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## **Inventory Model in a CVRP Distribution Network with Uniformly Distributed Demand**

*Lucía Cazabal-Valencia, Santiago-Omar Caballero-Morales,*

*José-Luis Martínez-Flores, Damián-Emilio Gibaja-Romero*

*Universidad Popular Autónoma del Estado de Puebla A.C., Postgraduate Department of Logistics and Supply Chain Management, 17 Sur 901, Barrio de Santiago, 72410, Puebla, México*

*lucia.cazabal01@upaep.edu.mx, santiagomar.caballero@upaep.mx, joseluis.martinez01@upaep.mx,*

*damiannemilio.gibaja@upaep.mx*

**Abstract.** At the present time, business and industrial organizations are given the task of good planning when determining any type of acquisition for their services. That is, identify and evaluate the different options of provision of inventory that allow the company to maintain and improve its offer. Within this evaluation, it can be noticed that the decision of the management and administration of resources, involves the problem of distribution (goods and services), being also of great importance within the supply chain. In this way, in this paper, we analyze the Capacitated Vehicle Routing Problem with ellipsoidal distances, which includes an inventory model with uniformly distributed demands, so that inventory costs and distances are minimized. With this contribution, the way to minimize total costs in a distribution period can be refined. For validation, computational evaluations obtained from the mathematical model and backed by the software are presented (LINGO 18.0 - MATLAB).

**Keywords:** CVRP, uniform demand, newsvendor, ellipsoid

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## **1 Introduction**

From an economic, industrial, financial and capital flow perspective, the problem of inventories has always been present, and this arises when it is realized that the costs of maintaining, incurring shortages and replenishing, associated with a particular product that is required for production, commercialization or consumption, they are found and, consequently, there is an optimum (in this case, a minimum cost) that must be located [1].

Inventory management is the process in charge of ensuring the number of adequate products in the organization, in such a way that the manufacturing and distribution operations do not stop, fulfilling the promises of delivery of products to customers. However, maintaining these inventories in organizations represents considerable costs, since for this activity it is required: capital investments in the goods, space to store them, personnel for their administration and care and technological and energy resources are required for their maintenance among others [2].

All firms (including JIT operations) maintain an inventory provision for the following reasons [3]:

- Maintain independence in operations.
- Adjust to the variation of product demand.
- Allow flexibility in production scheduling.
- Provide a safeguard for the variation in the delivery time of raw materials.
- Take advantage of the size of the economical purchase order.

Each of the reasons mentioned takes great importance, since when making any decision that affects the size of the inventory they generate undesirable costs [4]:

- Carrying costs
- Replenishment Costs

- System control costs
- Costs of insufficient supply in the short run

In this paper, the inventory of a certain item that is reviewed every fixed time interval will be addressed; that is, the order size varies with the behavior of the demand.

It is beneficial for any company to use logistics strategies, in terms of obtaining general corporate skills, since one of the most important characteristics of the logistics strategies must be to find a way to minimize the total cost in the distribution period, that is; achieve minimize distribution costs by determining the best route of the vehicle while serving the customer [5].

As already mentioned, maintaining a high number of inventories with which the customer is satisfied, but which incurs high costs is a dilemma for any organization, now if we add the costs for distribution of this matter this implies, even more, the management of a successful decision. For this reason, the Capacitated Vehicle Routing Problem (CVRP) is analyzed, which can be described in a simple way, such as a fleet of vehicles with homogeneous capacities that have to satisfy the uniformly distributed demand of a group of customers through a set of routes that start and end in a common center and that have as an objective, minimize the cost of the route that in turn is implied by the total coverage distance.

In section 2, a brief analysis of the single-period inventories with uniformly distributed demands will be addressed, as well as the CVRP proposed with an ellipsoidal geodesic length for distance minimization. Later, the influence and/or significance of the optimal quantities to be ordered in the Capacitated Vehicle Routing Problem will be seen with the customer demand distributed uniformly.

## 2 Technical Framework

### 2.1 Inventories: Single Period Problem

Liao et al. [6] they mention that the inventory and the possession of products in any distribution center are a subject of great importance in the supply chain and in their contribution they consider cross-docking as an efficient method to control the flow of inventory.

Within the stock management literature, the problem of newsvendor or also known as Single Period Problem (SPP) is mentioned, which consists of finding the quantity of a product that maximizes the expected profit under a random demand (probabilistic). The SPP model assumes that, if any inventory remains at the end of the period, a discount is used to sell it or liquidated, if the quantity of the order is less than the demand made, some profits are renounced [7].

The investigations on the problem newsvendor have followed two approaches for their solution:

- Minimize the expected costs of overestimating and underestimating demand.
- Maximize the expected benefit.

While this problem is used in many real-life situations, some of the most prominent are mentioned [2] [7]:

- Perishable or expired items.
- Articles at the end of their life cycle.
- Fashion items, sports and in general those sold by campaigns.
- Design and management of the adequate capacity of an installation.
- Evaluation of advanced reservation of orders in airline and hotel services.

The Newsvendor model is a well-known method for identifying the optimal order quantity for a single period replacement scenario with stochastic demand. The model aims to order very little what results in shortage or order too much which results in excess [8].

The optimal quantity of order is the one that covers the proportion of the demand up to a Critical Relationship (CR) that oscillates between 0 and 1; this can represent as:

$$CR = \frac{c_s}{c_s + c_e} \quad (1)$$

Where:

$$c_s = \text{cost of shortage}$$

$$c_e = \text{cost of excess}$$

The parameters that impact the cost of the shortage and the cost of the excess are the sale price, the cost, the recovery value and the shortage penalty, all at the unit level. Using these parameters, we can derive the cost of the shortage and the cost of the excess as:

$$c_s = p - c + B \text{ y } c_e = c - g.$$

Where:

$$c = \text{cost}$$

$$p = \text{selling price}$$

$$B = \text{shortage penalty}$$

$$g = \text{salvage value}$$

When we replace in equation (1) and assuming that there is no penalty for shortage, you have to  $CR = \frac{p-c}{p-g}$ . Keep in mind that, if there is no recovery value, this is further simplified:

$$CR = \frac{p - c}{p} \quad (2)$$

Since CR represents the accumulated demand up to the optimal order quantity, it follows that:

$$CR = CDF(Q^*) \quad (3)$$

$$Q^* = CDF^{-1}(CR)$$

Where:

$$Q^* = \text{Optimal Order Quantity}$$

As already mentioned, the objective of Newsvendor is to maximize profits from a fixed offer while the demand is uncertain or to minimize the expected costs of overestimating and underestimating the order. In this paper, we will focus on minimizing the expected costs. When using the Newsvendor model, we know that the values are stochastic. Therefore, we will calculate the expected cost by:

$$E(c) = cQ^* + BE(\text{shortage}) \quad (4)$$

$$\text{As: } \text{shortage} = \begin{cases} D - Q^*, & \text{si } D \geq Q^* \\ 0, & \text{si } D < Q^* \end{cases} \quad (5)$$

$$\text{So: } E(\text{shortage}) = \int_{x=D}^{\infty} (x - Q^*)f(x)dx \quad (6)$$

Take into account that  $E(\text{shortage})$  determines the number of insufficient units.

## 2.2 Uniform distribution

A uniform random variable is used to model the behavior of a continuous random variable whose values are uniform or exactly distributed in a given interval  $[a, b]$  [9]. The probability density function of the continuous uniform distribution is given by:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{si } a \leq x \leq b \\ 0 & \text{para } x < a \text{ o } x > b \end{cases} \quad (7)$$

The present work takes into account a set of uniformly distributed demands, the ranges considered will be mentioned later.

Now, if the demand follows a uniform distribution, it is obtained from (3) that the optimal quantity to be ordered is given by:

$$Q^* = CR(b - a) + (a) \quad (8)$$

Where:

$$\begin{aligned} a &= \text{lower bound of demand} \\ b &= \text{upper bound of demand} \end{aligned}$$

### 2.3 Capacitated Vehicle Routing Problem

The Capacitated Vehicle Routing Problem (CVRP) is an evolution of the Vehicle Routing Problem (VRP), which in turn has origins from the Traveling Salesman Problem (TSP) raised in 1956. The main objective of the TSP is to find the best possible route from different approaches (already be a process with some specific sequence or a logistic distribution) with criteria of economy in distances or costs [10].

The VRP type problems are the most frequent in combinatorial optimization, and most of them belong to the NP-Hard class. Rocha *et al.* [11] perform a revision of the taxonomy of these problems from the conceptual point of view of mathematical formulation and of the solution methods used. It should be mentioned that a formulation of the VRP type can include a large number of variables and various parameters, since this raises the search for the optimal solution with different restrictions such as; number of vehicles, capacity, places of destination, customer demand, among other.

For the CVRP the outstanding variable is the quantity of product, both for the demand of the customers and for the capacity of the vehicles. It is worth mentioning that the capacity restriction for vehicles is classified as a homogeneous and heterogeneous fleet. For the purposes of this study, vehicles with equal capabilities will be taken, that is; homogeneous fleets.

Before presenting the mathematical model of the CVRP it will be important to mention the characteristics that distinguish this problem [12]:

- Given a set of customers geographically distributed in a certain defined area, there will be a point that will deliver a single product to all customers.
- Each customer has a constant demand for a known product, and must be visited by a fleet of identical vehicles with limited capacity.
- The distance and / or cost between the customers and the point of supply is known.
- The vehicles are initially located in the warehouse where each of the routes starts, then they visit a subgroup of customers and finally return to the warehouse.
- The sum of the demands of customers who visit a vehicle must be equal to or less than the capacity of said vehicle.
- Each client must be visited by a single-vehicle.
- It is assumed that the amount of product available in the deposit is equal to or greater than the sum of the demands of all customers.
- The capacity of all vehicles is greater than or equal to the sum of the demands of all customers, i.e., All customers will be satisfied in the totality of their demand.
- The cost of the route may depend on time, distance, fuel consumption or others, however, the purpose will always be to minimize it.

In consideration of the aforementioned characteristics, the mathematical model of the CVRP assumes the following form [13]:

$$\text{Min} \sum_{i=0}^N \sum_{j=0}^N \sum_{k=0}^M d_{ij} x_{ijk} \quad (9)$$

$$\text{s. a.} \quad \sum_{k=1}^M \sum_{j=i}^N x_{ijk} \leq M \quad i = 0 \quad (10)$$

$$\sum_{k=1}^M \sum_{j=0}^N x_{ijk} = 1 \quad \forall i \in [1, N] \quad (11)$$

$$\sum_{j=1}^N x_{ijk} = \sum_{i=1}^N x_{ijk} \quad \forall k \in [1, M], \quad i = 0 \tag{12}$$

$$\sum_{i=0}^N \sum_{j=0}^N p_i x_{ijk} \leq P_k \quad \forall k \in [1, M] \tag{13}$$

$$\sum_{i=1}^N \sum_{j=1}^N x_{ijk} \leq |S| - 1 \quad |S| \geq 2, \quad \forall i, j \in [1, N] \tag{14}$$

$$x_{ijk} \in \{0,1\} \quad \forall k \in [1, M], \quad \forall i, j \in [1, N] \tag{15}$$

The restriction (10) indicates that from the supply center must depart maximum M vehicles. Restrictions (11) and (12) ensure that one and only one vehicle visits and abandons each customer, forming a TSP for each route. Equation (13) restricts vehicular capacity in terms of weight. Subsequently, the restrictions (14) and (15) establish, respectively, the absence of disconnected sub-routes and the binary values for the decision variables.

The objective function (9) establishes the minimization of costs, which for our purpose will be determined from the geodesic distances between the clients and the supply center. In this case, for the calculation of all the geodesic distances between two points, the ellipsoidal distance proposed by Cazabal [14] is considered:

$$d = s_{12} = bA(\sigma - \Delta\sigma) \tag{16}$$

The geodesic distance  $s_{12}$  connects two points  $A = (\varphi_1, \lambda_1)$  and  $B = (\varphi_2, \lambda_2)$  with latitude and longitude, Figure 1 shows the geodesic arc delimited by  $AB$ , known as the *inverse problem* on an ellipsoid.

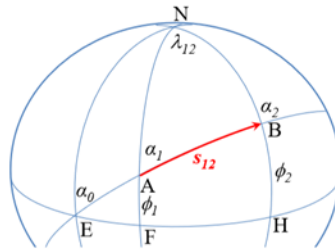


Figure 1. Geodesic distance  $AB$  over an ellipsoid (From Cazabal [11])

### 3 Evaluation of the model

For evaluation purposes, a set of 70 clients with their geographical location is considered, in addition to their respective demands distributed uniformly, see Table 1. It is worth mentioning that the demands approach was generated randomly.

Table 1. Location of clients and respective demands

	<i>Latitude [°]</i>	<i>Longitude [°]</i>	<i>Latitude [Rad]</i>	<i>Longitude [Rad]</i>	<i>Uniform Distribution [Interval]</i>	<i>Demand [Units]</i>
1	41.547270	-8.446441	0.725137	-0.147418	[73, 159]	122
2	41.162138	-8.656973	0.718415	-0.151093	[94, 143]	119
3	41.030641	-8.645966	0.716120	-0.150901	[38, 123]	92
4	41.811542	-6.783551	0.729749	-0.118395	[38, 100]	57
5	41.339482	-6.984215	0.721510	-0.121898	[152, 190]	161
6	39.811433	-7.603193	0.694841	-0.132701	[91, 144]	132
7	38.569799	-7.981050	0.673170	-0.139296	[176, 182]	181
8	37.920692	-8.452919	0.661841	-0.147531	[109, 143]	135
9	37.945347	-7.612149	0.662271	-0.132857	[69, 106]	70
10	37.534632	-8.491444	0.655103	-0.148204	[101, 150]	126
11	37.142411	-8.626138	0.648257	-0.150555	[84, 165]	131
12	37.129405	-7.670056	0.648030	-0.133868	[124, 175]	172
13	42.603604	-5.594901	0.743573	-0.097649	[91, 119]	115
14	40.963433	-5.704232	0.714947	-0.099558	[168, 218]	177
15	39.471631	-6.425739	0.688910	-0.112150	[64, 85]	78
16	37.375350	-6.025099	0.652323	-0.105158	[48, 91]	65
17	36.198922	-5.443021	0.631790	-0.094999	[114, 148]	115
18	36.500830	-4.935040	0.637060	-0.086133	[169, 202]	179
19	42.854032	-2.712178	0.747944	-0.047336	[156, 226]	184
20	42.344121	-3.729713	0.739044	-0.065096	[44, 96]	90
21	41.670065	-3.705820	0.727280	-0.064679	[25, 86]	67
22	41.651686	-0.930000	0.726959	-0.016232	[130, 178]	138
23	41.618337	0.584915	0.726377	0.010209	[87, 154]	134
24	41.150463	1.089659	0.718211	0.019018	[68, 113]	92
25	41.069698	1.030992	0.716801	0.017994	[179, 228]	196
26	41.041488	0.729902	0.716309	0.012739	[32, 90]	68
27	40.874229	0.707270	0.713390	0.012344	[139, 239]	189
28	40.719840	0.698240	0.710695	0.012187	[96, 143]	138
29	40.616140	0.576095	0.708885	0.010055	[123, 165]	155
30	40.552949	0.524836	0.707782	0.009160	[60, 103]	86
31	37.617407	-1.023527	0.656548	-0.017864	[184, 232]	186
32	39.407701	-0.501597	0.687794	-0.008755	[100, 185]	148
33	38.272222	-0.704102	0.667976	-0.012289	[74, 132]	111
34	37.804196	-0.845394	0.659808	-0.014755	[49, 120]	72
35	37.980527	-1.162195	0.662885	-0.020284	[26, 105]	55
36	40.297295	-3.847058	0.703320	-0.067144	[91, 161]	126
37	38.693238	-4.120831	0.675324	-0.071922	[150, 194]	167
38	38.546315	-3.002019	0.672760	-0.052395	[74, 122]	89

39	38.223347	-3.417911	0.667123	-0.059654	[132, 179]	173
40	38.614654	-4.143079	0.673953	-0.072310	[70, 78]	70
41	38.378132	-4.856056	0.669825	-0.084754	[132, 209]	181
42	37.780089	-3.831885	0.659387	-0.066879	[75, 115]	81
43	39.896177	-5.548242	0.696320	-0.096835	[57, 116]	98
44	38.992198	-1.878099	0.680542	-0.032779	[36, 113]	63
45	40.061967	-2.183044	0.699213	-0.038101	[19, 105]	62
46	41.522055	0.332200	0.724697	0.005798	[162, 199]	180
47	42.633452	0.213610	0.744094	0.003728	[98, 160]	139
48	43.108989	-0.150581	0.752394	-0.002628	[124, 156]	150
49	40.345008	-1.118475	0.704153	-0.019521	[42, 73]	62
50	40.162741	-1.235470	0.700972	-0.021563	[147, 07]	178
51	40.099300	-1.507270	0.699865	-0.026307	[25, 77]	63
52	39.725462	-1.905702	0.693340	-0.033261	[100, 132]	119
53	37.913575	-3.011814	0.661717	-0.052566	[109, 167]	117
54	37.760031	-3.204440	0.659037	-0.055928	[85, 115]	98
55	37.452975	-3.292870	0.653678	-0.057471	[71, 97]	92
56	37.677727	-3.569325	0.657600	-0.062296	[143, 191]	147
57	37.217693	-4.074055	0.649571	-0.071106	[98, 120]	107
58	38.743606	-9.230244	0.676203	-0.161098	[121, 142]	141
59	39.333652	-8.502518	0.686502	-0.148397	[85, 129]	114
60	40.362265	-8.058083	0.704454	-0.140640	[143, 194]	185
61	40.513534	-7.993203	0.707095	-0.139508	[46, 79]	75
62	41.168167	-7.798250	0.718520	-0.136105	[122, 166]	142
63	38.723209	-8.005261	0.675847	-0.139718	[163, 213]	178
64	38.092523	-7.424216	0.664840	-0.129577	[122, 141]	140
65	40.621422	-8.281128	0.708978	-0.144533	[10, 73]	57
66	41.250531	-7.964961	0.719958	-0.139015	[135, 191]	158
67	40.583234	-8.454206	0.708311	-0.147554	[124, 195]	154
68	40.840745	-8.427470	0.712805	-0.147087	[41 104]	55
69	42.569026	-0.433927	0.742970	-0.007573	[131, 170]	154
70	41.557099	1.188651	0.725308	0.020746	[74, 150]	119

The assumption is made to supply items with the specifications given in Table 2.

Table 2. Item data per unit

<b>c</b>	<i>Cost (purchase price)</i>	\$ 320.00
<b>p</b>	<i>Selling price</i>	\$ 500.00
<b>B</b>	<i>Shortage penalty</i>	\$ 200.00

It is necessary to emphasize again that the objective of the CVRP is to minimize the cost of the route, which in turn is implied by the total coverage distance. This distance is obtained considering the ellipsoid as the model for the surface of the Earth and with this making use of the geodetic distance (16).

**3.1. Results**

The tests were performed considering the location data of clients presented in Table 1 and Eq. 16 for the calculation of the distance matrix that is subsequently implemented in the CVRP model, which was used for the execution of the LINGO 18.0 Software. For all routes, vehicles with the same capacities were considered  $P_k = 500, k = 1, \dots, 18$ . Table 3 presents the 18 optimal routes with a total route of 210.7730 km, the assigned tour is also presented respectively, as the amount of total product supplied.

Table 3. Optimal solution of the CVRP model with ellipsoidal distance

<b>Route</b>	<b>Tour</b>							<b>Total Supply</b>
<b>1</b>	1	4	69	47	48	1		500
<b>2</b>	1	5	14	62	1			480
<b>3</b>	1	10	11	12	9	1		499
<b>4</b>	1	18	17	16	64	1		499
<b>5</b>	1	20	19	13	1			389
<b>6</b>	1	21	46	70	23	1		500
<b>7</b>	1	22	24	25	26	1		494
<b>8</b>	1	29	28	27	1			482
<b>9</b>	1	32	30	49	50	1		474
<b>10</b>	1	36	37	41	1			474
<b>11</b>	1	38	53	54	39	1		477
<b>12</b>	1	43	15	6	61	1		383
<b>13</b>	1	44	35	34	31	33	1	487
<b>14</b>	1	51	52	45	3	2	1	455
<b>15</b>	1	57	55	56	42	40	1	497
<b>16</b>	1	65	7	63	1			416
<b>17</b>	1	67	60	66	1			497
<b>18</b>	1	68	59	8	58	1		445

For the execution of the newsvendor model, MatLab Software was used with (2), (4) and (8) and considering all the points except the 1, since from there all the routes start and end (Hamiltonian cycle, that is, it is not possible to form sub tours [15]). Table 4 shows the optimal quantities to be ordered and, in turn, the expected costs that these would generate.

Table 4. Optimal solution of the Newsvendor model

<b>Points</b>	<b>(Q*) Optimal Order Quantity</b>	<b>(E(c)) Expected cost</b>	<b>Points</b>	<b>(Q*) Optimal Order Quantity</b>	<b>(E(c)) Expected cost</b>
<b>1</b>	----	----	<b>36</b>	117	\$ 39,240.00
<b>2</b>	112	\$ 37,140.00	<b>37</b>	166	\$ 54,320.00
<b>3</b>	101	\$ 32,320.00	<b>38</b>	92	\$ 29,440.00
<b>4</b>	61	\$ 19,520.00	<b>39</b>	149	\$ 48,980.00
<b>5</b>	166	\$ 53,120.00	<b>40</b>	73	\$ 23,360.00
<b>6</b>	111	\$ 36,820.00	<b>41</b>	160	\$ 53,300.00
<b>7</b>	179	\$ 57,280.00	<b>42</b>	90	\$ 28,800.00
<b>8</b>	122	\$ 39,840.00	<b>43</b>	79	\$ 26,780.00



<b>9</b>	83	\$ 26,560.00	<b>44</b>	64	\$ 20,480.00
<b>10</b>	119	\$ 39,380.00	<b>45</b>	50	\$ 18,400.00
<b>11</b>	114	\$ 38,580.00	<b>46</b>	176	\$ 57,220.00
<b>12</b>	143	\$ 47,060.00	<b>47</b>	121	\$ 40,320.00
<b>13</b>	102	\$ 33,240.00	<b>48</b>	136	\$ 44,320.00
<b>14</b>	186	\$ 59,520.00	<b>49</b>	54	\$ 17,980.00
<b>15</b>	72	\$ 23,540.00	<b>50</b>	169	\$ 55,680.00
<b>16</b>	64	\$ 21,580.00	<b>51</b>	44	\$ 15,480.00
<b>17</b>	127	\$ 40,640.00	<b>52</b>	112	\$ 36,640.00
<b>18</b>	181	\$ 57,920.00	<b>53</b>	130	\$ 41,600.00
<b>19</b>	182	\$ 60,040.00	<b>54</b>	96	\$ 31,520.00
<b>20</b>	63	\$ 21,560.00	<b>55</b>	81	\$ 26,520.00
<b>21</b>	47	\$ 16,740.00	<b>56</b>	161	\$ 51,520.00
<b>22</b>	148	\$ 47,360.00	<b>57</b>	106	\$ 34,520.00
<b>23</b>	112	\$ 37,540.00	<b>58</b>	129	\$ 41,780.00
<b>24</b>	85	\$ 28,300.00	<b>59</b>	101	\$ 33,520.00
<b>25</b>	197	\$ 63,040.00	<b>60</b>	162	\$ 53,140.00
<b>26</b>	53	\$ 18,560.00	<b>61</b>	58	\$ 19,460.00
<b>27</b>	175	\$ 58,800.00	<b>62</b>	138	\$ 45,360.00
<b>28</b>	113	\$ 37,460.00	<b>63</b>	181	\$ 57,920.00
<b>29</b>	139	\$ 45,480.00	<b>64</b>	129	\$ 41,780.00
<b>30</b>	76	\$ 25,420.00	<b>65</b>	33	\$ 12,260.00
<b>31</b>	202	\$ 64,640.00	<b>66</b>	156	\$ 51,320.00
<b>32</b>	131	\$ 44,220.00	<b>67</b>	150	\$ 49,900.00
<b>33</b>	95	\$ 32,000.00	<b>68</b>	64	\$ 20,480.00
<b>34</b>	75	\$ 24,000.00	<b>69</b>	146	\$ 47,620.00
<b>35</b>	55	\$ 19,700.00	<b>70</b>	102	\$ 34,640.00

When performing the analysis of results, considering a uniformly distributed demand for each point (client) and proposing the CVRP model for the determination of routes, it was observed that there is no great impact in terms of the variability of supplying the goods (optimal quantity to order  $Q^*$ ) with the tours obtained for each route. In Table 5, for example, route 2 supplies a total of 480 units and in case of taking the optimal quantity to be ordered by the customers of that route (5-14-62), a total of 490 units is obtained, which is allowed because the maximum capacity of the route is 500. Otherwise, for example, it can be seen that route 12 supplies a total of 383 units when an optimal quantity to be ordered of 320, by customers (43-15-6-61). The exception is route 4 and 15 that exceed capacity.

Table 5. Comparison of total supply - optimal quantity to order

<i>Route</i>	<i>Total Supply</i>	<i>Total Supply with (Q*)</i>	<i>Route</i>	<i>Total Supply</i>	<i>Total Supply with (Q*)</i>	<i>Route</i>	<i>Total Supply</i>	<i>Total Supply with (Q*)</i>
<b>1</b>	500	464	<b>7</b>	494	483	<b>13</b>	487	491
<b>2</b>	480	490	<b>8</b>	482	427	<b>14</b>	455	419
<b>3</b>	499	459	<b>9</b>	474	430	<b>15</b>	497	511
<b>4</b>	499	501	<b>10</b>	474	443	<b>16</b>	416	393
<b>5</b>	389	347	<b>11</b>	477	467	<b>17</b>	497	468
<b>6</b>	500	437	<b>12</b>	383	320	<b>18</b>	445	416

It should be kept in mind that the costs to be ordered are independent of the costs for shipping or distribution, for this reason, the optimal routes are sought. The benefit of performing the CVRP analysis with a periodic inventory model and with uniformly distributed demands is to identify the agreement that exists between them, in such a way that costs and distances are minimized. Despite having a set of random variables (demands) that are distributed uniformly, it can be noted that their behavior does not differ significantly.

#### 4. Conclusions

When facing economic-administrative decisions, the objective of every company is to maintain an acceptable service when minimizing the costs of operation, maintenance of inventory and transportation. Thus, in this article, we have presented the importance of the CVRP in distribution networks with a focus on inventory management and not only in terms of its solution techniques and taxonomy.

The CVRP with a periodic inventory analysis presents satisfactory results with 18 optimal routes that achieve the minimization of distances between 70 clients that must be satisfied, with a geodesic on an ellipsoidal surface, which resembles the most real form of the Earth.

When solving this problem, the problem of cost and service was attacked. That is, the transportation cost is reduced when distances are minimized and, on the other hand, by knowing the optimal quantity to be ordered, the cost to order is reduced, as well as the cost to maintain the inventory, therefore, provides a better service.

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