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Interval Type-3 Fuzzy-Fractal Approach for Arrhythmia Classification

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Abstract. In this paper a combination of interval type-3 fuzzy logic and the fractal dimension (FD) is employed for arrhythmia classification. FD is utilized to characterize the geometrical properties in the electrocardiogram (ECG) of a patient. The fractal dimension is a good representative of an ECG signal, as is able to model its complexity. A type-3 fuzzy system is used to convey the knowledge in image classification. Type-3 helps in capturing the uncertainty in medical classification. A new technique is introduced for arrhythmia classification using the FD. Results with the MIT dataset show the effectiveness of the proposal. In addition, type-3 results were shown to be statistically better than the ones obtained by type-2 and type-1. A limitation of this study is that arrhythmia classification is performed in an offline fashion, meaning that information of patients is contained in a dataset and we process the information in the computer, which is not real-world scenario in this case. Future work could be to optimize parameters for enhancing even further the performance. Also, a hardware implementation would help in making this approach closer to the real persons, as arrhythmias could be classified in a quicker fashion.
Keywords: Type-3 Fuzzy Logic, Fractals, Arrhythmia Classification.

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1 Introduction

It is well-known that fuzzy systems achieve amazing applications in many areas. Originally, fuzzy sets (now called type-1) were put forward in (Zadeh, 1965). Later fuzzy logic and fuzzy systems were also proposed in (Zadeh, 1989), and many applications follow, mainly in control (Cazarez-Castro et al., 2010; Castillo et al., 2020). Fuzzy Logic has evolved from the original works of Zadeh with type-1 fuzzy theory (Zadeh, 1965; Zadeh, 1989), to later the developments of type-2 and its applications (Cazarez-Castro et al., 2012; Mendel & John, 2002; Melin & Castillo, 2004; Castro et al., 2008), to now where type-3 and its applications are surging as a novel promissory area (Castillo et al., 2022; Castillo & Melin, 2022; Qasem et al., 2021; Mohammadzadeh et al., 2020). In addition, type-3 has been employed, with relative success, to handle uncertainty from sensor measurements, like in control (Liu et al., 2021; Singh et al., 2021; Wang et al., 2021; Alattas et al., 2021; Cao et al. 2021).

There have been some previous works applying the fractal dimension (FD) to diagnosis in diverse medical problems, as in (Chakravarty & Chakraborty, 2017; Hadzieva et al., 2016; Hadzieva et al., 2017; Lennon et al., 2015; Macgillivray et al., 2007; Lupo et al., 2016; Lv et al., 2009). Also, there exist some works using fuzzy logic in medical diagnosis, like in (Alpar, 2023; Alpar et al., 2022). Regarding in particular the case of Arrhythmia Classification, there have been some previous works, but not using fuzzy logic, as can be seen in (Sedielmaci & Reguig, 2013; Castiglioni et al., 2014; Oweis &Hijazi, 2006; Spasić et al., 2012; Pan & Tompkins, 1985; Mary & Singh, 2013). With this fact in mind, we offer a type-3 fuzzy-fractal approach for Arrhythmia Classification, which can be viewed as the contribution of this work. A dataset of Arrhythmia Images (Mary & Singh, 2013; Kiani & Maghsoudi, 2019) was used and the approach exhibits promissory results. A comparison versus type-2 and type-1 highlights the potential of the type-3 approach. The hardware implementation of the approach would make it usable in the real world, but remains as future work. However, for the moment this is the first step in achieving better results in the Arrhythmia classification problem. Finally, other kinds of fuzzy models could be applied, such as Fermatean, Pythagorean or hesitant fuzzy approaches are interesting alternatives.

The rest of the paper is: Section 2 offers type-3 concepts, in Section 3 the FD concept is offered, in Section 4 the type-1 fuzzy method is delineated, in Section 5 we offer the type-2 approach, in Section 6 the type-3 approach is postulated, in Section 6 results and comparisons are presented, and in Section 7 conclusions and future works are given.

2 Type-2 and Type-3 Fuzzy Sets

A General type-2 fuzzy set $(\tilde{A})$ comprises the primary variable x with domain X, and secondary variable u with domain J_x^u. The secondary membership grade $\mu_{\tilde{A}}(x,u)$ is a 3D membership function (MF) where $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$ (Mendel & John, 2002). It is expressed by (1)

$$\tilde{A} = \{((x,u),\mu_{\tilde{A}}(x,u)) | \forall x \in X, \forall u \in J_x^u \subseteq [0,1]\}. \tag{1}$$

The footprint of uncertainty (FOU) of $(\tilde{A})$ is the support of $\mu_{\tilde{A}}(x,u)$ and is given by (2)

$$FOU(\tilde{A}) = \{(x,u) \in X \times [0,1] | \mu_{\tilde{A}}(x,u) > 0\}. \tag{2}$$

If we assume that the secondary MF is fixed to 1, then we get reduced model that is called an interval type-2 set. The other concepts, like relations, implication and fuzzy models, can be build based on these definitions.

A type-3 fuzzy set (T3 FS) (Castillo et al., 2022), denoted by $A^{(3)}$, is represented by the membership function (MF) of $A^{(3)}$, in the Cartesian product $X \times [0,1] \times [0,1]$ in $[0,1]$, where X is the primary variable universe of $A^{(3)}$, x. The $\mu_{A^{(3)}}$ is a type-3 MF (T3 MF):

$$\mu_{A^{(3)}}: X \times [0,1] \times [0,1] \rightarrow [0,1]$$
$$A^{(3)} = \left\{\left(x, u(x), v(x,u), \mu_{A^{(3)}}(x,u,v)\right) \mid x \in X, u \in U \subseteq [0,1], v \in V \subseteq [0,1]\right\} \tag{3}$$

where U is universe for secondary variable u and V is universe for tertiary variable v. If T3MFs are used in the rules then a type-3 fuzzy logic system (T3FLS) arises. For more specific details on how the inference process and type reduction are postulated in this case, please check (Castillo et al., 2022).

3 Results and Discussion

Recently, significant progress has been achieved in comprehending the complexity of an object by employing fractal constructs (Hadzieva et al., 2017). For example, medical images exhibit properties suggesting a fractal structure (Castiglioni et al, 2014; Oweis &Hijazi, 2006; Spasić et al., 2012). There are several FD definitions, but here we consider the box counting FD as:

$$d = \lim_{r \to 0} [\ln N(r)] / [\ln(1/r)] \tag{4}$$

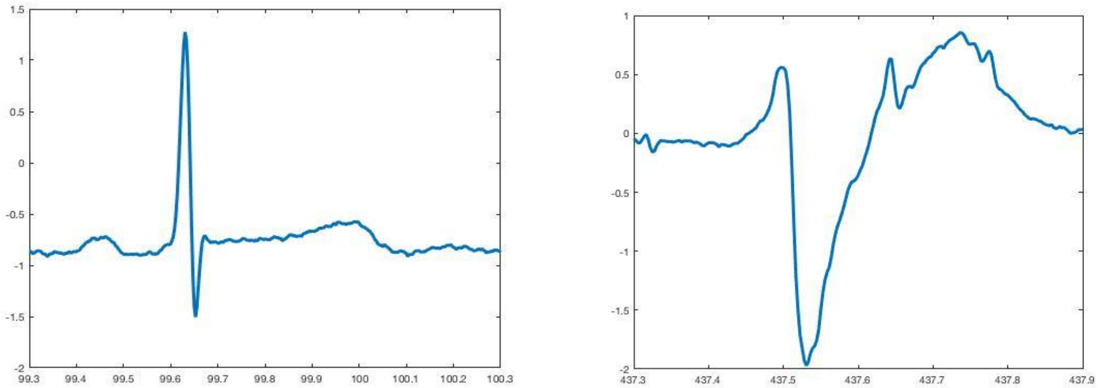
where $N(r)$ is for number of boxes of size r. The d in (4) is approximated with boxes for several r sizes and using regression for finding d.

The ECG depicts the electrical activity of the heart that includes the regular and calm contraction of the heart muscles. The ECG waveform includes 5 major waves of P, Q, R, S, and T (Mary & Singh, 2013). One of the most important parts of the ECG analysis is the measurement of RR-Interval and ST-segment, which are good representatives of the variety of heartbeat (Spasić et al., 2012; Pan & Tompkins, 1985; Mary & Singh, 2013). The FD of the RR-Interval is calculated with (4). Since the heart is nonlinear, severe diseases have a smaller FD. According to experiments, we can classify into 4 groups related to the FD.

Normal: FD >1.56, PAC: $1.37 <$ FD ≤ 1.56

PVC: $1.3 <$ FD ≤ 1.37, PSVT: $1 <$ FD ≤ 1.3

In this case, the corresponding arrhythmias are: Premature Ventricular Complex (PVC), Premature Atrial Contracture (PAC) and PSVT, which includes other types of arrhythmia (Kiani & Maghsoudi, 2019). We have to say that it is possible to only use FD in deciding if a person has a possible pathology, but the approach would only be based on a fixed numeric threshold (somewhat arbitrary) and the main problem with this alternative is it would not take into account the intrinsic uncertainty of medical decision making. As illustration, we show in Figure 1(a) a normal ECG signal and in Figure 1(b) a PVC signal that indicates an arrythmia. Of course, from Figure 1 there is a clear difference between a normal signal and problematic one, but if we want to automate the process of discriminating between normal and arrythmia signal we need to have a way to process the geometrical signal and distinguish between them. In this case, the FD provides a good way to measure the geometrical complexity and then a set of fuzzy rules complements the decision-making process to achieve arrythmia classification.



(a) (b)

Fig. 1 *Examples of ECG signals: (a) Normal beat, and (b) PVC beat*

4 Type-1 Fuzzy Approach

First, a type-1 FL approach for Arrhythmia Classification is offered. Figure 2 shows the structure of the system, where the image FD enters as input, and classification is the output.

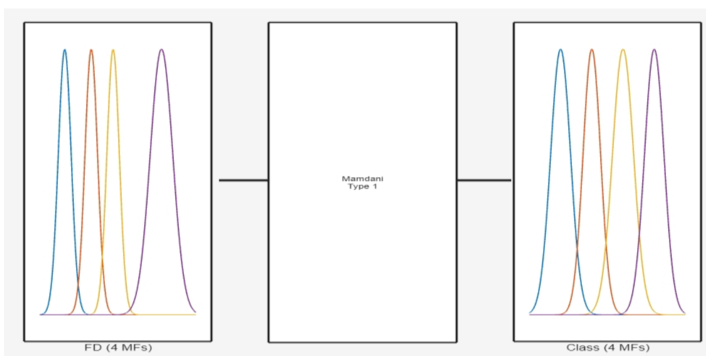


Fig. 2 *System for Arrhythmia Classification.*

Figure 3 depicts the MFs for FD, which are Low, Medium, High, and Very High. The shape of the MFs was decided based on previous studies, where the Gaussian MFs produce good results in medical problems (Melin & Castillo, 2025).

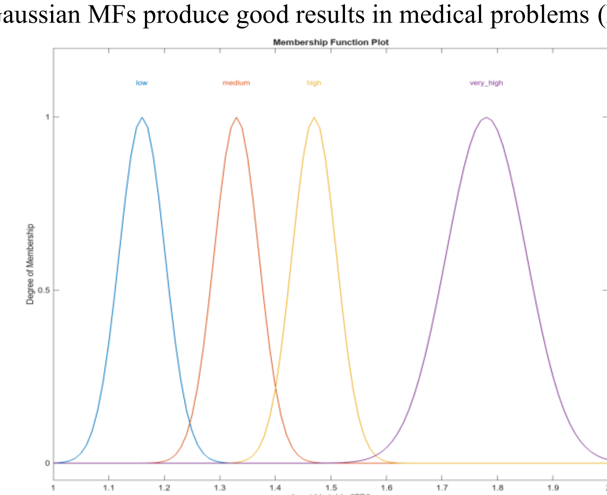


Fig. 3 *Input MFs.*

In Figure 4 we find the output MFs, for the classification (Class). In this case, there are four MFs, one for PSVT, another for PVC, another one for PAC and another for Normal.

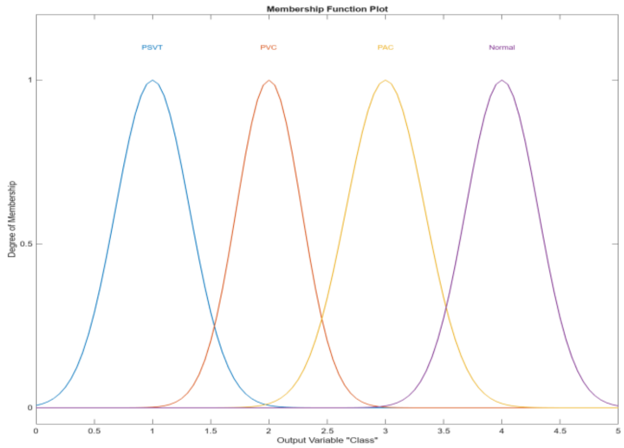


Fig. 4 *MFs for the Class.*

Table 1 lists the MF parameters for FD. The FD range is from 1 to 2, as experimentally the FDs of the images are in this range. Table 2 shows the MF parameters for the Class, which are in [0, 5]. All MFs are Gaussian, having two parameters, center (c) and deviation (s). These parameter values come from the experimental data obtained for the FD in (Kiani & Maghsoudi, 2019), meaning centers of Gaussians are the means in the experimens. The deviations were manually tuned until the best results were achieved, athough these could optimized in the future by using metaheuristics, which could enhance the results.

Table 1. Parameters of input MFs.

MFs	**c**	**s**
Low	1.16	0.04125
Medium	1.33	0.04027
High	1.47	0.03988
Very High	1.78	0.07212

Table 2. Parameters of output MFs.

MFs	**c**	**s**
PSVT	1.00	0.3179
PVC	2.00	0.2805
PAC	3.00	0.3382
Normal	4.00	0.3107

Based on knowledge, the Arrhythmia is classified as PSVT for low FD, classified as PVC for medium FD and is classified as PAC for high FD values. Finally, the signal is classified as normal if the FD is very high. Then the fuzzy rules are:

If FD is Low Then Class is PSVT

If FD is Medium Then Class is PVC

If FD is High Then Class is PAC

If FD is Very High Then Class is Normal

These rules basically are encapsulating the classification knowledge for this problem, but we could consider other variables and more rules for the system in the future. The complete method is summarized in Figure 5, where all the steps are shown.

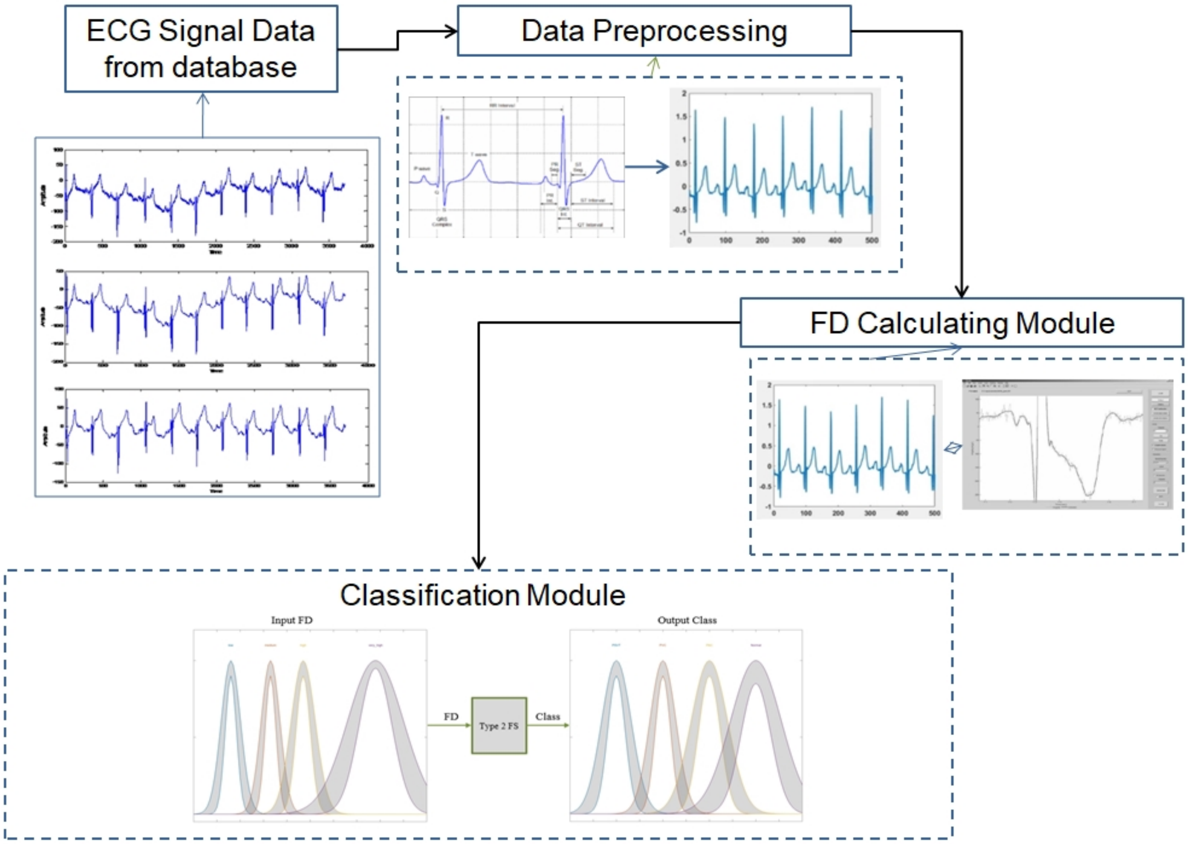


Fig. 5 *Proposed methodology.*

In Figure 5 we can note the part in which the image is preprocessed and then the FD value is calculated to proceed into the type-1 fuzzy system for evaluating the results. In the following sections the same approach is used, but the change would be in that the fuzzy system is replaced by a type-2 or type-3 variant.

5 Type-2 Approach

Here we augment the type-1 system to type-2 by utilizing T2MFs. The type-2 system structure is depicted in Figure 6. The difference with Figure 2 is that now we are using type-2 MFs (T2MFs) that handle higher uncertainty levels, which can enhance the classification. The input T2MFs are shown in Figure 7. The output T2MFs are shown in Figure 8.

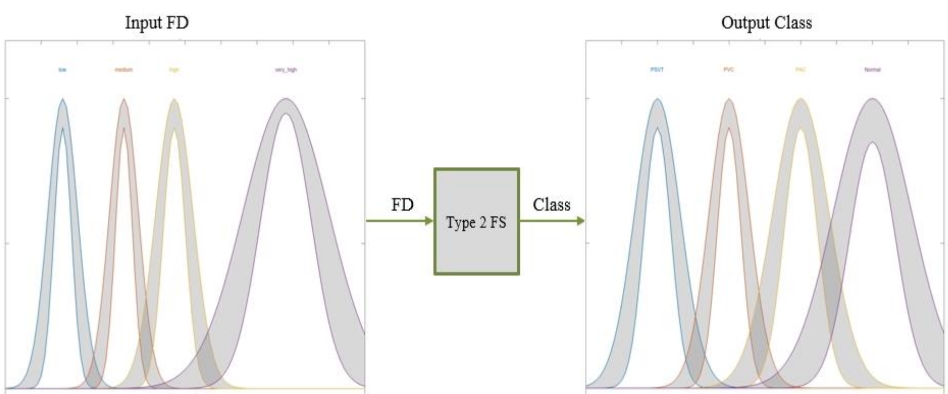
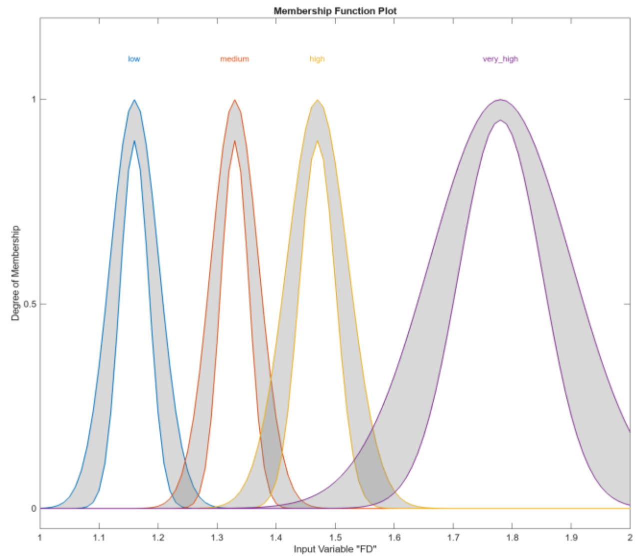


Fig. 6 *Type-2 approach for Arrhythmia Classification.*



Fig, 7 Input T2MFs

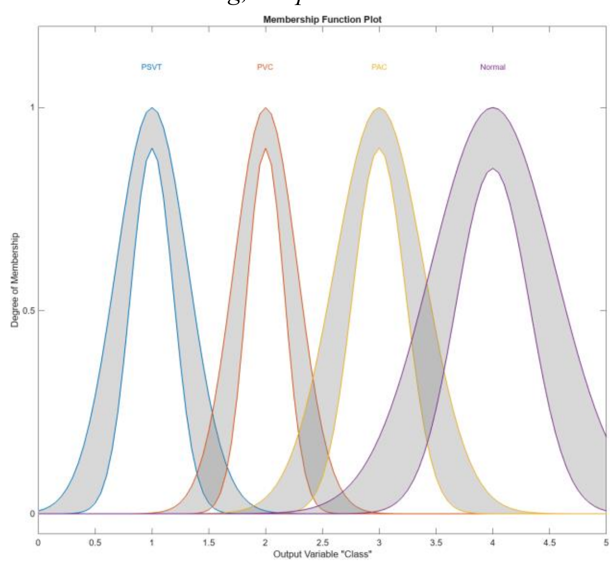


Fig. 8 *Output T2MFs.*

Table 3 lists the parameters for the T2MFs of the FD input. Table 4 lists the parameters for the T2MFs of Class, which are in [0, 5]. The T2MFs are scaled Gaussians, so we now have 4 parameters: c, s, lower lag (ℓ) and lower scale (λ) (Castillo et al. 2022). These values were obtained by trial and error until good results were achieved, but we expect to optimize them with metaheuristics in the future to enhance the results even further.

Table 3. Parameters of input T2MFs.

MFs	**c**	**s**	$\boldsymbol{\lambda}$	$\boldsymbol{\ell}$
Low	1.160	0.0412	0.900	0.200
Medium	1.330	0.0402	0.900	0.200
High	1.470	0.0518	0.900	0.200
Very High	1.780	0.1199	0.950	0.200

Table 4. Parameters of output T2MFs.

MFs	c	s	λ	ℓ
PSVT	1.000	0.3179	0.900	0.200
PVC	2.000	0.2805	0.900	0.200
PAC	3.000	0.3938	0.900	0.200
Normal	4.000	0.5472	0.850	0.200

The rules are the same as in type-1, as now the difference is that the T2FLS uses T2MFs. In Figure 9 we illustrate the inference process that shows how the rules are used and then their outputs are aggregated to finally obtain the results by type reduction.

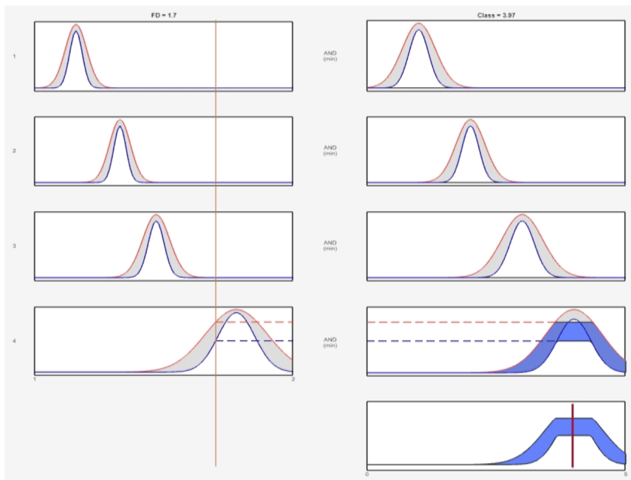


Fig. 9 *Illustration of the inference process.*

6 Type-3 Approach

The structure of the IT3FLS is depicted in Figure 10, which is similar to the previous cases. The rules are the same, but now the MFs are in 3D. In Figure 10, the FD value enters the input of the system and then goes into the rules for performing the inference and type reduction and then finding the classification. Figures 11 and 12 illustrate the nature of the input and output MFs in bidimensional view, respectively. On the other hand, Figures 13 and 14 show the input and output MFs in a 3D view, respectively.

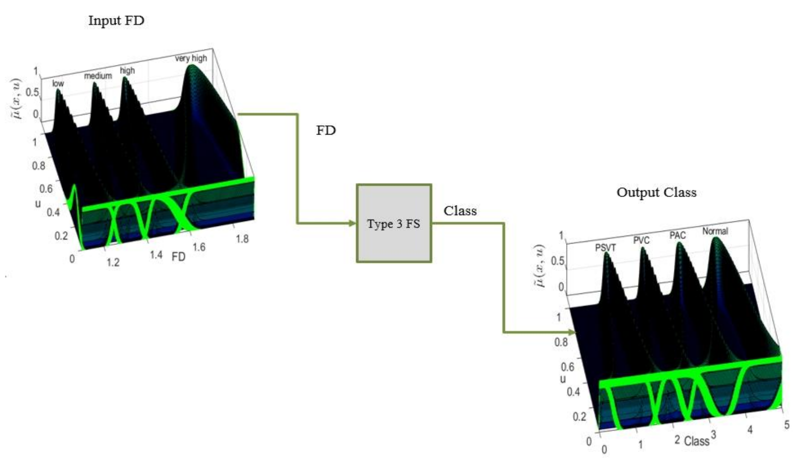


Fig. 10. *Structure of the system.*

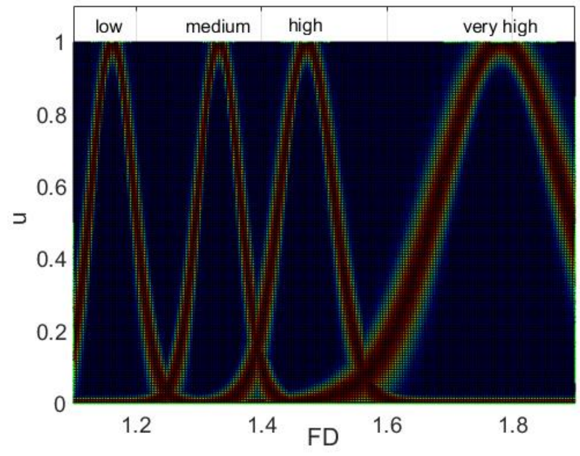


Fig. 11. *Input MFs.*

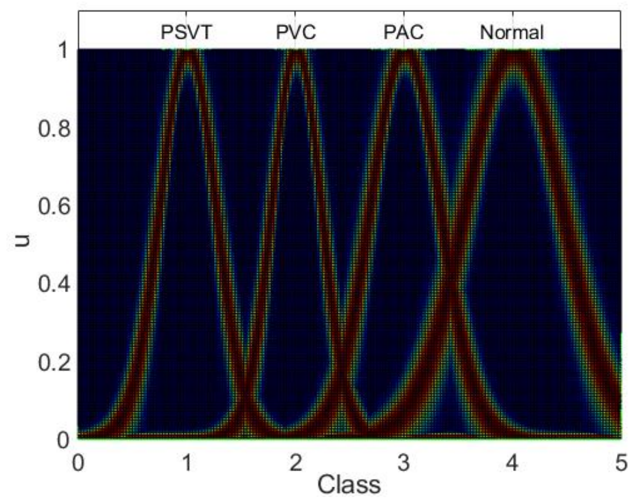


Fig. 12. *Output MFs.*

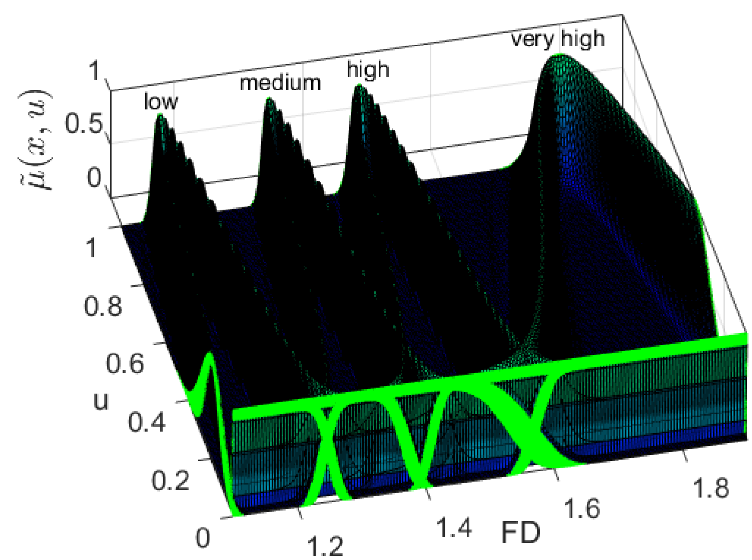


Fig. 13. *Input MFs in 3D.*

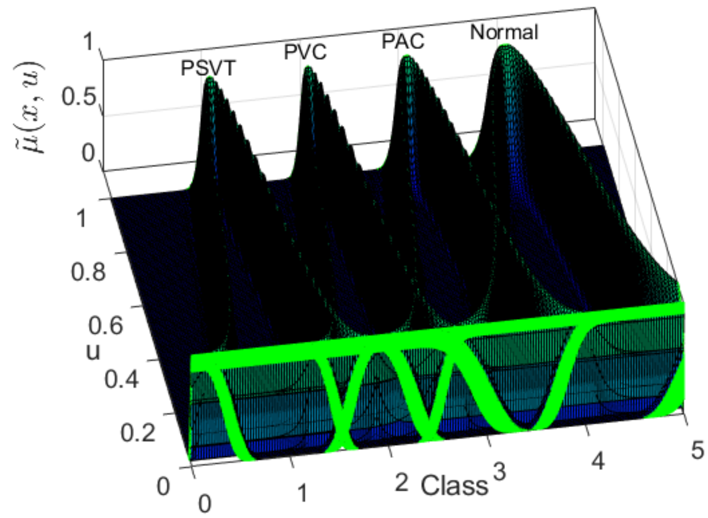


Fig. 14. *Output MFs in 3D.*

7 Application to Arrythmia Classification

Here we present the results of the approaches for Arrhythmia Classification. Table 5 shows a comparison, where column 1 shows the FDs, column 2 lists the classes labeled by experts (PSVT =1, PVC =2, PAC =3 and Normal=N=4), column 3 shows type-1 results, column 4 the results of interval type-2 FL and column 5 lists the type-3 results. We considered 15 cases from the MIT dataset, which are already labeled by experts, and calculated the FD value and then we did perform the inference process to obtain the corresponding classes. The classes obtained with the fuzzy systems are compared against the expert classes to verify the results. Table 5 summarizes the comparison of the fuzzy approaches with the ideal values of the experts.

Table 5. Comparison of Arrythmia Classification.

FD	Expert Class	Type-1	Type-2	Type-3
1.10	PSVT	1.00	1.00	1.1196
1.20	PSVT	1.01	1.01	1.0557
1.24	PVC	1.45	1.41	1.7411
1.25	PVC	1.56	1.49	1.8176
1.30	PVC	1.99	2.01	2.0173
1.35	PVC	2.03	2.11	2.0527
1.45	PAC	2.98	3.00	2.9952
1.50	PAC	3.00	3.06	3.0105
1.55	PAC	3.04	3.13	3.2485
1.56	PAC	3.25	3.31	3.3824
1.57	N	3.47	3.49	3.5150
1.571	N	3.49	3.51	3.5335

FD	Expert Class	Type-1	Type-2	Type-3
1.59	N	3.74	3.76	3.7898
1.60	N	3.89	3.86	3.8587
1.65	N	3.95	3.96	3.9433

From Table 5 we can note that type-2 misclassifies two cases of PVC (third and fourth rows), but type-1 produce an incorrect result in third row, which should be PVC. Also, type-1 has incorrect results in rows 11 and 12, which should be N. On the other hand, type-2 has another incorrect result in row 11, which should be N. Type-3 has the correct result in all 15 cases Also, numerically the type-3 results are closer to the experts.

We performed a Wilcoxon signed rank test to validate the obtained results and we did find the following statistical evidence: when comparing type-3 versus type-2 we obtain a p value of 0.05, which implies that there exists significant difference in favor of the type-3 method, and when comparing type-3 versus type-1 the p value is of 0.001, which indicates sufficient evidence that the type-3 results are better than the type-1 counterpart. Even if these results are encouraging, we still need to consider a larger sample of cases for achieving a better validation of the type-3 approach, and this work will be undertaken in the future. In addition, testing with real patients would be required in the future to really validate the possible real-world impact of the proposal, but for now the information of the MIT Arrhythmia dataset is only considered.

Actually, previous works that only used FD (not fuzzy logic) were used as a basis for the developments offered in this paper. After, developing the type-1 to type-3 FL systems we realized that fuzzy methods (added to the FD) actually improve the results. The main reason is that when only using FD for classification, the way to decide is to use a threshold (a crisp number) that arbitrarily decides (if we are below or higher) in which class the patient is considered to belong. However, the actual decision has uncertainty due to noise and other factors and then fuzzy logic helps to model this. The uncertainty sources are noise affecting instruments, instability caused by the environment and differences in the opinions from medical experts. A complete study of how these sources can affect the results can also be done, as it will also be interesting to understand this phenomenon. Also, we still need to optimize parameters (using metaheuristics) of the fuzzy systems and then perform a comparative study of the different variants.

8 Conclusions

In this paper a hybrid of type-3 fuzzy logic and FD was used for Arrhythmia Classification. FD was used to study the ECG geometry. An interval type-3 fuzzy system was used to convey the expert knowledge in classification. Results obtained with a sample of cases from an Arrhythmia dataset shows the proposal effectiveness. A statistical comparison versus type-1 and type-2 did show that there exists significant evidence that the type-3 approach offers better results. However, a limitation of the study is that the sample that was used is relatively small and more cases could be considered to achieve more evidence on the advantages of the proposal. Another limitation of this study is that arrhythmia classification is performed in an offline fashion, meaning that information of patients is contained in a dataset and we process the information in the computer, which is not real-world scenario in this case, as direct interaction with patients (online) would be needed in real situation. As future work we plan to use the approach to the classification of other problems, as we think that there are similar problems in which the general outlined architecture with small changes could also be successful. Also, the validation with real patients remains as a future endeavor for the research group, as for the moment the testing has only been done with information on the Arrhythmia dataset. Additionally, the hardware implementation of the approach would make it usable in the real world, as this would allow direct interaction with human and machine, but remains as future work to be undertaken. In addition, we plan to extend the approach to general type-3 fuzzy logic with the idea of enhancing the handling of uncertainty, as in (Castillo et al., 2026). Also, we will need to consider optimizing parameters in the fuzzy systems by means of efficient metaheuristic algorithms, as in (Martínez-Soto et al., 2014; Sanchez et al., 2020; Amador-Angulo et al., 2016). Also, quantum metaheuristics could be considered for this same task with the aim of enhancing even further the results, as has been found to be the case in other applications. In addition, other kinds of application areas could be considered, as in (Cazarez-Castro et al., 2010; Castillo et al., 2015; Soto et al., 2018). Also, combining fuzzy models with neural networks would be a good idea, as in (Ramirez et al., 2019). Finally, other kinds of fuzzy models could be applied, such as Fermatean, Pythagorean or hesitant fuzzy approaches that are interesting alternatives, and even hybrids of type-3 with these models could be considered. Additionally, the results of the proposal could be compared to other kinds of models to verify the possible proposal advantages.

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Data Availability Statement: The Dataset will be available on reasonable request.

Conflicts of Interest: The authors declare no conflicts of interest.

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