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Oil Platform Transport Problem (OPTP) is NP-hard

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Abstract. The Oil Platform Transport Problem is considered as a combination/interlink of the two well-studied NP-Hard/NP-Complete problems: the Helicopter Routing Problem-HRP (a generalization of the Split Delivery Vehicle Routing Problem) and the one-dimensional Bin Packing Problem (BPP-1). The Oil Platform Transport Problem consist of to minimize the cost of carry resources, goods or people from one location (airport/platform) to another location (airport/platform) using helicopters with some restrictions as capacity and time windows. We provide the proof that this problem is NP-Hard/NP-Complete Problem by the polynomial transformation using formal languages between the Vehicle Routing Problem and the Oil Platform Transport Problem. We propose a new mathematical model to the Oil Platform Transport problem, and we present the parameters or characterization of Oil Platform Transport Problem instances of Mexican state-owned petroleum company (PEMEX). We generated 5 instance set, each instance set has 50 cases of randomly generated instances and real instances (with GIS data) of PEMEX Oil Platforms. We use the CPLEX solver to find the optimal cost of carrier resources, goods or people contains in the Oil Platform Transport Problem.
Keywords. Oil Platform, Transport Problem, Split Delivery Vehicle Routing Problem.

I. INTRODUCTION

PEMEX [1] is the biggest company of Mexico and Latin America, and the most important fiscal contributor of the Mexico. It is of the few oil companies of the world that develops all the productive chain of the industry, from the exploration, to the distribution and commercialization of all the products. PEMEX has a total asset worth of $415.75 billion, and is the world's second largest non-publicly listed company by total market value, and Latin America's second largest enterprise by annual revenue, surpassed only by Petrobras (a Brazilian petrochemical company). PEMEX operates by the conduct of a Corporative Office and four Subsidiary entities: PEMEX Exploration and Production, PEMEX Refining, PEMEX Gas and Basic Petrochemicals and PEMEX Petrochemical.

The mission of PEMEX Exploration and Production is to maximize the reserves of the country, both in crude and natural gas economic value, on a long term basis, guaranteeing safety, both for personnel and facilities, in harmony with the community and the environment. Their main activities are oil and natural gas exploration and exploitation; conveyance, storage in terminals and first hand commercialization; these are carried out daily in four geographic regions comprising the total Mexican territory: North, South, Northeast Offshore and Southeast Offshore [1].

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PEMEX operates in the Southeast offshore (Campeche Sound is the maritime area corresponding to the submarine prolongation of the Yucatan Peninsula in the Gulf of Mexico) about 100 offshore platforms in living permanently-being rotated about 5000 people, often the facilities are cross-platform modular assemblies, one main and other satellites, joined by giant pipes that serve both structures for bridges form a remarkable geometry pipeline and connections. Most offshore platforms are based on extraction of crude oil and natural gas. Most offshore platforms are based on crude oil and natural gas, combined invariably arise. In some wells dominates liquid, but always with some percentage of gas, in others, the composition is reversed.

The platforms are self-sufficient in high measure: get drinking water through seawater desalination plants (treated sewage) have thermo-electric generators that run on natural gas; external supplies carry weekly the ship carrying perishables. Another group of platforms are exploration, which is precisely why they are not fixed but mobile platforms, hydraulic lifting legs that rest on the seabed, or pontoons filled or emptied of water by pumping, with a mechanism similar to submarines. A third group of platforms is supported both for technical and offshore pumping or other administrative needs-like, as in the case of an extraordinary floating hotel that hosts hundreds of workers at the drilling platforms and daily are moved by ship. Within this last group of structures stands the "platform brains", which is the telecommunications tower, equipped with radios and computerized radar equipment for heavy marine/air traffic control. The equipment includes radar with synthesizers that draw on screen the captured ship type, and a kind of telephoto or zoom for impressive approaches of the ship in question.

In the City of Carmen, Campeche, Mexico operates a modern heliport with capacity for 40 turbine apparatus, and an installation of our oil industry seems a public air terminal. The air terminal could be used to carry resources, goods or people from one location (airport/platform) to another location (airport/platform) using helicopters with some restrictions as capacity and time windows.

Fig. 1. Oil Platforms in Campeche Sound.

This paper propose a new mathematical model to the Oil Platform Transport Problem (interlink of the two well-studied NP-Hard/NP-Complete problems: the Helicopter Routing Problem-HRP, a generalization of the Split Delivery Vehicle Routing Problem, and the Bin Packing Problem - BPP), the proof that this problem is NP-Hard/NP-Complete Problem, the characterization of Oil Platform Transport Problem instances of Mexican state-owned petroleum company (PEMEX), and the solutions by the CPLEX solver. Section 2 presents the related works of the Oil Platform Transport Problem, Section 3 the formulation of the new Mathematical model, the proof of the polynomial transformation and the characterization of the problem instances, section 4 the experimentation of the problem by the CPLEX solver, and the last section presents the conclusions.

II. RELATED WORKS

Hensen et al. [2] formulated the problem of optimal location and sizing of offshore platforms for oil exploration of PETROBRAS (the Brazilian state-owned company in charge of oil prospection, exploitation and distribution). The problem is formulated as a multi-capacitated plant location problem and they presented the exact and its approximate solution by MIP/MPSX and by a Tabu Search heuristic.

Menezes et al. [3] propose an optimization model to reduce the number of offshore landings, total flight time of the helicopters, flight cost and annual savings of offshore oil platforms of PETROBRAS. They designed a column-generation algorithm that exploit the problem structure.

Barbarosoğlu et al. [6] Develop a mathematical model for helicopter mission planning during a disaster relief operation. The decisions inherent in the problem decompose hierarchically into two sub-problems where tactical decisions are made at the top level, and the operational routing and loading decisions are made at the base level.

Fiala Timlin and Pulleyblank [7] developed two heuristics for the 45 offshore oil platforms of the Mobil Producing Nigeria. The first heuristics ignore the problem of helicopter capacity, and the second handles it explicitly. Mobil has cited two benefits of the optimization software: all routing requirements are satisfied, and a significant reduction of total daily flying time has been achieved. Each day certain platforms must be visited to regulate flow rates and a number of people must be transported between specified pairs of platforms. They find a route for each daily set of stops that satisfied all the requirements and minimized the total distance flown.

Gribkovskaia et al. [8] introduce a pickup and delivery problem encountered in the servicing of offshore oil and gas platforms in the Norwegian Sea. They describe diverse heuristics as a Tabu algorithm to the Single Vehicle Pickup and Delivery Problem with Capacitated Customers consists of designing at least cost vehicle (vessel) route starting and ending at the depot (base), visiting each customer (platform), and such that there is always sufficient capacity in the vehicle and at the customer location to perform the pickup and delivery operations.

Fagerholt and Heimdal [9] show Algorithms for the transfer of ballast for an oil installation. Two alternative algorithms are evaluated; one mixed integer programming (MIP) model and one heuristic algorithm. The heuristic algorithm is installed in control systems in a platform operating in the North Sea.

Faiz Al-Khayyal and Seung-June Hwang [10] developed a mathematical model for planning the sailing routes and loading/unloading schedules for a fleet of ships carrying liquid bulk products across a network of harbours during a specified planning horizon (multi-ship pickup–delivery problem). The model is for the maritime chemical transport companies, including oil companies serving an archipelago of islands. The model is solved using CPLEX and other algorithms.

Grob [11] presents a mathematical formulation of the surveillance routing Problem (an extension of the on-line travelling salesman problem - OLTSP). The simulation model SURPASS (SURface Picture ASSessment) describes and visualizes the operations of surveillance operations of the Royal Netherlands Navy. SURPASS is a computer model which simulates the complicated process of maritime surface surveillance. The structure of SURPASS and the way in which it solves the surveillance routing problem is explained.

Iakovou [12] presents the development of a strategic multiobjective network flow model, allowing for risk analysis and routing, with multiple commodities, modalities and origin-destination pairs. He presents the development of a comprehensive maritime oil transportation model that integrates the risk assessment methodology. The development of an interactive solution methodology is presented by its implementation via a World Wide Web-based software package of the marine network of the Gulf of Mexico.

Halvorsen-Weare et al. [13] considered a fleet composition and periodic routing problem that appears in the offshore supply vessel service with Statoil, the leading operator on the Norwegian continental. They present a voyage-based solution method and a computational study shows how the solution method can be used to solve real-life problems.

Qian et al. [14] present a mathematical model of an offshore petroleum industry and a tabu search heuristic applied to the helicopter routing problem. Three routing policies are considered: a direct routing policy, a Hamiltonian routing policy, and a general routing policy. Computational experiments are conducted on instances derived from real data in order to assess and compare these policies under a travel time, a passenger risk and a combined passenger and pilot risk objective.

Velasco et al. [15] propose a method based on a Non-dominated Sorting Genetic Algorithm (NSGA-II) to solve a pickup and delivery problem (the case of oil companies that use helicopters to transport engineers, technicians and assistant personnel from platform to platform).

Agarwal and Ergun [16] present a model, a greedy heuristic, a column generation-based algorithm, a two-phase Benders decomposition-based algorithm, and an efficient iterative search algorithm to solve the ship-scheduling and the cargo-routing problems with up to 20 ports and 100 ships.

Iakovou et al. [17] propose an efficient two-phase solution approach to solve the strategic level routing problem of hazardous materials in marine waters over a multicommodity network with multiple origins–destinations of the marine transportation system of oil products in the Gulf of Mexico.

Shen et al. [18] present an inventory routing problem in crude oil transportation with a heterogeneous fleet of tankers. They developed a Lagrangian relaxation approach and an existing meta-heuristic algorithm to solve the problem.

Halvorsen-Weare et al. [19] considers a real-life Liquefied natural gas (LNG) ship routing and scheduling problem where a producer is responsible for transportation from production site to customers all over the world.

Taylor et al. [20] presents a study which shows that the offshore petroleum environment is ideally suited for air medical transport, as injuries are common and medical illnesses are to be expected. This study was undertaken to review the incidence of OSI-related incidents, injuries and deaths, and report the initial experience of a civilian hospital-based helicopter air transport program in the evacuation of offshore patients. One of the nine patients had been exposed to a potentially hazardous substance, requiring changes in the air medical team's operations, aircraft and equipment.

Galvão and Guimarães [21] discuss the design and implementation of a computerized system to control the helicopter operations (concentrating on an interactive algorithm designed to route the helicopters) used for the transportation of passengers between the continent and offshore oil exploration and production platforms in Brazil.

Moreno et al. [22] presents a column generation based heuristic algorithm for the problem of planning the flights of helicopters to attend transport requests among airports in the continent and offshore platforms on the Campos basin for the Brazilian State Oil Company (Petrobras).

Ozdamar [23] propose a system with a mathematical model and a Route Management Procedure (RMP) that post-processes the outputs of the model. The system is concerned with helicopter operations that involve the last mile distribution and pickups for post-disaster medical care and injured evacuation in a scenario that is based on the post-earthquake damage data provided by the Disaster Coordination Center of Istanbul.

Sierksma and Tijssen [24] present a Cluster-and-Route Heuristic to solve the flight schedule for helicopters to offshore platform locations for exchanging crew people employed on these platforms. The Cluster-and-Route procedure constructs a suitable clustering of the platforms and simultaneously forms the routes of the helicopter flights associated with the clusters.

Timlin and Pulleyblank [25] developed two heuristics that produce very good solutions to minimize the total distance flown for the Mobil Producing Nigeria (the company has an offshore oil field consisting of approximately 45 platforms). The first heuristics ignore the problem of helicopter capacity, and the second handles it explicitly.

Table 1. Related Works

Research	Problem	Contribution
Hensen et al. [2]	Multi-capacitated Plant Location Problem	The exact and its approximate solution MIP/MPSX and by a Tabu Search heuristic of PETROBRAS
Menezes et al. [3]	Helicopter Routing Problem	A column-generation algorithm to solve PETROBRAS instances.
Barbarosoğlu et al. [6]	Helicopter Routing Problem	Propose an optimization model to reduce the number of offshore landings, total flight time of the helicopters, flight cost and annual savings of offshore oil platforms of PETROBRAS
Fiala Timlin et al. [7]	Helicopter Routing Problem	Developed two heuristics for the 45 offshore oil platforms of the Mobil Producing Nigeria
Gribkovskaia et al. [8]	Pickup and delivery problem	Describe diverse heuristics as a Tabu algorithm to a pickup and delivery problem encountered in the servicing of offshore oil and gas platforms in the Norwegian Sea
Fagerholt and Heimdal [9]	The transfer of ballast for an oil installation	Two alternative algorithms are evaluated; one mixed integer programming (MIP) model and one heuristic algorithm. The heuristic algorithm is installed in control systems in a platform operating in the North Sea.
Faiz Al-Khayyal and Seung-June Hwang [10]	multi-ship pickup–delivery problem	A mathematical model for planning the sailing routes and loading/unloading schedules for a fleet of ships carrying liquid bulk products across a network of harbours during a specified planning horizon, and it is solved using CPLEX and other algorithms.
Grob [11]	Surveillance Routing Problem	A simulation model SURPASS (SURface Picture ASSessment) to describe and visualises the operations of surveillance operations of the Royal Netherlands Navy.
Iakovou [12]	multiobjective network flow model	He presents the development of a comprehensive maritime oil transportation model that integrates the risk assessment methodology. And the development of an interactive solution methodology is presented by its implementation via a World Wide Web-based software package of the marine network of the Gulf of Mexico.
Halvorsen-Weare [13]	Supply vessel planning problem	A voyage-based solution method and a computational study show how the solution method can be used to

		solve real-life problems. The Norwegian continental.
Qian et al. [14]	Helicopter routing problem	Three routing policies are considered: a direct routing policy, a Hamiltonian routing policy, and a general routing policy.
Velasco et al. [15]	A pick-up and delivery problem	They propose a method based on a Non-dominated Sorting Genetic Algorithm (NSGA-II) to solve a pickup and delivery problem (the case of oil companies that use helicopters to transport engineers, technicians and assistant personnel from platform to platform).
Agarwal and Ergun [16]	Ship-scheduling and the cargo-routing problems	A greedy heuristic, a column generation-based algorithm, a two-phase Benders decomposition-based algorithm, and an efficient iterative search algorithm to solve the ship-scheduling and the cargo-routing problems with up to 20 ports and 100 ships.
Iakovou et al. [17]	Single hazmat problem	An efficient two-phase solution approach to solve the strategic level routing problem of hazardous materials in marine waters over a multicommodity network with multiple origins–destinations of the marine transportation system of oil products in the Gulf of Mexico.
Shen et al. [18]	Inventory routing problem	They developed a Lagrangian relaxation approach and an existing meta-heuristic algorithm to solve the problem.
Halvorsen-Weare et al. [19]	Ship routing and scheduling problem	A real-life Liquefied natural gas (LNG) ship routing and scheduling problem where a producer is responsible for transportation from production site to customers all over the world.
Taylor et al. [20]	Helicopter air medical transport Problem	Presents a study which shows that the offshore petroleum environment is ideally suited for air medical transport, as injuries are common and medical illnesses are to be expected.
Galvão and Guimarães [21]	Helicopter Routing Problem	Discuss the design and implementation of a computerized system to control the helicopter operations (concentrating on an interactive algorithm designed to route the helicopters) used for the transportation of passengers between the continent and offshore oil exploration and production platforms in Brazil.

Moreno et al. [22]	Helicopter Routing Problem	A column generation based heuristic algorithm for the problem of planning the flights of helicopters to attend transport requests among airports in the continent and offshore platforms on the Campos basin for the Brazilian State Oil Company (Petrobras).
Ozdamar [23]	Helicopter Routing Problem	A system with a mathematical model and a Route Management Procedure (RMP) that post-processes the outputs of the model. The system is concerned with helicopter operations that involve the last mile distribution and pickups for post-disaster medical care and injured evacuation in a scenario that is based on the post-earthquake damage data provided by the Disaster Coordination Center of Istanbul.
Sierksma and Tijssen [24]	Split Delivery Vehicle Routing Problem	A Cluster-and-Route Heuristic to solve the flight schedule for helicopters to offshore platform locations for exchanging crew people employed on these platforms.
Timlin and Pulleyblank [25]	Helicopter Routing Problem	Developed two heuristics to minimize the total distance flown for the Mobil Producing Nigeria. The first heuristics ignore the problem of helicopter capacity, and the second handles it explicitly.

III. OIL PLATFORM TRANSPORT PROBLEM (OPTP)

The Oil Platform Transport Problem consist of to minimize the cost of carry resources, goods or people from one location (airport/platform) to another location (airport/platform) using helicopters with some restrictions as capacity and time windows. The Oil Platform Transport Problem is considered as a combination/interlink of the two well-studied NP-Hard/NP-Complete problems: the Helicopter Routing Problem-HRP (a generalization of the Split Delivery Vehicle Routing Problem [4,5]) and the one-dimensional Bin Packing Problem (BPP-1).

The Helicopter Routing Problem is NP-hard Problem [31, 32]. The Helicopter Routing Problem [30] is a directed graph $G = (V,A)$, where $V = \{0, \ldots, n\}$ is the node set, $A=\{(i,j):i,j\in V,i\neq j\}$ is the arc set, and $T = (t_{ij})_{n+1,n+1}$ is a travel time matrix defined on A. Node 0 corresponds to the depot (the onshore heliport), and the remaining nodes correspond to customers (the offshore installations). The offshore installations are served by a homogenous fleet composed of m helicopters, each having the same capacity Q, where m is a decision variable. Each installation i has a number p_i of workers to be picked up and brought back to the onshore heliport, and a number d_i of workers to be delivered to that installation from the onshore heliport. We denote by g the number of pilots who fly the helicopter.

The one-dimensional bin packing Problem is NP-Complete problem [26]. The one-dimensional bin packing problem [29] mention that given a finite set U of n items with sizes $w = \{w_1, \ldots, w_n\}$, a positive integer number

c that represents the bin capacity and a positive integer number K (maximal number of bins), the problem consists of determining if there exists a partition of U consisting of disjoint sets U_1, U_2, …, UK such that the sum of the items sizes in each U_i is c or less.

We propose a new mathematical model to the Oil Platform Transport problem. The mathematical model of Oil Platform Transport Problem is formed by the equations 1-10: The equation (1) is the objective function, it consists of minimizing the distance and cost of transportation of the helicopter; the equation (2) consist of a helicopter h belonging to a fleet H, serves only platforms assigned to the route passing once by each platform; the equation (3) is the number of helicopters serving each route to start the tour is the same at the end of the tour; the equation (4) is the loading and unloading time of packets assigned to each stop on the route must not exceed the total travel time allotted; the equation (5) mentions if the helicopter makes a tour R <= [1, ..., 7] one to seven nautical miles the flight altitude should be 1500 feet. The equation (6) mention If the helicopter makes a tour R <= [7, ..., 13] one to seven nautical miles the flight altitude should be 2500 feet. The equation (7) mention If the helicopter makes a tour R <= [13, ..., 32] one to seven nautical miles the flight altitude should be 3500 feet. The equation (8) mentions the vehicle capacity Q must not be exceeded by demand and the capacity is set based on the characteristics of the helicopter. The equation (9) is the total packages weight D_w of demand must not exceed weight allowance for the helicopter D_{wh} (weight allowed by the helicopter h). In the equation (10) is the Time Window of the model.

$$Min \sum_{i=0}^{n} \sum_{j=0}^{m} \sum_{h=1}^{H} C_{ij} X_{ijh} \tag{1}$$

$$\sum_{h=1}^{H} \sum_{i=0}^{n} X_{ijh} + X_{i0h} \quad j = 1, 2, 3, \ldots, m \tag{2}$$

$$\sum_{h=1}^{H} \sum_{j=1}^{m} X_{0jh} = \sum_{h=1}^{H} \sum_{i=0}^{n} X_{i0h} = H \tag{3}$$

$$\sum_{i=0}^{n} \sum_{j=1}^{m} X_{ijh} t_{ij} < \tau \tag{4}$$

$$\sum_{i=0}^{n} \sum_{j=1}^{m} X_{ijh} \leq [1,...,7mn] \rightarrow A_{X_{ijh}} = 1500 ft \tag{5}$$

$$\sum_{i=0}^{n} \sum_{j=1}^{m} X_{ijh} \leq [7,...,13mn] \rightarrow A_{X_{ijh}} = 2500 ft \tag{6}$$

$$\sum_{i=0}^{n} \sum_{j=1}^{m} X_{ijh} \leq [13,...,32mn] \rightarrow A_{X_{ijh}} = 3500 ft \tag{7}$$

$$\sum_{i=0}^{n} \sum_{j=1}^{m} X_{ijh} Q_h \leq Dp \quad \text{h=1, 2 ,3,…,} \ h \in H \tag{8}$$

$$\sum_{i=0}^{n} \sum_{j=1}^{m} X_{ijh} Dw_h \leq Dw \quad \text{h=1, 2, 3,…} \ h \in H \tag{9}$$

$$tw_i + tw_j + twX_{ijh} \leq tw \tag{10}$$

Where: C is the cost generated of the transportation from the i platform to the j platform, i, j = source and destination, X = distance from i to j in nautical miles, FH = fleet of helicopters (homogeneous, heterogeneous) with different sizes of helicopters, h = Helicopter k belongs to H (number of helicopters in the fleet), m is the last platform visited on the route before to get back to the deposit (airport/heliport), A = altitude at which the helicopter must fly, Q = capacity of the aircraft (relative to size), best combination found with the accommodation packages on demand and the ability of the helicopter, Dp = Demand (Number of packets or persons to be transported), Dw_h = maximum weight allowed by the helicopter h, $Dw_h = \{Dw_1, Dw_2, Dw_3, Dw_4\}$ the sum of all weights of the packages on each platform must not exceed the weight allowed for the helicopter to fly, t_l = time of arrival at node i, tp_i = starting time of node i, tp_d = time of loading and unloading of packages, τ = approximate total time to the next node in the route, tw = time attention, tw_i = time of service request to the platform i, tw_j = time attention to application platform j, z = total cost of transportation, mn = the nautical miles.

Theorem 1. The Oil Platform Transport Problem (OPTP) problem is NP-Hard.
Proof: This problem is NP-Hard with a polynomial transformation to the Vehicle Routing Problem (VRP).

The Polynomial transformations are used for proving that a problem is NP-complete or NP-Hard based on the different approaches [27]: using the theory of NP-completeness, using graph theory, and using formal language theory [28]. The usage of the theory of formal languages in polynomial Transformations consist [28]: a) Lexical analysis recognize and convert the character stream from the input source program or sequence of characters to valid words of the language or tokens, b) Syntactic analysis consider the sequence of tokens for possible valid constructs of the language, c) Semantic analysis: determine the meaning of the language, d) Error handling: detect the lexical, syntactic, semantic, and logical errors, and e) Language generation: generate the target code or the target language. The steps for polynomial transformation using formal language theory are:
1. Select an NP-complete problem A (OPTP).

2. Define a formal language L_1 for the NP-complete problem A. $L_1 = \{IN, N_1, L_1, Le_1, D_1, W_1, PE_1, Pqg_1, Pqm_1, Pqn_1, e_1, Du_1, RT_1, \ldots, N_n, L_n, Le_n, D_n, W_n, PE_n, Pqg_n, Pqm_n, Pqn_n, e_n, Du_n, RT_n\}$. Where: IN is the instance name, N the number of elements or platforms, L the Latitude, Le the Length, D the Demand, W the weight, PE one seat Measure, Pqg package of Big Size, Pqm package of normal size, Pqn package of medium size, e the earliest time, Di the Due Time, RT the Riding Time. Σ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, '{', '}', '.', ';', '=', '-', 'O', 'P', 'T'}, and the following BNF grammar:

```
<instance>:=<NameProblem> <Equal> <sentences>;
<sentences>:=<KOpen> <TNum> <semicolon>
<Num> <semicolon> <NumCap> <KClose>;
<TNum>:=<Num>;
<NumCap>:=<Num>;
<Num>:=<Num> <Comma> | <Integer> | <Decimal> ;
<Integer>:=<Digit>{<Digit>}*;
<Decimal>:= <Integer> <Dot> <Integer> | <Negative> <Integer> <Dot> <Integer> | <Integer> <Dot> | <Dot> <Integer>
<Digit>:=0|1|2|3|4|5|6|7|8|9;
<NameProblem>:='OPTP';
<Equal>:='=';
<KOpen>:='{';
```

```
<KClose>:=‘}’;
<semicolon>:=‘;’;
<Comma>:=‘,’;
<Negative>:=‘-’;
<Dot>:=‘.’;
```

3. Select an NP-complete problem B (VRP).

4. Define a formal language L_2 for the NP-complete problem B. $L_2 = \{VN, C, (CN_1, XCO_1, YCO_1, D_1, RT_1, DT_1, ST_1, \ldots, CNz, XCO_z, YCO_z, D_z, RT_z, DT_z, ST_z)\}$. Where: VN = Vehicle Number, C = Capacity, CN = Customer Number, XCO = X Coord., YCO = Y Coord., D = Demand, RT = Ready Time, DT = Due date, ST = Service Time. Σ={0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ‘{’, ‘}’, ‘,’, ‘;’, ‘=’, ‘.’, ‘V’, ‘R’, ‘P’}, and the following BNF grammar:

```
<instance>:=<NameProblem> <Equal> <sentences>;
<sentences>:=<KOpen> <TNum> <semicolon>
<Num> <semicolon> <NumCap> <KClose>;
<TNum>:=<Num>;
<NumCap>:=<Num>;
<Num>:=<Num> <Comma> | <Integer>;
<Integer>:=<Digit>{<Digit>}*;
<Decimal>:= <Integer> <Dot> <Integer> | <Negative> <Integer> <Dot> <Integer> | <Integer> <Dot> | <Dot> <Integer>
<Digit>:=0|1|2|3|4|5|6|7|8|9;
<NameProblem>:=‘VRP’;
<Equal>:=‘=’;
<KOpen>:=‘{’;
<KClose>:=‘}’;
<semicolon>:=‘;’;
<Comma>:=‘,’;
<Negative>:=‘-’;
<Dot>:=‘.’
```

5. Construct a compiler that transforms in polynomial time a source language L_1 into a target language L_2 ($L_1 \leq_P L_2$). In the phase of a lexical analysis of the compiler, a source language L_1 is transformed into tokens. The next information shows the token declaration for the GNU Flex software used in the lexical phase.

```
// Declarations
A [aA] B [bB] C [cC] D [dD] E [eE] F [fF]
G [gG] H [hH] I [iI] J [jJ] K [kK] L [lL]
M [mM] N [nN] O [oO] P [pP] Q [qQ]
R [rR] S [sS] T [tT] U [uU] V[vV]
W [wW] X [xX] Y [yY] Z [zZ] Digit [0-9]
// Rules
‘=’ return(EQUAL);
‘,’ return(COMMA);
‘;’ return(SEMICOLON);
‘.’ return(DOT);
‘{’ return(KOPEN);
```

```
'}' return(KCLOSE);
[Digit]+ return(DIGSEQ);
. { /* Ignore bad characters */ }
```

In the syntax analysis phase, tokens from source language L_1 are grouped into grammatical phrases. And in the semantic analysis phase detect the semantic errors (error handling) of the polynomial transformation from L_1 to L_2. In the phase of semantic analysis the restrictions were checked that allow a formal language L_2 (instance) to be obtained for VRP.

Finally, the L_1 was transformed into the L_2 ($L_1 \leq_P L_2$) or (OPTP $\leq_P$ VRP). Then, with the polynomial transformation of the OPTP instances to the VRP instances, we conclude that the Oil Platform Transport Problem (OPTP) problem is NP-Hard.

We present the parameters or characterization of Oil Platform Transport Problem instances of Mexican state-owned petroleum company (PEMEX). We generated 5 instances set; each instance set has 50 cases of randomly generated instances and real instances (with GIS data) of PEMEX Oil Platforms using the next algorithm. We used the nomenclature of instances: OPTP (name of the problem), P (number of platforms), T (type of transportation), # (number of case), .optp (extension of the file), for example: OPTP-36A-10.

```
Input: Size (Size of instance set), IX (Initial X GIS Area), FX (Final
X GIS Area), IY (initial Y GIS Area), FY (Final Y GIS Area), Fleet
Type (homogeneous or heterogeneous), Vehicle type (helicopters, ships
or both), the capacity of the helicopters (to select from a list), D
(total Demand), T (time of the work of the helicopters, ships or
both), Tr (approximate travel distance), A (altitude to select from a
list), p (number of platforms).
Output: instance set with size OPTP files.
Step 1. Repeat (size) //size of the instance Set
Step 2. To Create an OPTP file.
Step 3. Repeat (p) //number of platforms
Step 4. Random Latitude_p between (IX,FX).
Step 5. Random Length_p between (IY,FY).
Step 6. Random De between (0 to D/p).
Step 7. Random WEIGHT_p between (0-1.3). //Tons
Step 8. Random PE_p between (1,12). // Kilograms
Step 9. Random Pqg_p between (7-20). // Kilograms
Step 10. Random Pqm_p between (3-6). // Kilograms
Step 11. Random Pqn_p between (0-2). // Kilograms
Step 12. Random e_p between (900-1100) or (1100-1300) or (1500-1700).
Step 13. Random Du_p between (900-1100)+7 or (1100-1300)+7 or (1500-
1700)+7; //Due Time
Step 14. To Generate RT_p if node 0 (15) otherwise (7). // Riding Time
Step 15. While (m = p).
Step 16. Close the OPTP file.
```

```
Step 17. While (n = to size).
```

The depository of instances can be downloaded for other researchers for experimentation from the OPTPLib site. (www.diazparra.net/OPTPLib.html). In table 2 are the instance set characteristics of the OPTPLib.

Table 2. OPTP Library.

Instances Set	**Transportation Type**	**Fleet**	**Vehicle**	**Instances**
OPTP-36A	Air	Homogeneous	Helicopters	50
OPTP-36M	Maritime	Homogeneous	Ships	50
OPTP-36AH	Air	Heterogeneous	Helicopters	50
OPTP-36MH	Maritime	Heterogeneous	Ships	50
OPTP-36I	Intermodal	Homogeneous	Helicopters and ships	50

In Table 3 we present the parameters or characterization used for generating the OPTP instances.

Table 3. OPTP instances.

Instance Name											
FLEET	HELICOPTERS	CAPACITY	DEMAND	TIME	TRAVEL	ALTITUDE					
type	number	number	number	Number-Number	number	number					
					TYPE OF PACKAGE				TIME WINDOW		
No.	LATITUDE	LENGTH	DEMAND	WEIGHT	PE	Pqg	Pqm	Pqn	earliest	Due	Riding Time
N_1	L_1	Le_1	D_1	W_1	PE_1	Pqg_1	Pqm_1	Pqn_1	e_1	Du_1	RT_1
…	…	…	…	…	…	…	…	…	…	…	…
N_n	L_n	Le_n	D_n	W_n	PE_n	Pqg_n	Pqm_n	Pqn_n	e_n	Du_n	RT_n

Where: *N* is the number of the node 0 or the number of the platform, *L* is *X* coord. of the latitude, *Le* is the *Y* coord. of the length, *D* is the Demand, *W* is the Weight in Tons ($W = PE + Pqg + Pqm + Pqn$), *PE* is one seat Measure (weight of one person between 60-120 kilograms), *Pqg* package number of big size (each package between 7-20 kilograms), *Pqm* package number of normal size (each package between 3-6 kilograms), *Pqn* package number of medium size (each package between 0-2 kilograms), *e* is the earliest time, *Du* is the Due Time, *RT* is the Riding Time. In table 4 are the OPTP-36A-10 instances, where: *F* is the fleet, *HN* is the helicopter number, *Ca* is the capacity of each helicopter (number of packages or 12 persons maximum), *De* is the demand of users, *T* is the time of work of the helicopters, *Tr* is the travel distance, *A* is the altitude, *T* of *P* is the type of package.

Table 4. OPTP-36A-10 instance.

OPTP-36A-10											
F	*HN*	*Ca*	*D*	*T*	*Tr*		*A*				
H	36	20	720	9-11	1-7 NM		1500 Ft				
						T of P					
N	*L*	*Le*	*De*	*W*	*PE*	*Pqg*	*Pqm*	*Pqn*	*e*	*Du*	*RT*
0	18.62616	-91.85145	0	1.3	12	3	3	2	900	1100	15
1	19.3986111	-92.0397222	11	0.76	7	3	0	1	934	941	7
2	19.4172222	-92.0269444	12	0.7	5	3	2	2	1008	1015	7
3	19.4177778	-92.05	13	0.96	10	2	1	0	1048	1055	7
4	19.3980556	-92.0622222	7	0.46	4	2	0	1	1015	1022	7
5	19.38	-92.0505556	3	0.12	1	0	0	2	1024	1031	7
6	19.3738889	-92.0313889	9	0.56	4	2	3	0	926	933	7

7	19.3983333	-92.0155556	12	0.74	7	1	2	2	904	911	7
8	19.4613889	-92.0611111	11	0.84	9	2	0	0	947	954	7
9	19.4363889	-92.0613889	14	1.04	12	0	2	0	1001	1008	7
10	19.4058333	-92.0811111	9	0.64	7	1	0	1	1039	1046	7
11	19.3802778	-92.0733333	10	0.6	6	0	2	2	1048	1055	7
12	19.3475	-92.0475	7	0.46	4	2	0	1	1000	1007	7
13	19.3616667	-92.0694444	13	0.9	9	1	3	0	1019	1026	7
14	19.2952778	-92.1705556	3	0.18	2	0	0	1	1039	1046	7
15	19.3236111	-92.1858333	13	0.98	11	1	1	0	1015	1022	7
16	19.2986111	-92.2008333	11	0.66	5	2	3	1	1036	1043	7
17	19.2730556	-92.1852778	6	0.36	2	3	0	1	952	959	7
18	19.2738889	-92.155	10	0.56	4	2	2	2	908	915	7
19	19.2991667	-92.1394444	11	0.62	4	3	2	2	918	925	7
20	19.3413889	-92.2191667	5	0.28	2	1	1	1	1026	1033	7
21	19.2736111	-92.1322222	14	0.86	7	3	2	2	932	939	7
22	19.2833333	-92.23	7	0.36	1	3	2	1	946	953	7
23	19.2969444	-92.3172222	9	0.54	5	0	3	1	945	952	7
24	19.2219444	-92.0886111	13	0.96	10	2	1	0	1052	1059	7
25	19.1794444	-92.2866667	13	0.92	9	3	0	1	1020	1027	7
26	19.1458333	-92.3058333	8	0.52	5	1	1	1	1017	1024	7
27	19.5191667	-92.1886111	6	0.4	4	0	2	0	1022	1029	7
28	19.4941667	-92.1736111	10	0.78	9	1	0	0	1037	1044	7
29	19.5888889	-92.1997222	16	1	9	2	3	2	1051	1058	7
30	19.5625	-92.1841667	11	0.78	8	1	2	0	1012	1019	7
31	19.37	-92.0044444	10	0.68	7	0	3	0	1038	1045	7
32	19.3430556	-92.0044444	17	1.1	11	1	3	2	1049	1056	7
33	19.2366667	-92.2544444	7	0.34	2	1	2	2	1000	1007	7
34	19.2241667	-92.2622222	10	0.76	9	0	1	0	1000	1007	7
35	19.2427778	-92.2811111	16	1.16	12	2	2	0	1036	1043	7
36	19.0958333	-92.5330556	8	0.58	5	3	0	0	938	945	7

IV. EXPERIMENTATION AND RESULTS

In this section we show the results of the CPLEX solver for solving the instances of the Oil Platform Transport Problem. The results were obtained from the CPLEX solver to find the optimal cost of carrier resources, goods or people contains in the Oil Platform Transport Problem on a server IBM Proliant with 32 cores and 4 GB RAM.

We use the instance set OPTP-36 (Air transportation, Fleet: Homogeneous, Vehicles: Helicopters, a number of real instances: 50), and the mathematical model of OPTP (equations 1-10) for air transportation in the software of IBM.ILOG.CPLEX.Optimizer.v12. In Table 5 is the optimal cost and the number of helicopters of the OPTP 36-1 instance. The optimal cost of transportation of 720 elements (packages or workers) to the 36 oil platforms of PEMEX with 20 helicopters in one day is about USD $ 44,044.25. The nomenclature of the route is *P* (number of platforms), “:”, *h* (number of the helicopter). The number of platforms of the route must to fulfil the restriction of the capacity of the helicopters and the time windows.

Table 5. Optimal Cost of OPTP-36A-1.

Instance	Cost	h	Route
OPTP-36A-1	44,044.255	20	25:1; 27:2; 26:3; 4-5-10-11:4; 22:5; 33-34-35:6; 1-2-6-7:7; 29:8; 3-8-9:9; 23:10; 24:11; 12-13:12; 18-19-21:13; 31-32:14; 15-16:15; 36:16; 14-17:17; 28:18; 30:19; 20:20

In the figure 2 are the routes of the OPTP-36A-1 instance. We show the node 0 (airport / heliport), the 36 platforms of PEMEX, the latitude (XCoord.) and length (YCoord.) of the platforms, and the routes of the helicopters.

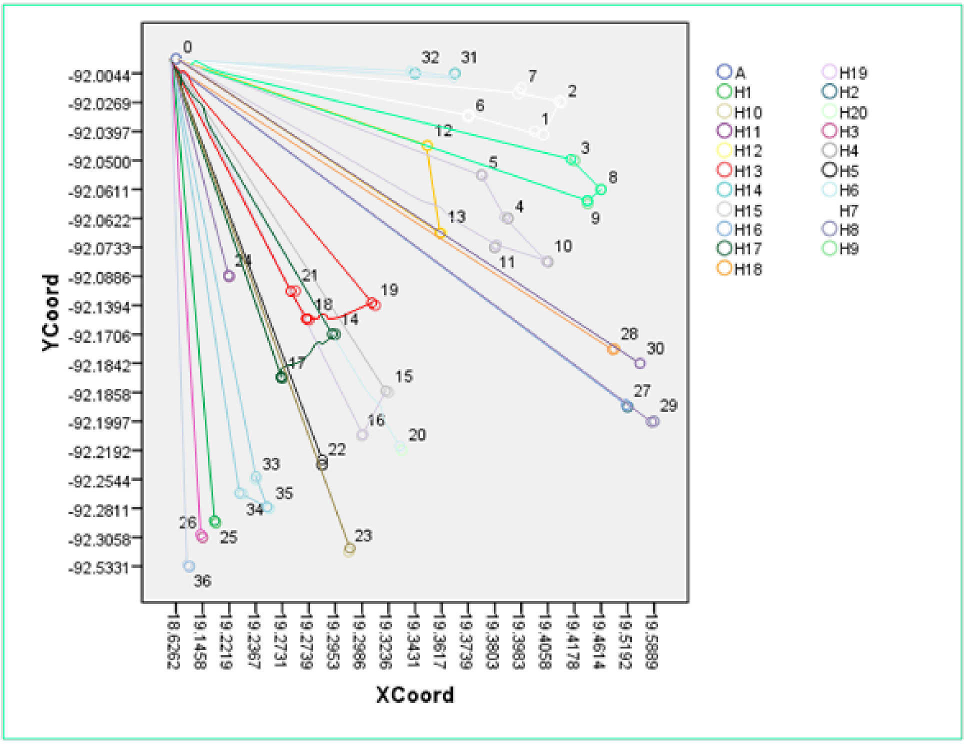

Fig. 2. Routes of the OPTP-36-1.

In Table 5 are some results of the experimentation of the OPTP problem instances (the optimal costs and the number of helicopters of the OPTP 36 instance set of type Air Transportation). In Table 6 are the Mean of Cost and Number of helicopters of the OPTP-36 Air instances set.

Table 5 Optimal Cost of OPTP instances (OPTP-36 Air instances set).

Instances	Cost	Helicopters	Routes
OPTP-36A-1	44,044.255	20	25:1; 27:2; 26:3; 4-5-10-11:4; 22:5; 33-34-35:6; 1-2-6-7:7; 29:8; 3-8-9:9; 23:10; 24:11; 12-13:12; 18-19-21:13; 31-32:14; 15-16:15; 36:16; 14-17:17; 28:18; 30:19; 20:20
OPTP-36A-2	43, 539.91	20	25:1; 27:2; 26:3; 4:4; 5-10-11:4; 22:5; 33-34-35:6; 1-2-6-7:7; 29:8; 3-8-9:9; 23:10; 24:11; 12-13:12; 18-19-21:13; 31-32:14; 15-16:15; 36:16; 14-17:17; 28:18; 30:19; 20:20

OPTP-36A-3	44,112.883	20	25:1; 27:2; 26:3; 4-5-10-11:4; 22:5; 33-34-35:6; 1-2-6-7:7; 29:8; 3-8-9:9; 23:10; 24:11; 12-13:12; 18-19-21:13; 31-32:14; 15-16:15; 36:16; 14-17:17; 28:18; 30:19; 20:20
OPTP-36A-4	44,010.359	19	26:1; 1-2-7:2; 23:3; 25:4; 18-19-21:5; 5-6:6; 24:7; 36:8; 3-4-8-9-10:9; 22:10; 20:11; 11-12-13:12; 29:13; 31-32:14; 15-16-33-34-35:16; 14-17:17; 27-28:18; 30:19
OPTP-36A-5	44,914.855	18	26:1; 1-2-7:2; 23:3; 25:4; 20:5; 6-31-32:6; 24:7; 36:8; 3-4-8-9-10:9; 18-19-21:10; 22:11; 5-11-12-13:12; 29:13; 30:14; 15-16:15; 33-34-35:16; 14-17:17; 27-28:18;
OPTP-36A-6	44,986.182	19	26:1; 1-2-7:2; 23:3; 25:4; 18-19-21:5; 5-6:6; 24:7; 36:8; 3-4-8-9-10:9; 22:10; 20:11; 11-12-13:12; 29:13; 31-32:14; 15-16:15; 33-34-35:16; 14-17:17; 27-28:18; 30:19
OPTP-36A-7	43,732.606	20	25:1; 27:2; 26:3; 4-5-10-11:4; 22:5; 33-34-35:6; 1-2-6-7:7; 29:8; 3-8-9:9; 23:10; 24:11; 12-13:12; 18-19-21:13; 31-32:14; 15-16:15; 36:16; 14-17:17; 28:18; 30:19; 20:20;
OPTP-36A-8	44,708.078	20	25:1; 27:2; 26:3; 4-5-10-11:4; 22:5; 33-34-35:6; 1-2-6-7:7; 29:8; 3-8-9:9; 23:10; 24:11; 12-13-18-19-21:13; 31-32:14; 15-16:15; 36:16; 14-17:17; 28:18; 30:19; 20:20;
OPTP-36A-9	44,066.962	19	26:1; 1-2-7:2; 23:3; 25:4; 18-19-21:5; 5-6:6; 24:7; 36:8; 3-4-8-9-10:9; 22:10; 20:11; 11-12-13:12; 29:13; 31-32:14; 15-16:15; 33-34-35:16; 14-17:17; 27-28:18; 30:19
OPTP-36A-10	44,631.085	19	26:1; 1-2-7:2; 23:3; 25:4; 18-19-21:5; 5-6:6; 24:7; 36:8; 3-4-8-9-10:9; 22:10; 20:11; 11-12-13:12; 29:13; 31-32:14; 15-16:15; 33-34-35:16; 14-17:17; 27-28:18; 30:19;

Table 6 Mean of Cost and Number of helicopters of the OPTP-36 Air instances set.

Instances set	Mean of Cost	Mean of Number of Helicopters
OPTP-36A	44,356.36	19

V. CONCLUSIONS

The main contribution of this work is proposal of a new problem called Oil Platform Transport Problem (OPTP), the proof that the OPTP is NP-Hard/NP-Complete Problem by the polynomial transformation using formal languages between the Vehicle Routing Problem and the OPTP, a mathematical model of the OPTP, the characterization of the OPTP instances of Mexican state-owned petroleum company (PEMEX), and the use the CPLEX solver to find the optimal cost of carry resources, goods or people contains in the OPTP-36-A instance set.

For future works, it is necessary to solve the instances sets of OPTP-36M, OPTP-36AH, OPTP-36MH, and OPTP-36I using CPLEX or a new meta-heuristics algorithm.

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