lot
R	Reorder point (units of product in inventory)
N	Number of orders through a planning horizon $N = D/Q$
T	Time between each order (Planning Horizon / N)
TC	Total Inventory Management Costs = $(D/Q) \times S + (Q/2) \times H$

For each customer j, through a planning horizon, N_j orders must be filled or supplied. As these orders require transportation, the distribution centre's total trips to a customer are determined as $2 \times N_j$ (inbound+outbound trips). Then, if d_{ij} is defined as the distance in km between the distribution center i and customer j, a transportation fare per km (f) can be applied to determine the transportation cost associated with each inbound/outbound trip. Note that f may include such concepts as: fuel consumption cost associated with the transportation vehicle, drivers' wage, and toll fare.

Finally, the transportation cost between any distribution center i and a customer j through a planning horizon (which is defined by the calculation of D) can be determined as:

$$C_{ij} = 2 \times N_j \times d_{ij} \times f \quad \textbf{(1)}$$

Note that C_{ij} is set in a cost matrix of dimensions $n \times m$ and the values within it depend on the location of the distribution centers if these are part of the decision problem (multi-facility location problem). Also, as d_{ij} is expected to be in km (kilometers), an approximate distance metric such as the spherical arc length metric can be considered. Figure 1 presents the model and calculations for this metric. Here, φ and λ represent the geophysical latitude and longitude respectively in radians of a location, and r is the radius of the Earth which is estimated as 6371 km.



$$d_{ij} = r \times Arcos(sin\varphi_i sin\varphi_j + cos\varphi_i cos\varphi_j \cos(\lambda_i - \lambda_j)$$

Fig. 1. Spherical model of the Earth with arc length metric.

Regarding the future value of this cost to specific periods, this can be estimated by:

$$FV = PV \times (1 + k)^t. \quad \textbf{(2)}$$

Here, PV is the present value of the economic entity, t is the number of future periods, and k is the interest rate for each period. Note that (2) implies a compound interest, leading to the FV increasing exponentially with time [9]. For this case:

$$FC_{ij} = C_{ij} \times (1 + k)^t, \quad \textbf{(3)}$$

where t is the number of periods based on the planning horizon. Thus, if D is estimated annually, then t would be the number of years in the future.

Finally, the objective function for the multi-facility allocation problem model can be defined as:

$$\text{Min } \Sigma^n_{i=1}\Sigma^m_{j=1} X_{ij} \times C_{ij} + \text{Initial Investment}, \quad \textbf{(4)}$$

where X_{ij} is a binary decision variable which is equal to 1 if the customer j is allocated (or assigned) to distribution center i and 0 otherwise, the *Initial Investment* is the economic resource required to open and operate the distribution centers, and n and m are the number of distribution centers and customers respectively. Then, the restrictions for the model are the following:

$$\Sigma^n_{i=1} X_{ij} = 1, \text{ for all } j=1, \dots, m \quad \textbf{(5)}$$

$$X_{ij} \in \{0, 1\} \text{ for all } i=1, \dots, n \text{ and } j=1, \dots, m \quad \textbf{(6)}$$

$$O_i \in \{0, 1\} \text{ for all } i=1, \dots, n \quad \textbf{(7)}$$

3 Application Case

The proposed model was applied to a case study with the following data:

- There are 170 branches (customers) of a company that sells various products ($P_1 - P_{10}$). Figure 2 presents the visualization of the locations of these branches.
- All branches are supplied by a single distribution center at geographical coordinates (-97.84167, 19.13648).
- The costs associated with transportation and inventory management are presented in Table 2.
- The two-week demand of customers for products $P_1 - P_{10}$ is presented in Table 3. Based on the inventory management costs reported in Table 2, the lost quantity Q and the supply frequency for each customer (N_j) through a planning horizon of one year is also reported in Table 3.
- The investment required to open and operate a distribution center is estimated at USD 50,000.
- The annual increase in operating costs is estimated at 7.5% per year.

From data of Table 2, the transportation *f* per km is estimated as 3.5 + 1.2 + 1.0 = $ 5.7 USD. Additionally, as presented in Table 3, the total inventory management costs associated with supplying all customers is estimated as USD 54,864.13. On the other hand, by using Eq. (1), the total transportation cost of supplying all customers from the distribution center located at (-97.84167, 19.13648) is estimated as USD 5,193,684.42.

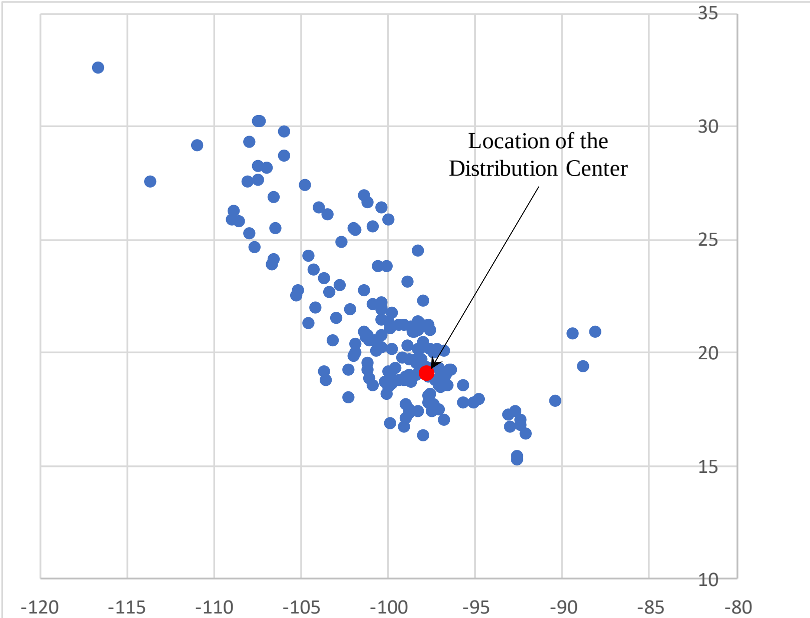


Fig. 2. Location of customers (branches) and current distribution center of the case study.

Table 2. Transportation and inventory management cost variables of the case study.

Variable	Value
Fuel consumption cost	$ 3.5 USD per km
Toll rate	$ 50.0 USD per 50 km
Drivers' wage	$ 1.2 USD per km
S – order cost	$ 30.0 USD per lot
H – maintenance cost	$ 0.1 USD per unit

At this point, these costs represent the baseline to evaluate the outcomes of the proposed approach. As previously presented, six new distribution centers are considered to improve supply to all customers. Based on the customers' location patterns presented in Figure 2, four centres were considered within this region with a clear concentration of branches in the central region. The remaining two centers were considered for the north and south region respectively. In this way, the following geographical coordinates were proposed to locate the six new distribution centers:

- Center 1 at (-106.005266799872, 28.7256313837228)
- Center 2 at (-103.721434054928, 23.2828422409267)
- Center 3 at (-103.162899197525, 20.5931860949597)
- Center 4 at (-99.0422696081225, 18.9716879632598)
- Center 5 at (-97.6283993446507, 21.0445494410414)
- Center 6 at (-90.387700643573, 17.930095196719)

With these centers, the multi-facility allocation problem was solved by using the mathematical formulation defined by Eq. (4) – (7). Here, two solving approaches were considered:

a) Solving based on a nearest-neighbour logic.
b) Solving through mixed linear integer programming (MILP). For this case, Eq. (4) – (7) were implemented with the optimization software Lingo v. 19.0.

Figure 3 presents the allocation of customers to each center based on both approaches. Also, the associated costs of both solutions are presented. Note that both solutions, involve an initial investment of six centers × $ 50,000 USD = $ 300,000.00 USD.

Table 3. Source inventory data and economic lot quantity and supply frequency for customers of the case study.

					two-weeks demands							one-year demand									two-weeks demands							one-year demand			
#	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	$\Sigma^{10}_{s=1} P_s$	D	Q	$N = D/Q$	TC	#	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	$\Sigma^{10}_{s=1} P_s$	D	Q	$N = D/Q$	TC
1	81	81	53	37	110	81	70	134	112	82	841	21866	3622	6.04	362.21	86	84	52	54	45	56	58	31	56	58	31	525	13650	2862	4.77	286.18
2	71	81	58	31	125	100	65	155	120	74	880	22880	3705	6.18	370.51	87	81	73	59	46	54	59	31	54	59	31	547	14222	2921	4.87	292.12
3	77	86	59	31	72	57	32	59	103	120	696	18096	3295	5.49	329.51	88	75	78	56	49	53	54	48	53	54	48	568	14768	2977	4.96	297.67
4	89	89	54	48	86	57	35	53	37	102	650	16900	3184	5.31	318.43	89	50	89	55	48	60	52	46	60	52	46	558	14508	2950	4.92	295.04
5	83	50	52	46	86	56	56	58	31	93	611	15886	3087	5.15	308.73	90	63	80	54	39	122	53	30	54	48	60	603	15678	3067	5.11	306.71
6	66	82	53	30	77	53	69	59	31	86	606	15756	3075	5.12	307.47	91	58	89	52	45	109	56	50	52	46	134	691	17966	3283	5.47	328.32
7	64	73	56	50	70	54	42	54	48	117	628	16328	3130	5.22	313.00	92	53	86	57	43	101	51	39	53	30	59	572	14872	2987	4.98	298.72
8	72	57	51	39	76	57	36	52	46	105	591	15366	3036	5.06	303.64	93	81	89	60	30	89	73	53	56	50	124	705	18330	3316	5.53	331.63
9	86	57	56	37	74	54	59	53	30	72	578	15028	3003	5.00	300.28	94	66	79	59	45	60	86	62	51	39	127	674	17524	3243	5.40	324.26
10	86	56	56	39	53	37	67	56	50	50	550	14300	2929	4.88	292.92	95	60	72	60	35	100	73	65	58	31	56	610	15860	3085	5.14	308.48
11	77	53	54	33	58	31	32	51	39	106	534	13884	2886	4.81	288.62	96	56	74	59	36	130	81	31	59	31	56	613	15938	3092	5.15	309.24
12	70	54	53	49	59	89	92	53	37	87	643	16718	3167	5.28	316.71	97	72	57	60	33	124	93	65	54	48	54	660	17160	3209	5.35	320.87
13	76	57	51	42	54	139	99	91	85	43	737	19162	3391	5.65	339.08	98	86	57	57	34	60	90	52	52	46	53	587	15262	3026	5.04	302.61
14	74	54	56	38	52	89	80	93	88	67	691	17966	3283	5.47	328.32	99	86	56	54	32	135	83	54	53	30	125	708	18408	3323	5.54	332.34
15	84	52	56	49	53	126	83	136	81	40	760	19760	3443	5.74	344.33	100	77	53	52	41	60	99	48	56	53	37	576	14976	2998	5.00	299.76
16	60	80	52	45	56	70	93	56	53	37	602	15652	3065	5.11	306.45	101	70	54	57	30	116	87	49	51	58	31	603	15678	3067	5.11	306.71
17	71	89	52	41	51	114	80	56	58	31	643	16718	3167	5.28	316.71	102	76	57	51	42	99	99	62	131	59	31	707	18382	3321	5.54	332.10
18	89	81	52	39	114	118	97	54	59	31	734	19084	3384	5.64	338.38	103	74	54	53	37	107	84	66	66	54	48	643	16718	3167	5.28	316.71
19	76	86	54	49	130	114	91	53	54	48	755	19630	3432	5.72	343.19	104	84	52	59	32	133	83	40	134	52	46	715	18590	3340	5.57	333.98
20	56	80	54	38	112	81	30	60	52	46	609	15834	3082	5.14	308.23	105	81	74	50	53	37	98	47	157	53	30	680	17680	3257	5.43	325.70
21	62	87	57	50	89	92	30	160	76	40	743	19318	3405	5.67	340.45	106	89	75	60	58	31	86	31	66	56	50	602	15652	3065	5.11	306.45
22	72	57	55	37	139	99	33	121	75	109	797	20722	3526	5.88	352.61	107	80	73	56	59	31	80	37	160	51	39	666	17316	3223	5.37	322.33
23	86	57	58	49	89	80	65	123	97	147	851	22126	3644	6.07	364.36	108	83	80	60	54	48	87	91	85	43	46	677	17602	3250	5.42	324.98
24	86	56	52	35	126	83	40	56	55	105	694	18044	3290	5.48	329.03	109	58	90	51	52	46	88	93	88	67	30	663	17238	3216	5.36	321.60
25	77	53	59	35	70	93	70	56	74	79	666	17316	3223	5.37	322.33	110	58	85	50	53	30	89	136	81	40	50	672	17472	3238	5.40	323.78
26	70	54	51	30	114	80	46	89	92	136	762	19812	3448	5.75	344.78	111	53	37	59	56	50	56	56	53	37	39	496	12896	2782	4.64	278.17
27	76	57	54	43	118	97	59	139	99	140	882	22932	3709	6.18	370.93	112	58	31	53	51	39	56	56	58	31	56	489	12714	2762	4.60	276.20
28	74	54	52	42	91	85	43	89	80	64	674	17524	3243	5.40	324.26	113	59	31	55	32	99	54	54	59	31	53	527	13702	2867	4.78	286.73
29	84	52	53	37	93	88	67	126	83	136	819	21294	3574	5.96	357.44	114	54	48	57	41	106	53	53	54	48	54	568	14768	2977	4.96	297.67
30	88	73	59	42	136	81	40	70	93	64	746	19396	3411	5.69	341.14	115	52	46	57	50	100	81	60	52	46	57	601	15626	3062	5.10	306.20
31	58	88	57	44	56	53	37	114	80	132	719	18694	3349	5.58	334.91	116	53	30	55	42	99	90	43	56	50	54	572	14872	2987	4.98	298.72
32	87	86	54	35	56	58	31	118	97	127	749	19474	3418	5.70	341.82	117	56	50	53	45	107	96	61	51	39	52	610	15860	3085	5.14	308.48
33	89	89	52	44	54	59	31	114	91	129	752	19552	3425	5.71	342.51	118	51	39	52	32	58	93	58	151	62	56	652	16952	3189	5.32	318.92
34	80	87	50	31	53	54	48	53	37	64	557	14482	2948	4.91	294.77	119	70	54	52	34	121	83	53	37	102	56	662	17212	3214	5.36	321.36
35	53	73	50	48	60	52	46	58	31	141	612	15912	3090	5.15	308.99	120	76	57	55	44	129	84	58	31	119	54	707	18382	3321	5.54	332.10
36	65	83	56	40	139	99	39	59	31	148	759	19734	3441	5.73	344.10	121	74	54	60	39	115	98	59	31	98	53	681	17706	3259	5.43	325.94
37	73	87	51	42	89	80	51	54	48	40	615	15990	3097	5.16	309.74	122	84	52	54	48	58	82	54	48	71	101	652	16952	3189	5.32	318.92
38	55	87	60	36	126	83	45	52	46	125	715	18590	3340	5.57	333.98	123	75	88	52	43	120	98	52	46	73	115	762	19812	3448	5.75	344.78
39	89	89	58	44	70	93	44	53	30	111	681	17706	3259	5.43	325.94	124	90	85	60	36	116	84	53	91	85	43	743	19318	3405	5.67	340.45
40	72	79	53	32	114	80	70	56	50	74	680	17680	3257	5.43	325.70	125	52	70	60	32	130	81	56	93	88	67	729	18954	3372	5.62	337.23
41	72	57	57	37	118	97	31	51	39	64	623	16198	3117	5.20	311.75	126	70	82	55	50	58	84	51	136	81	40	707	18382	3321	5.54	332.10
42	86	57	51	43	114	91	50	139	55	148	834	21684	3607	6.01	360.70	127	55	85	50	46	100	96	59	56	53	37	637	16562	3152	5.25	315.23
43	86	56	53	33	59	94	53	53	37	127	651	16926	3187	5.31	318.68	128	63	88	56	53	37	87	47	56	58	31	576	14976	2998	5.00	299.76
44	77	53	51	43	124	85	46	58	31	74	642	16692	3165	5.27	316.47	129	88	81	56	58	31	88	57	54	59	31	603	15678	3067	5.11	306.71
45	70	54	52	46	127	99	45	89	92	108	782	20332	3493	5.82	349.27	130	78	72	58	59	31	97	50	53	54	48	600	15600	3059	5.10	305.94
46	76	57	55	37	64	80	61	139	99	116	784	20384	3497	5.83	349.72	131	66	42	120	54	48	97	61	60	52	46	646	16796	3175	5.29	317.45
47	74	54	51	46	160	89	69	89	80	104	816	21216	3568	5.95	356.79	132	72	31	82	52	46	95	43	80	87	121	709	18434	3326	5.54	332.57
48	84	52	60	35	82	83	31	126	83	64	700	18200	3305	5.51	330.45	133	86	32	74	53	30	85	32	71	108	143	714	18564	3337	5.56	333.74
49	61	81	57	40	74	84	42	70	93	74	676	17576	3247	5.41	324.74	134	86	39	120	56	50	82	58	71	72	57	691	17966	3283	5.47	328.32
50	56	76	57	41	120	95	57	114	80	104	800	20800	3533	5.89	353.27	135	77	50	102	51	39	91	54	72	86	57	679	17654	3255	5.42	325.46
51	71	90	58	44	102	83	56	118	97	37	756	19656	3434	5.72	343.42	136	70	44	93	94	105	95	59	86	86	56	788	20488	3506	5.84	350.61
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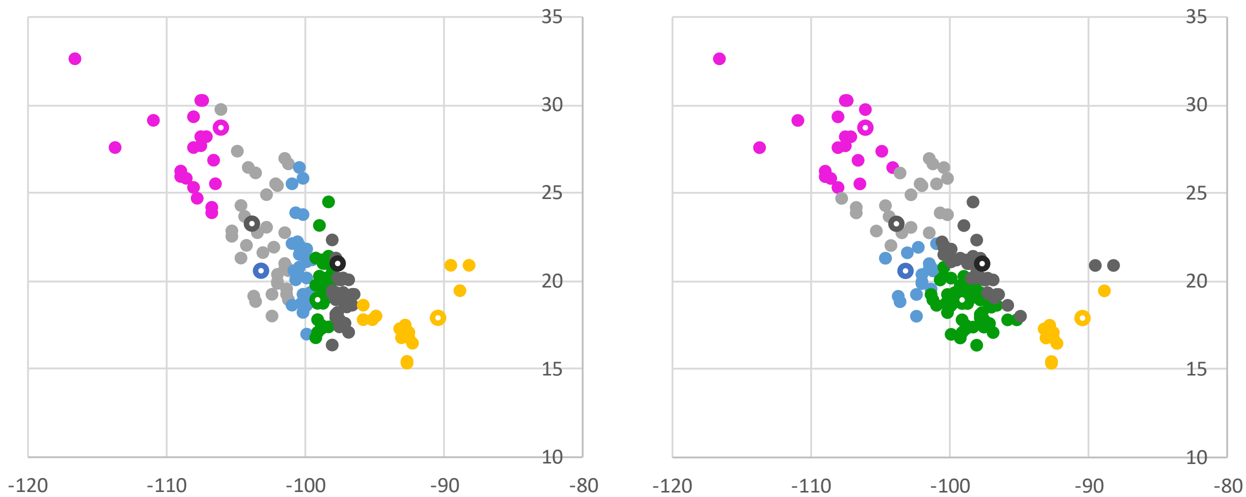
53	89	72	60	50	139	99	50	117	59	31	766	19916	3457	5.76	345.68	138	74	36	117	95	108	94	74	77	70	54	799	20774	3530	5.88	353.05
54	50	70	56	30	89	80	68	105	54	48	650	16900	3184	5.31	318.43	139	84	43	105	89	111	89	120	70	76	57	844	21944	3629	6.05	362.86
55	82	74	51	46	126	83	38	72	52	46	670	17420	3233	5.39	323.30	140	67	73	50	34	119	90	102	76	74	54	739	19214	3395	5.66	339.53
56	84	79	55	36	70	93	36	50	53	30	586	15236	3024	5.04	302.35	141	86	90	57	50	112	100	93	91	85	43	807	20982	3548	5.91	354.81
57	73	72	57	35	114	80	49	106	56	50	692	17992	3286	5.48	328.56	142	61	80	55	31	122	89	86	93	88	67	772	20072	3470	5.78	347.03
58	53	72	53	47	118	97	32	87	51	39	649	16874	3182	5.30	318.19	143	56	73	55	50	58	100	117	136	81	40	766	19916	3457	5.76	345.68
59	53	87	58	42	114	91	60	63	110	64	742	19292	3402	5.67	340.22	144	74	73	56	30	129	88	105	56	53	37	701	18226	3307	5.51	330.69
60	84	71	57	50	63	80	51	92	91	141	780	20280	3488	5.81	348.83	145	61	76	59	50	108	99	72	56	58	31	670	17420	3233	5.39	323.30
61	72	57	50	48	92	83	53	37	54	124	670	17420	3233	5.39	323.30	146	82	81	51	34	58	95	50	54	59	31	595	15470	3047	5.08	304.66
62	86	57	60	31	91	80	58	31	115	103	712	18512	3333	5.55	333.27	147	83	71	52	31	82	83	106	53	54	48	663	17238	3216	5.36	321.60
63	86	56	56	33	93	53	59	31	60	114	641	16666	3162	5.27	316.22	148	90	72	55	37	74	80	87	60	52	46	653	16978	3192	5.32	319.17
64	77	53	57	33	64	58	54	48	119	64	627	16302	3127	5.21	312.75	149	51	77	58	41	53	37	63	127	86	53	646	16796	3175	5.29	317.45
65	70	54	51	88	114	59	52	46	105	88	727	18902	3368	5.61	336.77	150	88	42	120	82	58	31	92	132	117	89	851	22126	3644	6.07	364.36
66	76	57	55	81	140	54	53	30	111	81	738	19188	3393	5.66	339.31	151	69	31	82	92	59	31	91	53	37	135	680	17680	3257	5.43	325.70
67	74	54	58	72	136	52	56	50	92	72	716	18616	3342	5.57	334.21	152	60	32	74	100	54	48	93	58	31	155	705	18330	3316	5.53	331.63
68	84	52	59	53	37	53	51	39	51	72	551	14326	2932	4.89	293.18	153	55	39	120	89	52	46	54	59	31	120	665	17290	3221	5.37	322.09
69	72	88	50	58	31	56	50	56	71	79	611	15886	3087	5.15	308.73	154	59	50	102	86	53	30	60	54	48	128	670	17420	3233	5.39	323.30
70	86	77	56	59	31	51	39	54	68	74	595	15470	3047	5.08	304.66	155	73	44	93	94	56	50	54	52	46	57	619	16094	3107	5.18	310.75
71	59	76	54	54	48	99	39	53	64	80	626	16276	3125	5.21	312.50	156	72	42	86	80	51	39	44	53	30	57	554	14404	2940	4.90	293.98
72	76	83	52	52	46	81	57	53	37	71	608	15808	3080	5.13	307.97	157	86	36	117	95	120	89	64	56	50	56	769	19994	3464	5.77	346.36
73	67	72	53	53	30	88	68	58	31	71	591	15366	3036	5.06	303.64	158	86	43	105	89	102	86	44	51	39	53	698	18148	3300	5.50	329.98
74	56	53	37	56	50	93	53	59	31	79	567	14742	2974	4.96	297.41	159	77	53	57	44	93	94	57	86	70	54	685	17810	3269	5.45	326.89
75	73	58	31	51	39	97	58	54	48	73	582	15132	3013	5.02	301.32	160	70	54	57	42	86	80	63	86	76	57	671	17446	3235	5.39	323.54
76	72	59	31	86	72	98	59	52	46	86	661	17186	3211	5.35	321.12	161	76	57	56	36	117	95	64	77	74	54	706	18356	3319	5.53	331.87
77	74	54	48	73	50	89	54	53	30	73	598	15548	3054	5.09	305.43	162	74	54	53	43	53	37	35	91	85	43	568	14768	2977	4.96	297.67
78	90	52	46	40	106	97	52	56	50	138	727	18902	3368	5.61	336.77	163	84	52	54	37	58	31	69	93	88	67	633	16458	3142	5.24	314.24
79	72	53	30	34	87	84	53	51	39	89	592	15392	3039	5.06	303.89	164	52	76	57	42	59	31	60	136	81	40	634	16484	3145	5.24	314.49
80	86	56	50	48	63	96	56	50	92	127	724	18824	3361	5.60	336.07	165	76	74	54	40	54	48	63	56	53	37	555	14430	2942	4.90	294.24
81	86	51	39	44	92	85	51	39	58	89	634	16484	3145	5.24	314.49	166	57	84	52	45	52	46	54	56	58	31	535	13910	2889	4.81	288.89
82	77	53	55	34	91	85	43	91	85	43	657	17082	3201	5.34	320.14	167	78	79	57	47	53	30	43	54	59	31	531	13806	2878	4.80	287.81
83	70	54	52	37	93	88	67	93	88	67	709	18434	3326	5.54	332.57	168	63	84	54	37	56	50	52	53	54	48	551	14326	2932	4.89	293.18
84	76	57	52	45	136	81	40	136	81	40	744	19344	3407	5.68	340.68	169	68	74	52	41	51	39	30	60	52	46	513	13338	2829	4.71	282.89
85	74	54	56	38	56	53	37	56	53	37	514	13364	2832	4.72	283.17	170	84	73	58	41	93	80	70	158	50	89	796	20696	3524	5.87	352.39
																														Total =	**54864.13**



$ 3,071,705.56 USD

$ 2,209,228.93 USD

(a) (b)

Fig. 3. Multi-facility allocation with (a) nearest neighbour logic, and (b) optimization of Eq. (4) – (7) through Lingo.

The results presented in Figure 3 corroborate the suitability of the mathematical model defined by Eq. (4) – (7) and the use of MILP. Finally, the future values of these costs and the expected savings are considered to assess the pertinence of this decision. Table 4 presents the future value of C_{ij} through a period of 15 years for the following scenarios: BS (baseline with one distribution center), NN (allocation with nearest neighbour logic and six centers), and MLP (optimal allocation with MILP and six centers). The initial investment (*InitInv*) due to the infrastructure required to open the new six distribution centers is also included. As presented, if the decision of opening the six distribution centers is delayed, may increase from $2,209,200 at year 0 to $6,536,800 at year 15. The analysis presented in Table 5 shows that, within the first year, *InitInv* can be compensated by the savings obtained with six centers, independent of the allocation's optimality.

Table 4. Future value of C_{ij} through a period of 15 years.

t	0	1	2	3	4	5	6	7
BS	$5,193,700	$5,583,200	$6,002,000	$6,452,100	$6,936,000	$7,456,200	$8,015,400	$8,616,600
NN	$3,071,700	$3,302,100	$3,549,700	$3,816,000	$4,102,200	$4,409,800	$4,740,600	$5,096,100
MLP	$2,209,200	$2,374,900	$2,553,000	$2,744,500	$2,950,400	$3,171,600	$3,409,500	$3,665,200
InitInv	$300,000	$322,500	$346,688	$372,689	$400,641	$430,689	$462,990	$497,715

t	8	9	10	11	12	13	14	15
BS	$9,262,800	$9,957,500	$10,704,000	$11,507,000	$12,370,000	$13,298,000	$14,295,000	$15,367,000
NN	$5,478,300	$5,889,200	$6,330,900	$6,805,700	$7,316,100	$7,864,800	$8,454,700	$9,088,800
MLP	$3,940,100	$4,235,600	$4,553,300	$4,894,800	$5,261,900	$5,656,500	$6,080,800	$6,536,800
InitInv	$535,043	$575,172	$618,309	$664,683	$714,534	$768,124	$825,733	$887,663

Table 5. Future value of savings through a period of 15 years.

t	0	1	2	3	4	5	6	7
BS	-	-	-	-	-	-	-	-
NN	2,122,000	2,281,100	2,452,200	2,636,100	2,833,800	3,046,400	3,274,900	3,520,500
MLP	2,984,500	3,208,300	3,448,900	3,707,600	3,985,600	4,284,600	4,605,900	4,951,400

t	8	9	10	11	12	13	14	15
BS	-	-	-	-	-	-	-	-
NN	3,784,500	4,068,300	4,373,500	4,701,500	5,054,100	5,433,100	5,840,600	6,278,700
MLP	5,322,700	5,721,900	6,151,100	6,612,400	7,108,300	7,641,400	8,214,500	8,830,600

4 Conclusions and Future Work

The effective delivery of items throughout the supply chain is crucial for the production process. Likewise, the costs associated with the delivery of raw materials and inventory costs are crucial to maintain the company with a high level of competitiveness. In this context, the proposed multi-facility allocation model with future value assessment can provide companies with useful

decision-making resources when important economic resources are compromised. Under this scenario, the development of comprehensive cost metrics and MILP models can provide effective solutions to reduce costs and improve profits.
Allocation is an important aspect of distribution services. It must be studied considering the dynamic behavior of costs because an optimal solution in the present may not be economically sustainable. Also, economic losses due to delayed decisions may reduce the potential economic benefits of the planned actions or investments.
As future work, the present work can be extended on the following points:

- Develop a capacitated multi-facility location-allocation model with a periodic vehicle routing scheme (CMFLAwPVRP).
- Improve the cost metric with additional aspects such as asymmetric distances and delays.
- Design a metaheuristic to solve the CMFLAwPVRP model.
- Consider future value with variable interest rates.

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