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Decomposition Based on Models that use Bandwidth Minimization Heuristics in Matrices for Scheduling Problems

Mayra S. Hernández G.[1], Federico Gamboa R.[1], Nelson Rangel V. [2], Laura Cruz R.[1], Claudia G. Gómez S.[1], Daniel A. Martínez V.[1]
[1]Tecnológico Nacional de México, Instituto Tecnológico de Ciudad Madero, [2] Catedrático CONACyT, División de Estudios de Posgrado e Investigación, Tecnológico Nacional de México, Instituto Tecnológico de Ciudad Madero

selenehdz@live.com.mx, fgamboa91@hotmail.com, nrangelva@conacyt.mx, lauracruzreyes@itcm.edu.mx, cggs71@hotmail.com, adalbertovega@gmail.com

Abstract. Scheduling Activity (SA) is a highly complex optimization problem that is relevant nowdays, both in the private sector and in the public sector. In state of the art are reported several approaches for SA. However, the number of jobs that involve decomposition strategies in their solution is very limited. According to the literature, the decomposition of a problem favors the efficient use of computing resources, improving execution time and reaching competitive solutions compared to those reported in the literature. This paper proposes a new model-based decomposition method, which uses bandwidth minimization algorithms (BMA) in scattered matrices for the generation of angular structures in linear programming models. The study seeks to demonstrate that, depending on the BMA, there is a difference in the results obtained by the proposed method. The results obtained from the proposed experiment demonstrate that there are differences in the decomposition obtained by the method, using two particular heuristics that minimize the bandwidth in scattered matrices. The differences are regarding the reduction in the number of additional variables generated, when the bandwidth is lower. This fact suggests to look for better BMA, and to compare its effects under the proposed method, against other strategies of decomposition.

1. Introduction

The problem of scheduling activities (denoted PSA) is present in almost all public or private organizations, being one of the most common and difficult to solve, because it seeks to allocate diverse activities and resources in a limited space of time [1]. The scheduling can be applied in many problems such as project portfolio [2], parts production lines [3], personnel management and resources [4] [5] [6], among others.

A strategy used to solve problems considered difficult, such as PSA, are decomposition methods [7]. The decomposition based on optimization models has the objective of solving difficult problems (from 100 to 10,000 variables and constraints) and allows the identification of the structures implicit in the mathematical model that can satisfy the high-level design requirements, such as concurrency, modularity, and robustness, as well as the availability of computational resources.

In this article, we propose to design a method to construct an angular matrix equivalent to the constraint matrix that models an instance of the PSA, to decompose the problem into a set of simpler subproblems involving a small number of additional variables in his construction. To achieve this, it is proposed to use heuristics of bandwidth minimization as part of the method in scattered matrices. The research focuses on proving that depending on the heuristics there maybe a difference in the quality of the angular matrix produced. The method was validated using instances of the PSA created by a random generator provided in the literature; the results corroborate that significant differences exist depending on the selected heuristic, and that this impacts on the amount of additional variables needed.

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This document is organized as follows. Section 2 presents basic concepts related to the research topic. Section 3 presents the contribution of this work, which is the method of construction of the angular matrix based on heuristics of bandwidth minimization. Section 4 shows the experimental design used to validate the proposed method, and the results obtained. Finally, Section 5 contains the conclusions reached from this research paper.

2. Conceptual Framework

2.1 Decomposition Techniques

The algorithms of decomposition apply the methodology of divide and conquer. This methodology has the objective of subdividing the problem into parts for its solution. In the literature, we can find different decomposition strategies such as Benders, Dantzig-Wolfe, Lagrange, among others.

Decomposition of Benders

Benders in [8], proposes to separate in subproblems the decisions made in different stages. This requires that decisions at one stage only depend on the consequences of the decisions taken in the previous stage. With this decomposition, a problem is posed by each stage, and that problem includes both the part corresponding to the stage itself and the part that links that stage to the decisions made in the previous stage.

Decomposition of Dantzig-Wolfe

The Dantzig-Wolfe method [9] is also known as dual decomposition since the master problem sends dual variables to the subproblems. It is often used when a set of constraints make it difficult to solve the problem. The algorithm relaxes the constraints facilitating the solution of the problem. In this algorithm, the subproblem generates a lower bound and the master problem generates an upper bound, if these are significantly equal, it is said that the algorithm has achieved convergence.

Lagrangian decomposition

Lagrangian decomposition [10] refers to the way to partition the originally disordered incidence matrix into a well-ordered matrix known as the block angular matrix in which each block indicates a subproblem of the original problem. In general, there are two patterns of decomposition: ideal and based on coordination.

The ideal decomposition assumes that the incidence matrix can be perfectly decomposed into independent problems in decoupled blocks without interactions between them as shown in figure 1. For this reason, the present work looks for different alternatives to be able to contribute to the solution of this problem.

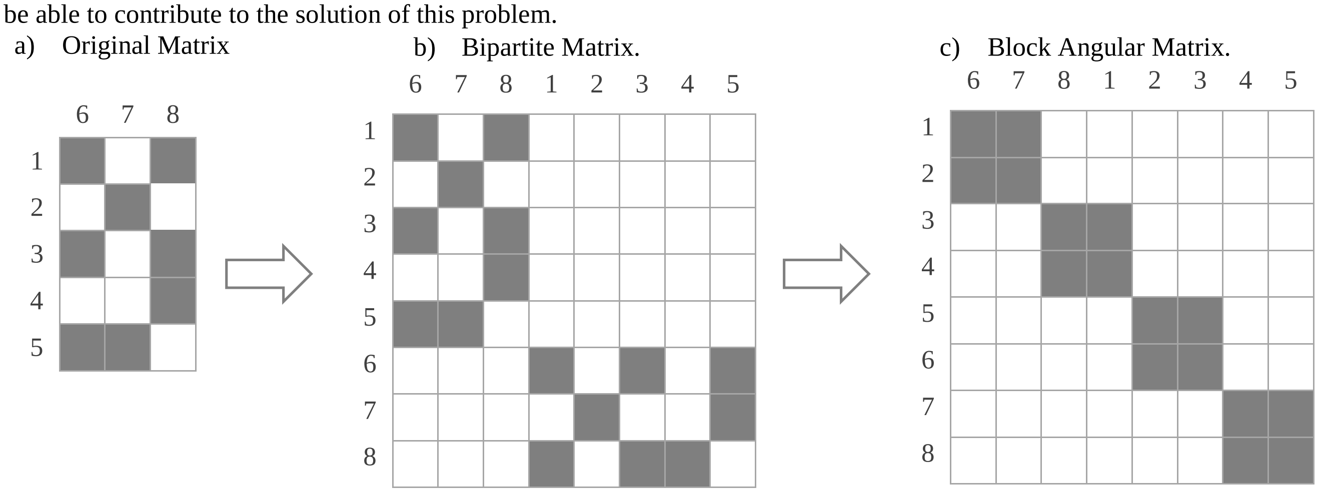


Figure 1. Scatter Matrix Decomposition.

2.2 Heuristics for Bandwidth Minimization in Scattered Matrices

The heuristics that served as the basis for the method proposed in this work were taken from the scientific literature. These algorithms are classical to approach the bandwidth minimization problem in sparse matrices; the problem consists of finding the permutation of rows and columns that allow the non-zero elements be as close as possible to the main diagonal of the matrix [11]. The heuristics are described below.

The Reverse Cuthill-McKee algorithm, or RCM [12], is a well-known algorithm used to reduce the bandwidth of scattered matrices and makes use of level structures. A level structure is a partition of a graph $G = (V, E)$ at levels $L_1, L_2, \dots, L_k$ subject to the neighbors $N(v)$ of a node v; these are assigned to the same level of v, or at contiguous levels.

RCM makes use of the level structure to reduce the bandwidth of a given graph $G = (V, E)$, based on four steps: a) Generates a level structure that has as root each vertex $v \in V$ with a degree $\Delta(v) \leq k$, where k can be a fixed value or a parameter of the algorithm; b) for each level structure generated in the first step, RCM numbers each node according to the level of its already numbered neighbors, and according to its degree (the ties are decided arbitrarily); c) RCM chooses the level structure that produces the lowest bandwidth for the graph G, according to the numbering of nodes developed in step number two; Finally, d) The enumeration of the level structure chosen in the previous steps is reverted, in the sense that a node v with label i is assigned to the label $n - i + 1$.

Gibbs, Poole, and Stockmeyer developed an algorithm, called GPS [13], to reduce the bandwidth in a scattered matrix; this algorithm is also based on level structures. The algorithm was initially created as a two-level structure using the path that describes the diameter of the graph instance. The initial and final nodes, which define the diameter of the graph, were assigned to the first level L_1 of each structure. The rest of the nodes are organized in such a way that the adjacent vertices are at the same level or at a contiguous level. The combination of the levels of the structure results in a new level in which the nodes are tagged according to their level and their degree.

3. Solution Proposal

When optimization problems of great size that cannot be solved easily in the available equipment are approached, decomposition techniques are often used, which allow to fragment the problem and to coordinate the solution of its subproblems to solve the complete problem [7].

According to the above, a new model-based decomposition method is proposed. This method is based on the use of heuristics that minimize bandwidth in sparse matrices. The overall presentation of its steps is shown in Figure 2, as well as its incorporation into a more general problem-solving strategy.

The proposed decomposition method is defined in six basic steps:

Step 1. *Modeling Problem.* This step consists of identifying an Integer Linear Programming (or ILP) model that solves the problem studied, e.g., Nadehri's [2] or Chen [1] model for PSA.

Step 2. *Generate Adjacency Matrix of the Model.* This activity focuses on expanding the ILP model identified in the previous step, for a particular instance. As a result, we will obtain the coefficient matrix that describes it, which is known as the Adjacency Matrix (see Fig. 1.a).

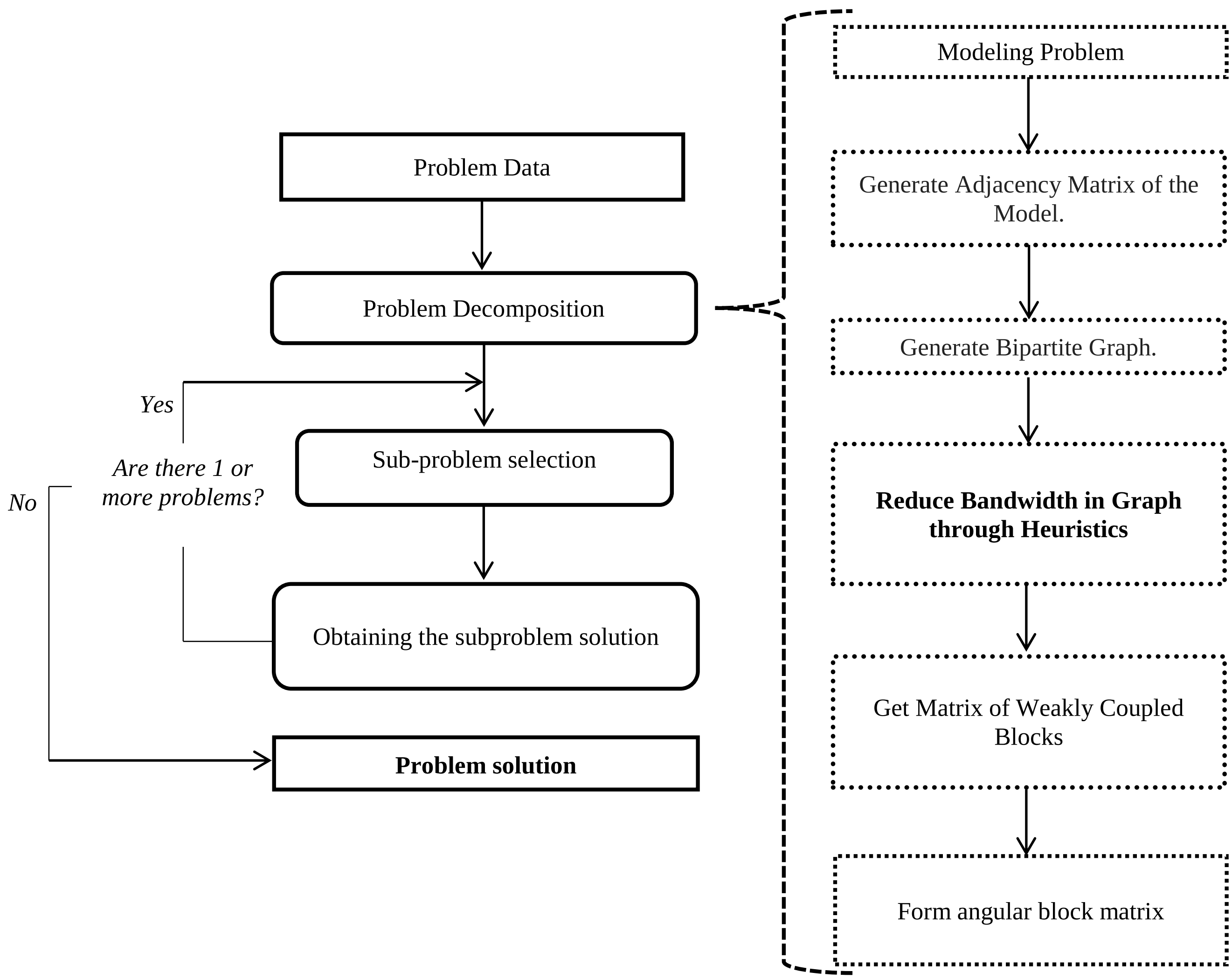
Step 3. *Generate Bipartite Graph*. This step converts the matrix, possibly sparse and represents an instance of the problem in the mathematical model studied, in a bipartite symmetric graph (see Fig. 1.b).

Step 4. *Reduce Bandwidth.* This activity involves usindescrig heuristics that minimize the bandwidth in the graph obtained from the previous step. As a result, it is obtained a permutation of rows and columns, and the bandwidth of the matrix (see Fig. 1.c).

Step 5. *Obtain Matrix of Weakly Coupled Blocks.* This step will take the permutation obtained in the previous step, and will use it to swap rows and columns of the Adjacency Matrix of the Model obtained in step 2. The resulting matrix will be known as Weakly Coupled Block Matrix and will be processed in the next step.

Step 6. *Form Angular Blocks Matrix*. This last activity will take as input the Weakly Coupled Blocks matrix obtained in the previous step, and will form angular blocks by grouping constraints according to the bandwidth found by the heuristic used. For this, the symmetric graph is grouped in blocks of a size equal to the bandwidth, and then each node that is associated with a constraint in the original matrix will be part of that same block. In this step, the quality of the decomposition method is measured by identifying the number of blocks, and the number of additional variables, according to Chen [10], which would be required for transformation to an angular block matrix.

Figure 2. Model-based decomposition method, defined through bandwidth minimization algorithms, for the generation of an angular block matrix.



4. Experimentation

To validate the proposed decomposition method, the following experimental design is proposed. For this experiment, the heuristics were implemented in C language, and the instances were solved in a machine with the following specifications: laptop with Intel Core i7 at 2.6 GHz (6th generation), 16 GB RAM, and under the Windows Operating System 10.

According to the proposed decomposition method, the instances must be represented by a mathematical model, which in this work will be Chen's model [1], for its versatility and generality of application to various problems, including PSA. The instances considered were random, created through the generator proposed by Naderi [2]. Its characteristics include, number of projects $|P|$, number of activities per project $|A|$, and the planning horizon T.

Table 1. Test instances.

a) Original Characteristics according to Chen´s Model [4].

	Characteristics				
Instance	**|*P*|**	**|*A*|**	***T***	**|*V*|**	**|*R*|**
I-6-3-30	6	3	30	726	96
I-6-5-35	6	5	35	1266	200
I-8-3-35	8	3	35	1128	116
I-8-5-45	8	5	45	2168	255
I-10-3-45	10	3	45	1810	147
I-10-5-60	10	5	60	3610	286

b) Characteristics of the symmetrical graph.

	Characteristics	
Instance	**|*N*|**	**|*M*|**
I-6-3-30	642	3194
I-6-5-35	1256	6344
I-8-3-35	964	5341
I-8-5-45	2065	12020
I-10-3-45	1507	7927
I-10-5-60	3296	20601

Following the methodology, the next step was to model the instance as an ILP, using the model proposed by Chen [1]. As a result, the number of variables $|V|$, and constraints $|R|$, involved in the models are shown in Table 1.a, columns 5 and 6, respectively. Then, the adjacency matrix was converted into a symmetric graph; the number of nodes and edges, $|N|$ and $|M|$ respectively, of the resulting graphs per instance are shown in Table 1.b.

Once the graphs of the PSA instances are defined, we proceeded to minimize the bandwidth (denoted by Bw) using the RCM and GPS heuristics; these are two state-of-the-art algorithms recognized for their simplicity and performance in Bw in graphs. Table 2.a shows the bandwidth obtained for each instance.

Table 2. Summary Results.

a) Bw Bandwidths

Instance	**GPS**	**RCM**	**Optimal**
I-6-3-30	310	308	14837
I-6-5-35	708	710	8911
I-8-3-35	503	493	19686
I-8-5-45	1273	1264	14675
I-10-3-45	826	824	21367
I-10-5-60	2176	2176	18306

b) Evaluation of the quality of the angular structure

GPS			RCM		
B1	**B2**	**Var.**	**B1**	**B2**	**Var.**
$\|R_1\|$	**$\|R_2\|$**	**$\|V^+\|$**	**$\|R_1\|$**	**$\|R_2\|$**	**$\|V^+\|$**
66	30	300	6	90	180
150	50	700	74	126	553
74	42	490	9	107	315
185	70	1259	75	180	786
93	54	810	13	134	581

Then, since the objective is to compare the existing difference between RCM and GPS heuristics, we proceed to exchange the resulting matrix of Chen´s model [1] and evaluate their differences. To do this, the permutations obtained through the heuristics RCM and GPS were used to reorganize the graphs of the instances, and the original adjacency matrices; producing weakly coupled blocks matrices.

These matrices are formed by blocks of size Bw with common variables among them. Finally, those nodes that are associated exclusively with constraints in the original adjacency matrix are identified from the blocks, and their clusters will represent separable sets that will serve as the basis for the definition of an angular block matrix. The quality of the angular structure of the matrix obtained by this procedure is measured, for each permutation obtained by the heuristics used, according to the number of blocks obtained, and to the number of additional variables required for the original model to become angular. The results of this experiment are shown in Table 2.b; In this table, for each instance and each GPS and RCM strategy the blocks $|B_1|$, ..., $|B_n|$ obtained, the number of constraints $|R_i|$ grouped by block, and the number of additional variables $|V^+|$ derived from this block reorganization are reported.

It can be observed, from the results, that RCM obtained a better bandwidth than GPS in all the considered instances; additionally, both RCM and GPS obtained two blocks from the obtained bandwidth, which implies the division of the initial problem into two subproblems, and that to achieve this purpose, RCM used a smaller number of additional variables in all cases. With this, we conclude that the bandwidth reduction in the adjacency matrix generated by ILP mathematical models impacts on its redefinition as an angular block matrix, and that it is possible to find permutations that increase the number of blocks (independent subproblems), And reduce the number of additional variables required to obtain such a structure.

5. Final Comments

This paper presents a study of the impact of heuristics to minimize bandwidth in graphs, or sparse matrices, in the decomposition of complex optimization problems. As a result, a model-based decomposition method was proposed, employing bandwidth minimization algorithms. According to the proposed experimentation, it was observed that the permutations produced by different heuristics impact on the quality of the angular block matrix created during the decomposition, and that in the particular case of RCM and GPS heuristics, the first one had a better performance considering the independent blocks generated, and the number of additional variables required.

It is important to mention that this work opens a new research line that involves studying more efficient strategies for the bandwidth minimization problem, and its effect on the construction of angular block matrices and evaluate the generality of the proposal with different models of mathematical programming of other optimization problems.

As future work we intend to improve the quality of the angular matrix, with the intention of obtaining a greater number of blocks facilitating the application of decomposition strategies that help to reduce the problem in subproblems with less complexity. Improving the condition of the matrix and applying decomposition strategies will solve the problem with parallel methods in a natural way.

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References

1 Chen, J., & Askin, R. G.. Project selection, scheduling and resource allocation with time dependent returns. European Journal of Operati Research, 193(1), 23-34 (2009).
2 Ghahremani, P., & Naderi, B.. Solution algorithms for the project selection and scheduling problem with resource constraints and time dependent returns. International Journal of Industrial and Systems Engineering, 19(3), 348-363 (2015).
3 Giraldo, J. A., Castrillón, O. D., & Sarache, W. A.. Generador de modelos de simulación. Aplicaciones en scheduling. Revista de Matemática: Teoría y Aplicaciones, 20(2), 231-242 (2013).
4 Ribeiro, C. C.. Sports scheduling: Problems and applications. International Transactions in Operational Research, 19(1-2), 201-226 (2012).
5 Yildiz, H.. Methodologies and applications for scheduling, routing & related problems. ProQuest (2008).
6 Novoa-Hernández, P.. Optimización evolutiva multi-objetivo en la planificación de controles a clase en la educación superior cubana. Computación y Sistemas, 19(2), 321-335 (2015).
7 Canteli, J. M. L.. Optimización bajo incertidumbre. Técnicas de descomposición y aplicación en Grid. In anales de mecánica y electricidad (2008).
8 Baena Mirabete, D.. Aplicación del método de Benders para ajuste controlado de tablas (CTA) (2008).
9 Ramos, A., & Cerisola, S.. Optimización estocástica. Pontificia Universidad COMILLAS de Madrid (2016).
10 Chen, L., Ding, Z., & Li, S.. A formal two-phase method for decomposition of complex design problems. Journal of Mechanical Design,127(2), 184-195 (2005).
11 Lim, A., Rodrigues, B., & Xiao, F.. Heuristics for matrix bandwidth reduction. European Journal of Operational Research, 174(1), 69-91. (2006).
12 Cuthill, E., & McKee, J.. Reducing the bandwidth of sparse symmetric matrices. In Proceedings of the 1969 24th national conference (pp. 157-172). ACM (1969).
13 Gibbs, N. E., Poole, Jr, W. G., & Stockmeyer, P. K.. An algorithm for reducing the bandwidth and profile of a sparse matrix. SIAM Journal on Numerical Analysis, 13(2), 236-250 (1976).