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Computational Framework for Zbus Matrix Construction in Power Systems Based on Case-Based Algorithms

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Abstract.
The analysis of Electric Power Systems requires computational tools capable of handling admittance and impedance matrices accurately and efficiently. This work presents the development and implementation of MATLAB software for the construction of the Ybus matrix by inspection and the derivation of the Zbus matrix using the Zbus building algorithm together with Kron reduction. The proposed methodology applies the four fundamental cases of impedance matrix expansion, allowing the Zbus matrix to be updated as the network topology changes without performing complete matrix inversion processes, which helps reduce computational effort and improve processing efficiency.

The software also includes matrix visualization and topological representation functions for the analysis of symmetrical and asymmetrical electrical networks. In addition, a comparative analysis was performed between the proposed implementation and conventional matrix inversion methods in order to evaluate their computational performance during matrix construction and reduction. To validate the methodology, the algorithm was tested on a five-bus power system, verifying the correct formation and reduction of the matrices. The obtained results show that the implementation provides an efficient and practical tool for matrix processing in applications such as load flow analysis, fault studies, and stability analysis in Electric Power Systems.

Keywords: Electric Power System, Zbus, admittance, transmission line, matrix, inductive reactance, resistance, algorithm,

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1 Introduction

The application of the Y-bus in power system modeling allows constructing a matrix related to the design of the power system where the buses are taken as interconnection buses to other buses, whereby the branches represented are lines, so between them they build this matrix, which as a result allows the study of load flows in the system, as well as applying it to detect faults or instability in the system. The Y-bus matrix built by applying impedance conversion to admittances solves the inverse, thereby obtaining Zbus, also considering that the buses have n number in the power system, there is symmetry in the system, so the diagonal elements of the matrix, for Y11, the result is the sum of the lines connected through the other buses, and for Ynn the same procedure occurs, including elements connected to the reference bus 0.

In the construction of the admittance matrix when considering a real model, within its series impedance it has a resistance or real part and an inductive reactance or imaginary part, this relates to its physical part where, being long-distance conductors, the generation of a magnetic field turns out to be significant, leading to an inductive reactance with values greater than the resistance. To solve it, the admittance is considered as the inverse of the impedance, applying the mathematical operation of multiplying by the conjugate, which inverts its sign. Physically interpreted, this focuses on the conductance (g), which turns out to be a positive value since it indicates the real behavior as heat losses due to the Joule effect, and the susceptance (-jb), which is responsible for interpreting alternating current and, in response, is negative because it represents an inductive element indicated as the absorption of reactive power that creates magnetic fields and is the dominant component in power system networks.

Based on various studies, it is important to be able to apply methods that improve the power system in real time to avoid supply failures and also address problems from the generation stage, so applying methods such as Dynamic State Estimation (DSE) has become pivotal in power system regulation and real-time contingency analysis, thanks to advancements in Phasor Measurement Units (PMUs) and Wide-Area Measurement Systems (WAMS) (Riahinia et al., 2025), that are based on applying real-time and accurate information, the effectiveness of the systems and methods addressed in their application must be verified to corroborate theoretical data, including computational methods such as the application of MATLAB software (MathWorks, n.d.), which allows obtaining data accurately and assisting in power system models so that scalability and numerical precision for systems of increasing complexity (Hsieh et al., 2017), enabling the use of methods such as construction algorithms optimized through case-based solving, which allows an evolution of current methods that are available and often limited in construction.

It is important to consider that the invertibility of the YBus is especially important since it allows for computation of load-flow solutions through the Z-Bus method (Chen et al., 1991), it is important to consider that within the space to set of MATLAB (MathWorks, s.f.), scripts is provided online that takes as input the data files for the IEEE 37-bus, the IEEE 123-bus, the 8500-bus feeders, and the European 906-bus low voltage feeder (IEEE PES Test Feeder Working Group, s.f.), therefore methods for building systems where expansion is related to real growth modify the solution patterns and thus present reliable systems from which Zbus is expected to be obtained without errors, in addition, the use of multiconductor transmission lines (MTLs) consisting of more than two conductors is becoming more widespread because of the increasing need for high-volume and high-speed data and signal transmission. Signal integrity, the effect of the transmission line on the signal transmission, is becoming a critical aspect of high-speed digital system performance (Paul, 2007).

For practical purposes, the application of methods such as Brown's method and the Sherman-Morrison method under the implementation of matrix identities, first-hand applying Brown's method to find Zbus by the common way of inverting results in having two disadvantages: 1. matrix inversion is computationally intensive and requires a lot of memory and 2. ill-conditioning and round-off errors. Thus, for large systems, it is almost always better to build Zbus by analyzing the relationship between currents and voltages. Even though it is utterly impossible to build Zbus directly for a pre-existing system, it is still possible to build it by 1) starting from one bus connected to the reference bus (usually the ground) via a shunt admittance, 2) expanding Zbus by adding one series impedance or shunt admittance at a time, and 3) each time we add a new element, we augment or modify the preceding matrix to an updated one. Such procedure is used in Brown's method, which finds Zbus by direct construction (Brown, 1985; Makram & Girgis, 1988, 1989; Grainger & Stevenson, 1994). To begin the Brown process, the four considered cases must be used, through them Zbus will be reached, following the procedure of: 1. From a new bus k to the reference bus 2. From an existing bus k to a new bus p 3. From an existing bus k to the reference bus 4. From an existing bus k to an existing bus p (Aldaoudeyeh & Wu, 2019), by applying this method, its solution does not allow a new bus to be added because once the Power System is obtained, the design does not allow omitting the already solved cases, so the solution would have to be built from the section that is being added and referring to which point the bus is located, there are examples where this application is not addressed as in (Lee, 2016; Wafaa & Dessaint, 2017; Javadi et al., 2017), and have the situation of being more updated documents. Within our methodology, the cases are explained and in such a way that from the fourth case we detail how Brown's method is applied, where the equivalent impedance that exists between k and p becomes a parallel impedance and becomes called new, thereby considering a change in its impedance that, being already in the fourth case, is applied, which can affect the overall behavior of the system, due to the change between buses since the original model did not have that impedance and this situation now considers that a previously unapplied case pre-existed.

The Sherman–Morrison–Woodbury method allows providing a solution to matrices so formally defined in the vectorized form of the problem, but applied in the matrix setting. This allows one to solve medium size dense problems with computational costs and memory requirements dramatically lower than with a Kronecker formulation (Hao & Simoncini, 2021). In a real way, it can be considered that the available methods achieve approximation, so the application of the Kronecker method allows, in the field of Power Systems, moving from matrices of limited spaces to matrices of a larger size, as well as converting a complex matrix equation into a linear system to solve for the main matrix as if it were a vector. Although Kronecker is related to the Sherman–Morrison–Woodbury method, applying its methods computationally may be limited by the design of the matrix where its memory spaces would be full, and when a maintained system is required, it would be constrained by its natural structure, so we arrive at this formula being a basic component of many numerical linear algebra methods, such as preconditioning and quasi-Newton methods, only for its computational behavior (Bru et al., 2003; Malyshev & Sadkane, 2011; Li & Saad, 2013). Other applications indicate that electrical systems can solve conductive dielectric matrices, to speed up the direct solution of the matrix equation, the dense impedance matrix is transformed into a product of several block diagonal matrices via the SMW formula. In the grouping process, the situation that the elements of an array simultaneously belong to two different subgroups at peer level is avoided in order to promote the efficiency. The SMWA conducts the calculation with a respectable reduction in the computational time as well as memory (Zhang et al., 2016).

The Kron reduction method is ubiquitous in classic circuit theory and in related disciplines such as electrical impedance tomography, smart grid monitoring, transient stability assessment, and analysis of power electronics. Kron reduction is also relevant in other physical domains, in computational applications, and in the reduction of Markov chains. Related concepts have also been studied as purely theoretic problems in the literature on linear algebra (Dorfler & Bullo, 2013), by applying these computational tools, we were able to implement software that uses a reliable analysis system for solving these problems, especially the use of algorithms for the formation of Zbus and Kron reduction that facilitates analysis, thus achieving a way to facilitate the study and analysis of symmetric and asymmetric problems, optimizing data processing and obtaining results.

Through the application of the methods, it is essential to recognize that the Z-Bus matrix construction method by branch addition is a progressive and systematic technique that allows building this matrix as elements (branches and buses) are added to the system. This method is especially useful in contingency studies or network expansion, since it does not require recalculating the entire matrix from scratch every time the system topology is modified (Hernández Jiménez et al., 2025). This allows us to apply power systems academically and professionally, reducing theoretical procedures and applying computerized methods, and through this writing the mathematical methodology is broken down where the equations are described for their importance in application, the experimentation procedures that allow the user to understand the software development and the results by applying an exercise as an experiment that combines matrix algebra mathematics with computational methods.

2 Methodology

The procedure for solving Power Systems using Y_{bus} and Z_{bus} methods can lead to answers for studies such as power flows, symmetrical fault solution, asymmetrical faults, short-circuit analysis, power system stability, or protection coordination. A system is designed using buses also known as buses, which are the interconnection points in the network, and impedances representing a transmission line in the system, making a compact way to exemplify. The application of the Z_{bus} algorithm construction allows, as mentioned, reducing errors that may occur in Y_{bus}, adding a case-based solution that evolves as required in the procedure, and this can be applied to obtain a response. In addition, the use of Kron reduction facilitates the matrix design applying the equations for the bus equation of the electrical system:

Bus equation for a Power System:

$$I = Y_{bus} * V \tag{1}$$

Since to find the voltaje at any bus:

$$V = Y_{bus}^{-1} * I = Z_{bus} * I \tag{2}$$

Therefore:

$$Z_{bus} = Y_{bus}^{-1} \tag{3}$$

And applying that Y_{bus} symmetric, is Z_{bus} is equally symmetric.

Applying Y_{bus} inversely requires processing more data in reduced spaces, which leads to considering that can be applied to shape Thevenin impedance. Kron reduction is applicable, and it states that when buses related to current values that are null have been eliminated, it allows analysis without integrating external values, and the application is integrated by:

$$Y_{jk(new)} = Y_{jk} - \frac{Y_{jp} * Y_{pk}}{Y_{pp}} \tag{4}$$

The construction of Z_{bus} applying cases in order from 1 to 4, being Case 1.

Case 1: Connection between the reference bus and a new bus in the Power System.

It occurs when a new bus is added to the system with an impedance connecting them, one terminal is connected to the reference bus and the other end to the new bus as shown:

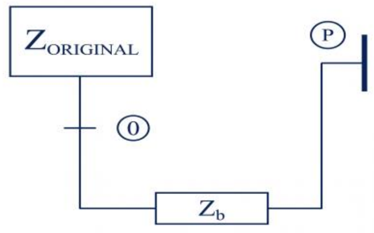


Fig. 1. Visual expression of case 1 between the reference bus and a new bus.

Where the matrix is represented as:

$$Z_{bus} = \left[\begin{array}{ccc|c} & & & 0 \\ & Z_{orig} & & \vdots \\ & & & 0 \\ \hline & & & \\ 0 & .. & 0 & Z_b \\ & & & \end{array}\right] \quad \textbf{(5)}$$

Within the evolution, it presents:

Case 2: From an existing bus k to a new bus p, defined as the procedure of adding a new bus where a specific impedance exists between the two buses, so for the constructed matrix it is modified, in addition a row and a column are added representing the updated connection, and through a visual construction it is shown as:

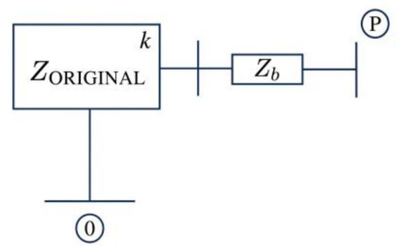


Fig. 2. Development of case 2 between existing bus k and a new bus p.

Expressing the matrix as:

$$\begin{array}{cc} & \begin{array}{cc} (k) & \qquad (p) \end{array} \\ Z_{bus} = \begin{array}{c} (k) \\ \\ (p) \end{array} & \left[\begin{array}{c|c} Z_{orig} & columns\ k \\ \hline rows\ k & Z_{kk} + Z_b \end{array}\right] \end{array} \quad \textbf{(6)}$$

Within the evolution of the cases we add:

Case 3: Represents that from an existing bus k to the reference bus, this case does not involve adding a new bus, however an impedance is added between two existing buses belonging to the electrical system; within the matrix its space is adjusted to the added one, which implies attaching a row and a column showing the recent connection of the system. And the matrix of equation (6), since P and 0 are connected, leads to applying Kron reduction, case 2 occurs and it is applied to eliminate row P and column P.

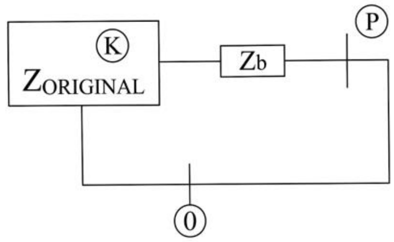


Fig. 3. Position diagram of case 3 between an existing bus k and the reference bus.

And reaching the final step of the algorithm construction, apply:

Case 4: Where it applies that from an existing bus j to an existing bus k, simply an interconnection in the electrical system, and it establishes that the interconnection of the two buses j and k is carried out through a defined impedance, incorporating it and updating the matrix with the modification of the buses.

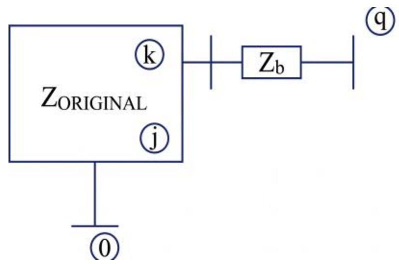


Fig. 4. Design of the diagram for Case 4 between the connection of an existing bus j and an existing bus k.

Matrix show:

$$
\begin{array}{rc}
 & \qquad\qquad\qquad\qquad (q) \\
Z_{bus} = & \left[\begin{array}{c|c} Z_{orig} & \begin{array}{c} columns\ j - \\ columns\ k \end{array} \\ \hline rows\ j - rows\ k & Z_{thevenin,jk} + Z_b \end{array}\right] \\
(q) &
\end{array}
\tag{7}
$$

Construction algorithm through the application of MATLAB Software, which implements a code built by applying the Zbus algorithm and applying Kron reduction for the required conditions:

The system is started through the compilation software, showing a work window with tabs to indicate the type of activity or procedure to be performed.

Applying the Power System construction option, the specifications regarding the electrical system diagram are loaded and each case is selected according to the impedance network requirements.

The software applies matrix visualization through the process of adding, reducing, or modifying matrices to obtain the Zbus or Ybus matrix, as the case may be, together with the generation of the single-line diagram of the network topology, which allows viewing the number and size of the buses, as well as the layout algorithm Sugiyama, Spring, or radial used as part of the dynamic representation.

3 Experimental procedures

3.1 Environment for Modeling and Software Design

The analysis of software for Power Systems requires tools capable of obtaining results through computational simulation or mathematical modeling. Therefore, using MATLAB software, a tool capable of building and solving Power System models based on the Zbus building algorithm was designed and implemented, providing the data required to perform short-circuit studies, load flow analysis, and symmetrical or asymmetrical fault analysis. Its architecture based on a set of windows that start when running the designed program, which takes advantage of matrix algebra, the application of iterations with which solution time is optimized.

3.2 Implementation

The design is divided into evolving activities such as Phase 1 or topology initiation, which defines the required parameters for system modeling, the selection of resolution according to the Kron Reduction or Sparse Inspection Algorithm, and the option to implement between direct complex impedances or physical ACSR-type transmission line models. Phase 2 evolves to the union and step-by-step progression as it determines the topological incidence of each new element entered by the user through the evaluation of its connection buses, applies to the electrical system network one of the four fundamental network addition cases, and in this applies the corresponding algebraic transformations to expand the matrix dimension or apply its reduction. Phase 3 applies matrix resolution or stability verification according to the loaded system, routinely stores the matrices made, which leads to processing the information, and within Phase 4 it manages to apply a matrix engine and map the system, applies a vector system that allows the user to move or analyze topology through an interactive environment. And obtaining data within Phase 5, which allows compiling and exporting data, matrices, topology, and also obtaining a spreadsheet in HTML format in which the data can be navigated and visualized, and in special cases, a total reset where it starts again and clears the process.

3.3 Algorithm and Application of Cases

Within the solution during application time, iterations are applied so that cases are solved sequentially, where once the exercise has been loaded and the program starts, it leads to a solution process where:

Case 1 in the implementation of a reference bus towards a new bus, introducing a branch and where its original matrix is built and expands according to the case applied, since mathematically the matrix expansion is applied, considering its design contemplates being 1 x 1 and is reflected as

$$1 * 1 = (Z_{orig}) \tag{8}$$

$$Z_{orig} = [1 , \quad 1] = Z_{11} * j \tag{9}$$

Case 2 where an existing bus is connected to a new bus, a branch is introduced and the dimension copies the initial matrix of equation (8), and it is copied in the new row and column except the diagonal at row 2 column 2, and is expressed mathematically as:

$$Z_{new} = Z_{kk} * Z_b * j \tag{10}$$

Within Case 3 linking an existing bus to the reference bus, its link branch applies an expansion where the matrix expansion is applied as in the second case in the row and column, and its diagonal applies the mathematics of equation (10), and finally Kron reduction is implied to eliminate the last bus as:

$$Z_{improved}[i, \ j] = Z_{orig} = [i, \ j] - \frac{Z_{orig}[i, \ n] * Z_{orig}[n, \ j]}{Z_{orig}[n, \ n]} \tag{11}$$

Case 4 applies a parallel link between an existing bus and another existing bus, the algorithm requires an expansion that adds one more row and column to the matrix, and the Thevenin equivalent is applied as:

$$Z_{thevenin} = Z_{i,j} + Z_{kk} - 2Z_{j,k} \tag{12}$$

Provide:

$$Z_{new,\,new} = Z_{thevenin} + Z_b * j \tag{13}$$

Applying as in Case 3 Kron reduction by the equation:

$$Z[: , \ j] - Z[: , \ j] \tag{14}$$

4 Results and Discussion

Applying the Power System, where a 5-bus system is described, in addition to a reference bus, it has two sources feeding bus number 1 with 1.10 < 0° and the other connected to bus number 5 with its value of 0.90 < -30°, there is a branch of j 1.0 between bus 1 and reference, bus 1 and bus 2 with a branch of j 0.4, bus 1 and bus 3 with a branch of j 0.5, for the bus 2 connect to bus 3, bus 2 to bus 5 with a branch of j 0.2, bus 3 to bus 4 with a branch value of j 0.125, bus 4 to bus 5 with a branch of j 0.5, and as a closure from bus 5 to reference a bran]ch value of j 1.25.

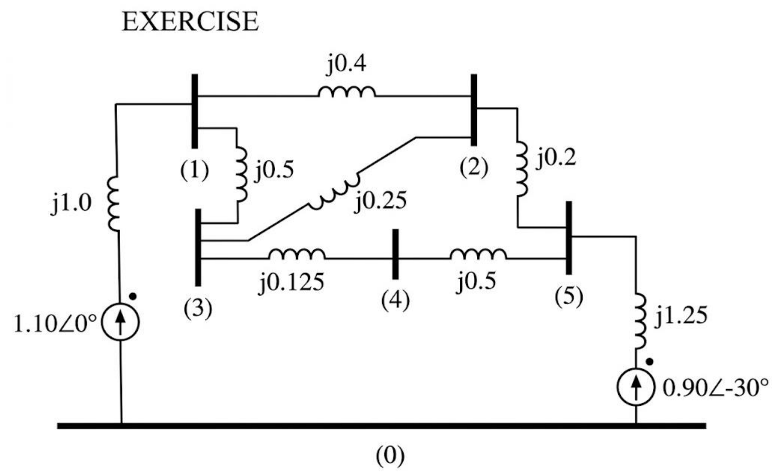


Fig. 5. Electric Power System Exercise.

The first part where the exercise with the corresponding cases is loaded into MATLAB.

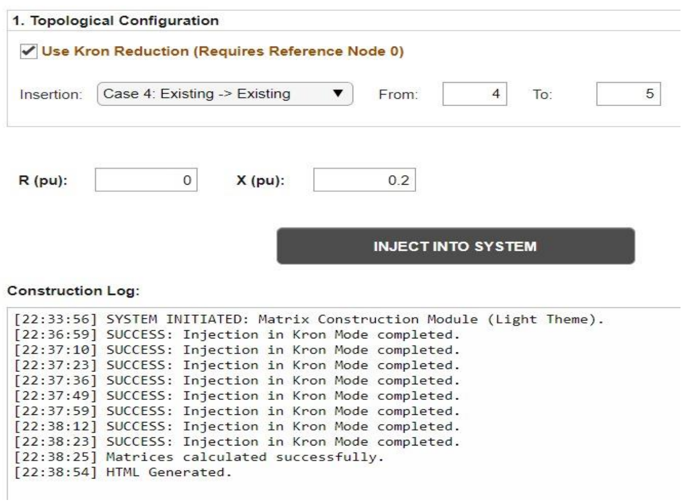


Fig. 6. The data and the topology are loaded and organized within the system.

The process of applying the cases begins, solving the first Case 1 of Electric Power System:

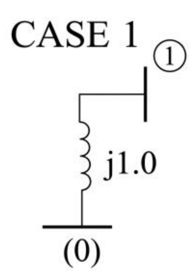


Fig. 7. Case 1 reference bus to bus one.

Giving a 1x1 matrix due to its solution of case 1.

$$Z_{bus\,1} = \left[j1.0 \right] \tag{15}$$

[22:36:59] [KRON_EXP] Case 1: Node 1 from Reference.

Bus	1
1	0 + j1

Fig. 8. The first case in the MATLAB System.

The evolution in the matrix, where the column and row are increased and the new bus with its branch adds a value to the 2x2 matrix.

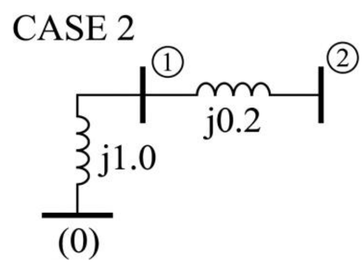


Fig. 9. Case 2 connect bus one to bus two.

$$Z_{bus\ 2} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.4 \end{bmatrix} \quad \textbf{(16)}$$

[22:37:10] [KRON_EXP] Case 2: Node 2 branched from 1.

Bus	1	2
1	0 + j1	0 + j1
2	0 + j1	0 + j1.4

Fig. 10. The matrix built within the MATLAB software is shown, demonstrating that the performed mathematics corresponds.

For the System where it moves to another bus and requires the adjustment of space in the matrix and grows to 3*3.

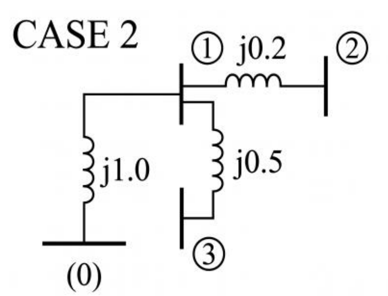


Fig. 11. Case 2 connect bus one to bus three.

$$Z_{bus\ 3} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.4 & j1.0 \\ j1.0 & j1.0 & j1.5 \end{bmatrix} \quad \textbf{(17)}$$

[22:37:23] [KRON_EXP] Case 2: Node 3 branched from 1.

Bus	1	2	3
1	0 + j1	0 + j1	0 + j1
2	0 + j1	0 + j1.4	0 + j1
3	0 + j1	0 + j1	0 + j1.5

Fig. 12. The procedure applies case 2 where the impedance matrix increases when entering the new bus, maintaining its structure in the 3*3 block.

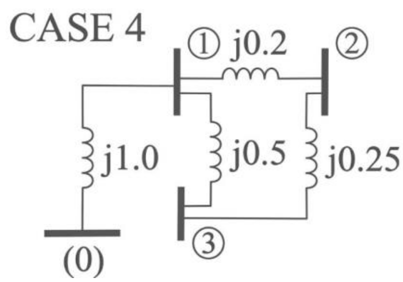


Fig. 13. Case 4 connect bus two for the bus three.

$$Z_{bus\ 4} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.261 & j1.174 \\ j1.0 & j1.174 & j1.283 \end{bmatrix} \tag{18}$$

[22:37:36] [KRON_RED] Case 4: Loop between Nodes 2 and 3.

Bus	1	2	3
1	0 + j1	0 + j1	0 + j1
2	0 + j1	0 + j1.261	0 + j1.174
3	0 + j1	0 + j1.174	0 + j1.283

Fig. 14. Within the matrix made by the software, it relates to the mathematical system of the 3*3 matrix, where bus 3 has already been updated with the data.

Case 2 is applied again and bus number 4 is added with an interconnection branch to bus 3.

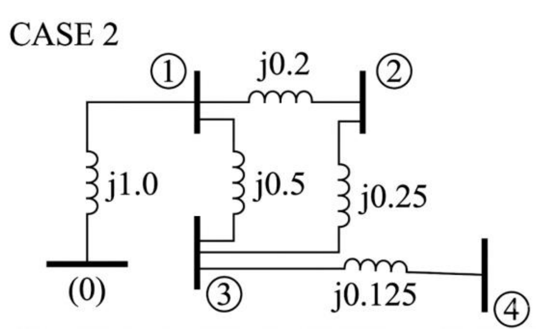


Fig. 15. Case 2 connect bus three to the new bus four.

$$Z_{bus\ 5} = \begin{bmatrix} j1.0 & j1.0 & j1.0 & j1.0 \\ j1.0 & j1.261 & j1.174 & j1.174 \\ j1.0 & j1.174 & j1.283 & j1.283 \\ j1.0 & j1.174 & j1.283 & j1.408 \end{bmatrix} \tag{19}$$

[22:37:49] [KRON_EXP] Case 2: Node 4 branched from 3.

Bus	1	2	3	4
1	0 + j1	0 + j1	0 + j1	0 + j1
2	0 + j1	0 + j1.261	0 + j1.174	0 + j1.174
3	0 + j1	0 + j1.174	0 + j1.283	0 + j1.283
4	0 + j1	0 + j1.174	0 + j1.283	0 + j1.408

Fig. 16. The interpretation of the method by means of the software with the matrix 4*4.

he procedure applies case 2 to connect an existing bus to a new bus as shown.

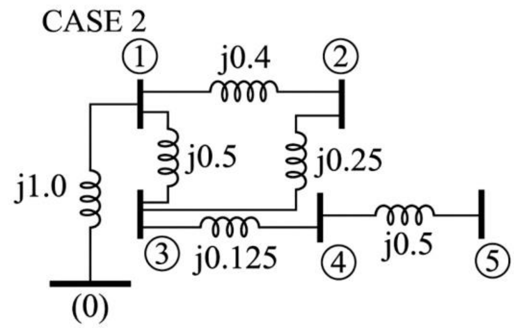


Fig. 17. Applying Case 2 interconnect bus four and bus five.

The matrix is built and increases to 5x5, and Kron reduction is applied.

$$Z_{bus\ 6} = \begin{bmatrix} j1.0 & j1.0 & j1.0 & j1.0 & j1.0 \\ j1.0 & j1.261 & j1.174 & j1.174 & j1.174 \\ j1.0 & j1.174 & j1.283 & j1.283 & j1.283 \\ j1.0 & j1.174 & j1.283 & j1.408 & j1.408 \\ j1.0 & j1.174 & j1.283 & j1.408 & j1.908 \end{bmatrix} \quad \textbf{(20)}$$

[22:37:59] [KRON_EXP] Case 2: Node 5 branched from 4.

Bus	1	2	3	4	5
1	0 + j1	0 + j1	0 + j1	0 + j1	0 + j1
2	0 + j1	0 + j1.261	0 + j1.174	0 + j1.174	0 + j1.174
3	0 + j1	0 + j1.174	0 + j1.283	0 + j1.283	0 + j1.283
4	0 + j1	0 + j1.174	0 + j1.283	0 + j1.408	0 + j1.408
5	0 + j1	0 + j1.174	0 + j1.283	0 + j1.408	0 + j1.908

Fig. 18. This matrix that is built using MATLAB software through the algorithm accompanies the mathematical procedure of the matrices that were performed manually.

Case 3 is applied, which indicates the connection of an existing bus to the reference bus as shown in the diagram.

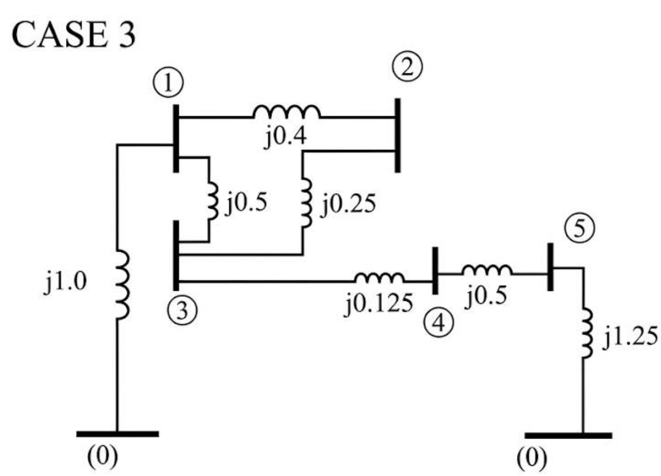


Fig. 19. Implication of the power system built with case 3 to make the connection to the reference bus.

$$Z_{bus\,6} = \begin{bmatrix} j0.6833 & j0.6282 & j0.5938 & j0.5542 & j0.3959 \\ j0.6282 & j0.8244 & j0.6971 & j0.6506 & j0.4647 \\ j0.5938 & j0.6971 & j0.7616 & j0.7108 & j0.5077 \\ j0.5542 & j0.6506 & j0.7108 & j0.7801 & j0.5572 \\ j0.3959 & j0.4647 & j0.5077 & j0.5572 & j0.7552 \end{bmatrix} \quad \textbf{(21)}$$

[22:38:12] [KRON_RED] Case 3: Node 5 to Reference.

Bus	1	2	3	4	5
1	0 + j0.6833	0 + j0.6282	0 + j0.5938	0 + j0.5542	0 + j0.3959
2	0 + j0.6282	0 + j0.8244	0 + j0.6971	0 + j0.6506	0 + j0.4647
3	0 + j0.5938	0 + j0.6971	0 + j0.7616	0 + j0.7108	0 + j0.5077
4	0 + j0.5542	0 + j0.6506	0 + j0.7108	0 + j0.7801	0 + j0.5572
5	0 + j0.3959	0 + j0.4647	0 + j0.5077	0 + j0.5572	0 + j0.7552

Fig. 20. The software processes case 3 specifically, where the existing bus is connected to the reference bus.

The diagram represents the closure through the last connection by means of case 4, where two buses exist and the branch joins them.

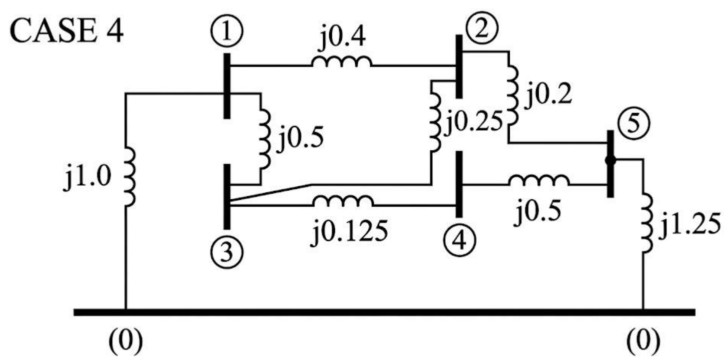


Fig. 21. By applying case 4, the diagram shows the closure between bus 2 and bus 5.

$$Z_{bus\,7} = \begin{bmatrix} j0.6198 & j0.5299 & j0.5421 & j0.5287 & j0.4753 \\ j0.5299 & j0.6722 & j0.617 & j0.6111 & j0.5876 \\ j0.5421 & j0.617 & j0.7195 & j0.69 & j0.5724 \\ j0.5287 & j0.6111 & j0.69 & j0.7699 & j0.5891 \\ j0.4753 & j0.5876 & j0.5724 & j0.5891 & j0.6559 \end{bmatrix} \quad \textbf{(22)}$$

[22:38:23] [KRON_RED] Case 4: Loop between Nodes 2 and 5.

Bus	1	2	3	4	5
1	0 + j0.6198	0 + j0.5299	0 + j0.5421	0 + j0.5287	0 + j0.4753
2	0 + j0.5299	0 + j0.6722	0 + j0.617	0 + j0.6111	0 + j0.5876
3	0 + j0.5421	0 + j0.617	0 + j0.7195	0 + j0.69	0 + j0.5724
4	0 + j0.5287	0 + j0.6111	0 + j0.69	0 + j0.7699	0 + j0.5891
5	0 + j0.4753	0 + j0.5876	0 + j0.5724	0 + j0.5891	0 + j0.6559

Fig. 22. The software, as a final case, gives the solution through the link between bus 2 and bus 5.

Once the diagram is completed, the network becomes fully connected, and the last case matches the result and is represented through matrix algebra and the software.

$$Z_{bus} = \begin{bmatrix} j0.6198 & j0.5299 & j0.5421 & j0.5287 & j0.4753 \\ j0.5299 & j0.6722 & j0.617 & j0.6111 & j0.5876 \\ j0.5421 & j0.617 & j0.7195 & j0.69 & j0.5724 \\ j0.5287 & j0.6111 & j0.69 & j0.7699 & j0.5891 \\ j0.4753 & j0.5876 & j0.5724 & j0.5891 & j0.6559 \end{bmatrix} \quad \textbf{(23)}$$

Upon completing the Zbus matrix shown in equation (23), it is verified by constructing Ybus in the software's matrix viewer, loading the data, and again obtaining Zbus.]

[22:38:25] [SOLVER] Matrices resolved.

Bus	1	2	3	4	5
1	0 + j0.6198	0 + j0.5299	0 + j0.5421	0 + j0.5287	0 + j0.4753
2	0 + j0.5299	0 + j0.6722	0 + j0.617	0 + j0.6111	0 + j0.5876
3	0 + j0.5421	0 + j0.617	0 + j0.7195	0 + j0.69	0 + j0.5724
4	0 + j0.5287	0 + j0.6111	0 + j0.69	0 + j0.7699	0 + j0.5891
5	0 + j0.4753	0 + j0.5876	0 + j0.5724	0 + j0.5891	0 + j0.6559

Fig. 23. Verification of the results successfully through the software, adding that during the process the evolution of its solution is presented in real time.

Y-Bus Matrix

	1	2	3	4	5
1	0-j5.5	0+j2.5	0+j2	0 + j0	0 + j0
2	0+j2.5	0-j11.5	0+j4	0 + j0	0+j5
3	0+j2	0+j4	0+j14	0+j8	0 + j0
4	0 + j0	0 + j0	0+j8	0+j10	0+j2
5	0 + j0	0+j5	0 + j0	0+j2	0-j7.8

Fig. 24. The values of the Y_{bus} matrix of the power system already loaded into the matrix viewer.

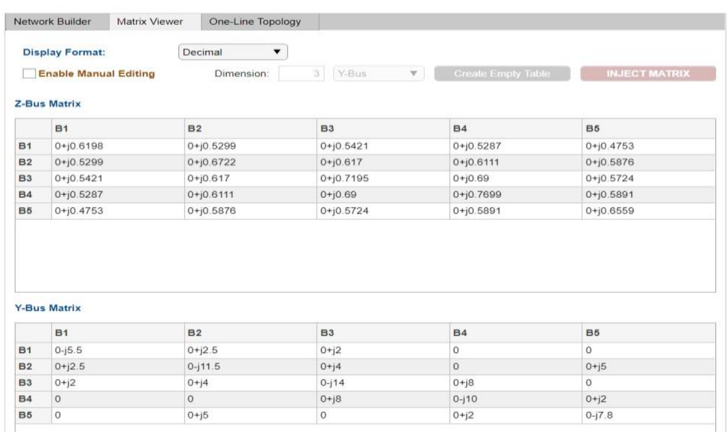
It yields the following Z_{bus} matrix.

Z-Bus Matrix

	B1	B2	B3	B4	B5
B1	0+j0.6198	0+j0.5299	0+j0.5421	0+j0.5287	0+j0.4753
B2	0+j0.5299	0+j0.6722	0+j0.617	0+j0.6111	0+j0.5876
B3	0+j0.5421	0+j0.617	0+j0.7195	0+j0.69	0+j0.5724
B4	0+j0.5287	0+j0.6111	0+j0.69	0+j0.7699	0+j0.5891
B5	0+j0.4753	0+j0.5876	0+j0.5724	0+j0.5891	0+j0.6559

Fig. 25. Resulting Z_{bus} matriz

The result obtained by injecting the matrix into the Y_{bus} space and starting the procedure to obtain the data in Z_{bus} is consistent with what is shown by equation (23), the MATLAB program, and also relates to Fig. 26.



	B1	B2	B3	B4	B5
B1	0+j0.6198	0+j0.5299	0+j0.5421	0+j0.5287	0+j0.4753
B2	0+j0.5299	0+j0.6722	0+j0.617	0+j0.6111	0+j0.5876
B3	0+j0.5421	0+j0.617	0+j0.7195	0+j0.69	0+j0.5724
B4	0+j0.5287	0+j0.6111	0+j0.69	0+j0.7699	0+j0.5891
B5	0+j0.4753	0+j0.5876	0+j0.5724	0+j0.5891	0+j0.6559

	B1	B2	B3	B4	B5
B1	0-j5.5	0+j2.5	0+j2	0	0
B2	0+j2.5	0-j11.5	0+j4	0	0+j5
B3	0+j2	0+j4	0-j14	0+j8	0
B4	0	0	0+j8	0-j10	0+j2
B5	0	0+j5	0	0+j2	0-j7.8

Fig. 26. Integrated viewer in the MATLAB software as a matrix viewer, also including a manual editing space.

Demonstration of the single-line topology where the power system is presented, also being an implicit process within the MATLAB program, allowing it to be generated automatically, by applying the software to obtain Zbus through the case algorithm and applying Kron reduction, it led to obtaining the expected result, manually, using MATLAB software.

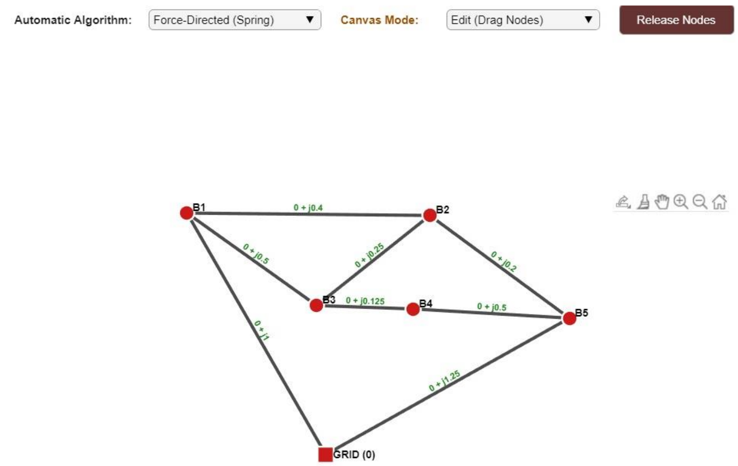


Fig. 27. The software as a last case gives the solution through the link between bus 2 and bus 5.

5 Conclusions

The implementation of the Zbus construction algorithm using MATLAB allowed the development of a computational tool capable of modeling and solving Power Systems in a structured, efficient and adaptable way to the different existing network configurations. The integration of the four fundamental construction cases, together with the application of Kron reduction, made it possible to update and modify impedance matrices without continuously resorting to complete matrix inversion processes, thus reducing computational cost and possible numerical errors that may arise.

The software development demonstrated that the application of evolutionary matrix methods improves the analysis of electrical systems, especially those used in studies where network expansion or topological modification requires fast and reliable solutions. Likewise, the incorporation of topological visualization and iterative processing tools facilitates the interpretation of results and the monitoring of system behavior during the construction process of Ybus and Zbus matrices.

The use of software to implement the algorithm leads to academic development in such a way that one can visualize the mathematical methods, the design of the systems through which others of the same category can be verified. The results obtained in the test system successfully validated the performance of the implemented algorithm, showing consistency in the representation of impedances and stability during matrix reduction and expansion. This allows us to consider the proposal as a useful alternative for academic and research applications related to power flow, symmetrical and asymmetrical fault analysis, stability and protection studies in Power Systems.

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