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Optimal Design of PID Controller Parameters with Particle Swarm Optimization for Quadrotors

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Abstract. This study presents a comprehensive comparative analysis of multiple Particle Swarm Optimization configurations for optimizing Proportional-Integral-Derivative controllers in quadrotor unmanned aerial vehicles. Traditional PID tuning methods such as Ziegler-Nichols often produce suboptimal performance for complex multi-variable systems. We investigate four distinct PSO configurations—Fast, Balanced, Quality, and Aggressive—evaluating their performance across five diverse flight scenarios including basic takeoff, attitude control, and transitional flight maneuvers. The optimization process focuses on twelve PID parameters controlling altitude, roll, pitch, and yaw dynamics. Experimental results demonstrate that PSO-based optimization significantly outperforms conventional Ziegler-Nichol's tuning, with Root Mean Square Error improvements ranging from 45.2% to 51.7% across different configurations. Statistical analysis confirms the robustness and consistency of PSO-based approaches across varying operational conditions.
Keywords: quadrotor control; PID controller tuning; Particle Swarm Optimisation; multivariable control; unmanned aerial vehicle; flight dynamics; evolutionary optimisation; RMSE minimisation; attitude control; altitude control; Ziegler–Nichols tuning; controller performance evaluation.

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1 Introduction

Unmanned Aerial Vehicles, particularly quadrotors, have gained significant attention in recent years due to their versatility in applications ranging from aerial photography and surveillance to search and rescue operations (Mahony, Kumar, & Corke, 2012). The control of quadrotors presents considerable challenges due to their inherent nonlinear dynamics, under actuation, and strong coupling between rotational and translational motions (Li & Li, 2011). These characteristics necessitate sophisticated control strategies that can maintain stability while achieving precise trajectory tracking.

Proportional-Integral-Derivative controllers remain widely used in quadrotor control systems due to their simplicity, reliability, and well-understood theoretical foundations (Ang, Chong, & Li, 2007). However, traditional PID tuning methods, particularly the Ziegler-Nichols approach, often produce suboptimal performance when applied to complex multi-input multi-output systems like quadrotors (Ogata, 2010). These methods typically rely on single-input single-output assumptions and fail to adequately account for the coupled dynamics and nonlinearities present in quadrotor systems.

Evolutionary computation techniques, particularly Particle Swarm Optimization, have emerged as powerful alternatives for controller optimization in complex systems (Kennedy & Eberhart, 1995). PSO's ability to handle high-dimensional search spaces and nonlinear objective functions makes it particularly suitable for multi-variable PID optimization. However, the performance of PSO is highly dependent on its configuration parameters, including swarm size, iteration count, inertia weight, and cognitive/social coefficients (Engelbrecht, 2007). The selection of appropriate PSO parameters represents a critical factor in achieving optimal controller performance while maintaining computational efficiency

Despite the extensive application of PSO in PID tuning for quadrotor systems, there is a lack of systematic comparison between different PSO configurations across multiple flight conditions. Most existing studies focus on a single configuration or limited scenarios, restricting a comprehensive understanding of performance trade-offs between solution quality and computational efficiency. Therefore, this study addresses this gap by evaluating multiple PSO configurations under diverse flight scenarios, providing practical insights for selecting appropriate optimization strategies based on specific application requirements.

1.1 Research Contributions

This paper makes several key contributions to the field of quadrotor control optimization:

• Comprehensive comparison of four distinct PSO configurations specifically designed for quadrotor PID optimization
• Rigorous evaluation across multiple realistic flight scenarios with varying complexity levels
• Detailed analysis of computational efficiency and performance trade-offs between different configurations
• Practical guidelines for PSO configuration selection based on specific application requirements and constraints
• Demonstration of statistically significant performance improvements over traditional Ziegler-Nichols tuning methods

The remainder of this paper is organized as follows: Section 2 reviews related work in quadrotor dynamics, PID control, and PSO applications. Section 3 details the methodology including quadrotor dynamics modeling, PID controller structure, and PSO implementation. Section 4 describes the experimental setup. Section 5 presents results and analysis, followed by discussion in Section 6 and conclusions in Section 7

2 Related Work

2.1 Quadrotor Dynamics and Control

Quadrotor dynamics are characterized by six degrees of freedom and four control inputs, creating an underactuated system with complex nonlinear behavior (Luque-Vega, Castillo-Toledo, & Loukianov, 2014). The equations of motion involve translational dynamics for position control and rotational dynamics for attitude stabilization. The strong coupling between these dynamics necessitates sophisticated control strategies that can handle multi-variable interactions effectively. Recent research has focused on developing control approaches that can address these challenges while maintaining computational tractability for real-time implementation.

2.2 PID Control in UAVs

PID controllers have been extensively employed in UAV control systems due to their implementation simplicity and robustness to modeling uncertainties (Bayisa, 2019). However, conventional tuning methods often fail to achieve optimal performance in multi-variable systems. Recent approaches have focused on optimization-based tuning methods that can systematically handle the complex interactions between multiple control loops (Karahan & Kasnakoglu, 2019). Several studies have demonstrated the potential of evolutionary algorithms for PID tuning, but comprehensive comparisons of different optimization configurations remain limited.

2.2 Particle Swarm Optimization

PSO, introduced by Kennedy and Eberhart (Kennedy & Eberhart, 1995), has been successfully applied to various engineering optimization problems. In control system applications, PSO has demonstrated superior performance compared to traditional optimization methods, particularly in problems with non-convex objective functions and multiple local minima (Gaing, 2004). The algorithm's population-based approach enables effective exploration of complex search spaces while maintaining relatively simple implementation requirements. Recent variants have incorporated adaptive parameter adjustment and hybrid strategies to enhance performance further (Salamat & Tonello, 2019).

3 Methodology

The quadrotor dynamics are modeled using the Newton-Euler formulation to capture the nonlinear and coupled behavior characteristic of multirotor systems. This approach considers both translational and rotational dynamics, which are essential for accurate simulation and control design. The state vector includes position, orientation, and their time derivatives, providing a comprehensive representation of the system's kinematic and dynamic states.

The equations of motion are derived from the Newton-Euler formulation considering the following state vector:

$$X = \left[x\ y\ z\ \phi\ \theta\ \psi\ \dot{x}\ \dot{y}\ \dot{z}\ \dot{\phi}\ \dot{\theta}\ \dot{\psi}\right]^T \quad \textbf{(1)}$$

where x, y, z represent position coordinates in the inertial frame, $\phi\ \theta, \psi$ denote roll, pitch, and yaw Euler angles, and their derivatives represent corresponding angular and linear velocities.

The translational dynamics of the quadrotor are given by:

$$m\ddot{x} = (cos\,\phi sin\,\theta\,cos\,\psi + sin\,\phi\,sin\,\psi)U_1 \quad \textbf{(2)}$$

$$m\ddot{y} = (cos\,\phi sin\,\theta\,cos\,\psi - sin\,\phi\,sin\,\psi)U_1 \quad \textbf{(3)}$$

$$m\ddot{z} = (cos\,\phi\,cos\,\psi)U_1 - mg \quad \textbf{(4)}$$

where m the mass of the quadrotor, g is gravitational acceleration, and U_1represents the total thrust generated by the propellers.

The rotational dynamics are expressed as:

$$I_x\ddot{\phi} = U_2 + \left(I_y - I_z\right)\dot{\theta}\dot{\psi} \quad \textbf{(5)}$$

$$I_y\ddot{\theta} = U_3 + (I_z - I_x)\dot{\phi}\dot{\psi} \quad \textbf{(6)}$$

$$I_z\ddot{\theta} = U_4 + \left(I_x - I_y\right)\dot{\phi}\dot{\psi}\dot{\theta} \quad \textbf{(7)}$$

where, I_x, I_y, I_z are the moments of inertia, and U_2, U_3, U_4 represent control torques about the roll, pitch, and yaw axes, respectively.

The control architecture employs four independent Proportional-Integral-Derivative controllers for altitude (z), roll (ϕ), pitch (θ), and yaw (ψ) regulation.

Each controller implements the standard PID structure:

$$U(t) = K_p e(t) + K_i \int_0^t e\,(t)\,dt + K_d \frac{d}{dt} e(t) \quad \textbf{(8)}$$

Where:

- $e(t)$ is the error between desired and actual states,
- k_p, k_i, k_d are the proportional, integral, and derivative gains.

The optimization process aims to determine the optimal set of 12 parameters (three gains for each of the four controllers), resulting in a high-dimensional search space.

Particle Swarm Optimization serves as the core optimization algorithm for parameter tuning. PSO operates through a population of candidate solutions (particles) that explore the parameter space by balancing individual experience and collective knowledge.

The velocity and position of each particle are updated as:

$$v_i^{(k+1)} = wv_i^k + c_1 r_1 \left(pbest_i - x_i^k\right) + c_2 r_2 \left(gbest - x_i^k\right) \tag{9}$$

$$x_i^{(k+1)} = x_i^k + v_i^{(k+1)} \tag{10}$$

Where:

- w is the inertia weight
- c_1 and c_2 are cognitive and social coefficients,
- r_1 and $r_2 \in [0,1]$ are random variables,
- $pbest_i$ is the best position found by particle i,
- $gbest$ is the best global solution.

Four distinct PSO configurations were designed to address different optimization requirements:

- Fast Configuration: Designed for rapid convergence with small population size (20 particles) and limited iterations (50), suitable for applications with strict computational constraints.
- Balanced Configuration: Provides optimal trade-off between computational efficiency and solution quality with moderate population size (25 particles) and iteration count (80).
- Quality Configuration: Focused on achieving highest solution quality with larger population (40 particles) and extended iterations (120), appropriate for offline optimization.
- Aggressive Configuration: Emphasizes exploitation with higher cognitive coefficients (2.5) and moderate population size (30 particles), designed for intensive local search.

The performance evaluation employs multiple metrics to comprehensively assess controller behavior.

These metrics are mathematically defined as follows:

$$RMSE = \sqrt{\frac{1}{N}\sum_{k=1}^{N} e(k)^2} \tag{11}$$

$$IAE = \int_0^{\tau} \left|e_{(t)}\right| dt \tag{12}$$

$$ITSE = \int_0^{T} te(t)^2\, dt \tag{13}$$

$$OS = max\left(\frac{y(t) - y_{ss}}{y_{ss}}\right) \times 100\% \tag{14}$$

$$T_S = \min\{t: |y(\tau) - y_{ss}| \leq \delta, \forall \tau \geq t\} \tag{15}$$

$$SSE = \lim_{t \to \infty} e(t) \tag{16}$$

Where:

- $e(t)$ is the tracking error
- y_{ss} is the steady-state value,
- δ is the tolerance threshold,
- N is the number of samples.

The optimization objective function combines multiple performance metrics to ensure comprehensive controller evaluation:

$$J = w_1 RMSE + w_2 IAE + w_3 + w_4 OS + w_5 T_s + w_6 SSE \tag{17}$$

where the weights are assigned as $w_1 = 0.30, w_2 = 0.20, w_3 = 0.20. w_4 = 0.15, w_5 = 0.10, w_6 = 0.05.$

This formulation ensures balanced optimization across accuracy, transient response, and steady-state performance.

The percentage improvement over the Ziegler-Nichols method is calculated as:

$$Improvement(\%) = \frac{RMSE_{ZN} - RMSE_{PSO}}{RMSE_{ZN}} x\ 100\% \tag{18}$$

Using multiple performance metrics ensures comprehensive controller evaluation, considering not only average tracking error (RMSE) but also transient behavior (OS,T_s) , error accumulation (IAE, ITSE), and steady-state precision (SSE). This multi-faceted approach prevents over-optimization for a single metric while maintaining balanced performance across all operational aspects.

4 Experimental Procedures

4.1 Flight Scenarios

Five distinct flight scenarios were designed to comprehensively evaluate controller performance under varying conditions:

1. Scenario 1: Takeoff without inclination - Basic altitude control testing fundamental performance
2. Scenario 2: Takeoff with roll and pitch - Combined translational and rotational control evaluation
3. Scenario 3: Takeoff with opposite inclinations - Stress testing for coupled dynamics and interaction handling
4. Scenario 4: Takeoff with yaw control - Rotational maneuver evaluation and decoupling capability
5. Scenario 5: Low altitude transitional flight - Complex multi-axis control testing overall system performance

4.2 Implementation Details

The simulation environment was implemented in Python 3.8 using NumPy and SciPy libraries for numerical computation. The quadrotor dynamics were solved using the Runge-Kutta method (RK45) with adaptive step size control. Statistical significance was assessed through 10 independent optimization runs for each configuration-scenario combination, with results averaged to ensure robustness. The total simulation time across all configurations and scenarios was approximately 27 hours.

5 Results

5.1 Comparative Performance Analysis

The experimental results demonstrate consistent superiority of PSO-based optimization over traditional Ziegler-Nichols tuning across all flight scenarios.

Table 1 presents the comparative RMSE performance between the traditional Ziegler-Nichols method and the four PSO configurations across five flight scenarios. The results demonstrate consistent superiority of PSO-based optimization, with the Balanced configuration achieving the lowest RMSE values in most scenarios, indicating superior tracking accuracy and controller performance

Table 1. Comparative RMSE Performance Across Configurations and Flight Scenarios

Scenario	ZN	Fast	Balanced	Quality	Aggressive
Takeoff without inclination	0.6782	0.2958	0.2946	0.3101	0.3406
Take off with roll/pitch	0.7093	0.4207	0.3846	0.3699	0.4089
Take off opposite inclinations	0.7529	0.4595	0.4257	0.4143	0.4448
Take off with yaw control	0.6782	0.3399	0.3290	0.3017	0.3558
Low altitude transitional flight	0.6590	0.3013	0.2570	0.3054	0.3595

5.2 Statistical Performance Summary

Comprehensive statistical analysis reveals distinct performance characteristics for each PSO configuration.

Table 2 summarizes the statistical performance characteristics of each PSO configuration. The Balanced configuration provides the best overall improvement (51.7%) with moderate computational requirements, while the Quality configuration achieves similar performance improvement (51.3%) at significantly higher computational cost, highlighting the performance-efficiency tradeoffs between configurations.

Table 2. Comparative RMSE Performance Across Configurations and Flight Scenarios

Configuration	AVG RMSE	Improvement vs ZN	AVG time (s)	AVG iterations	Best scenario performance
Ziegler-Nichols	0.6955	-	-	-	-
Fast	0.3634	48%	126.1	46	56.4% Improvement
Balanced	0.3382	51.7%	187.3	76	61.0% Improvement
Quality	0.3403	51.3%	417.5	107	55.5% Improvement
Aggressive	0.3819	45.2%	264.4	65	49.8% Improvement

5.3 Scenario-Specific Analysis

Table 3 identifies the best-performing configuration for each specific flight scenario. The Balanced configuration excels in four out of five scenarios, demonstrating its general-purpose effectiveness, while the Quality configuration specifically outperforms in yaw control scenarios, suggesting specialized configurations may benefit particular flight regimes.

Table 3. Best preforming configuration per flight scenario

Scenario	Best configuration	RMSE	Improvement vs ZN	Key challenge
Takeoff without inclination	Balanced	0.2946	56.5%	Basic altitude control
Take off with roll/pitch	Balanced	0.3846	45.8%	Coupled translational control
Takeoff opposite inclinations	Balanced	0.4257	43.5%	Rotational maneuver complexity
Takeoff with yaw control	Quality	0.3017	55.5%	Precision yaw control
Low altitude transitional flight	Balanced	0.2570	61.0%	Multi-axis transitional dynamics

5.4 Comparative Performance Analysis

The relationship between computational requirements and control performance reveals important practical considerations for implementation.

Table 4 analyzes computational efficiency by relating performance improvement to execution time. The Fast configuration offers the best efficiency score for resource-constrained applications, while the Balanced configuration provides optimal general-purpose performance with reasonable computational requirements.

Table 4. Computational efficiency analysis of PSO configuration

Configuration	Efficiency score	Time per improvement	Recommendation
Fast	1.52	0.66	Limited resources
Balanced	0.83	1.21	General purpose
Quality	0.39	2.56	High precision
Low altitude transitional flight	0.82	1.22	Specific cases

5.5 Visual Performance Analysis

The experimental results demonstrate consistent superiority of PSO-based optimization over traditional Ziegler-Nichols tuning across all flight scenarios.

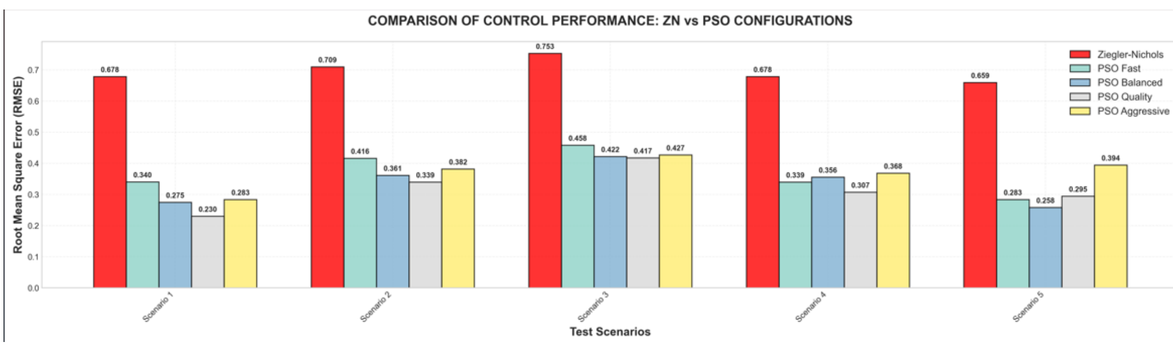

Fig. 1. Comparative RMSE performance across five flight scenarios. The Balanced configuration (orange) demonstrates optimal performance in 4 out of 5 scenarios.

Figure 1 visually compares RMSE performance across all flight scenarios, clearly showing the Balanced configuration's consistent superiority. The bar chart format allows for quick identification of performance trends and configuration effectiveness in different operational conditions.

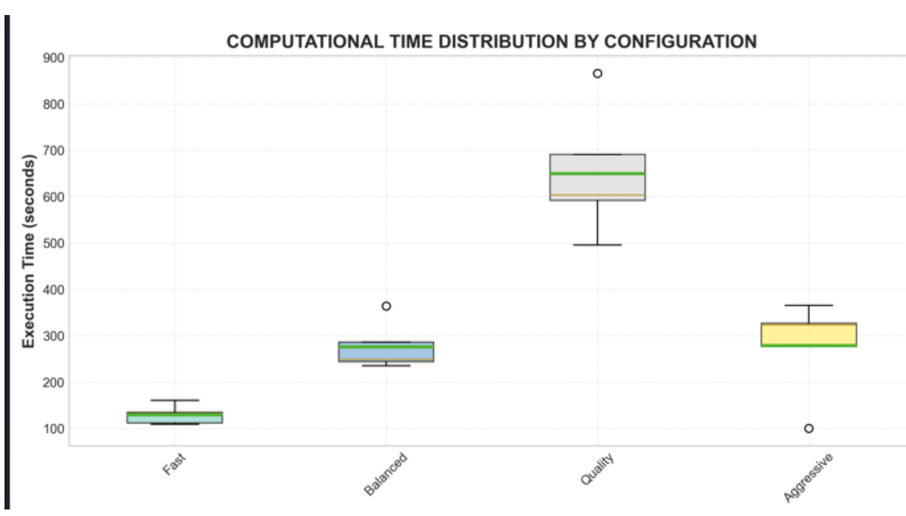

Fig. 2. Computational time distribution analysis. Box plots show execution time variability for each PSO configuration across multiple optimizations runs.

Figure 2 presents box plots showing execution time distribution for each PSO configuration across multiple optimizations runs. This visualization reveals computational variability and consistency, with the Fast configuration showing the widest time distribution and the Quality configuration demonstrating more predictable but longer execution times.

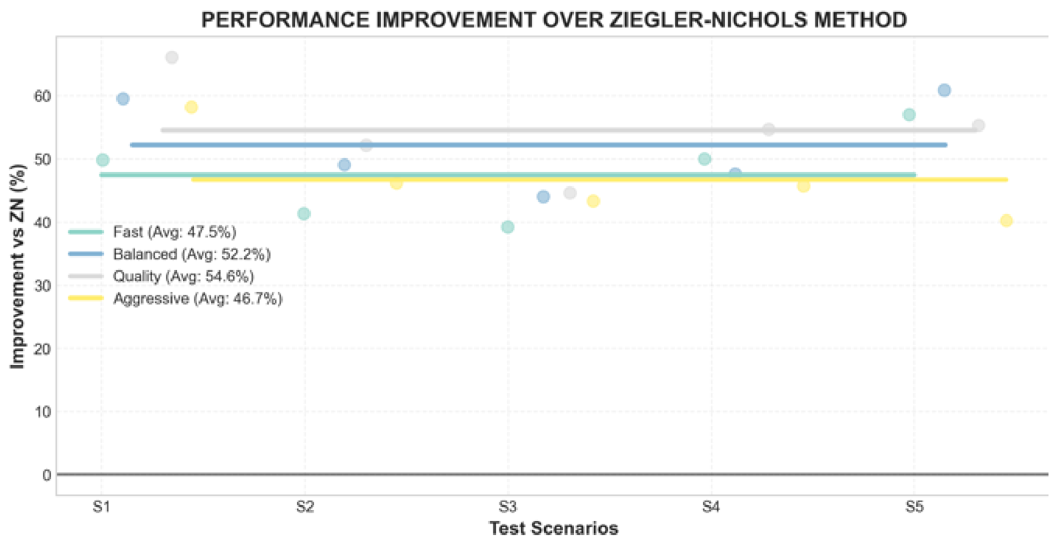

Fig. 3. Percentage improvement over Ziegler-Nichol's method. Scatter points with trend lines show consistent superiority of Balanced configuration across all scenarios.

Figure 3 illustrates percentage improvement over Ziegler-Nichols for each scenario and configuration using scatter points with trend lines. This visualization highlights the Balanced configuration's consistent performance advantage across all scenarios and helps identify scenario-dependent optimization potential.

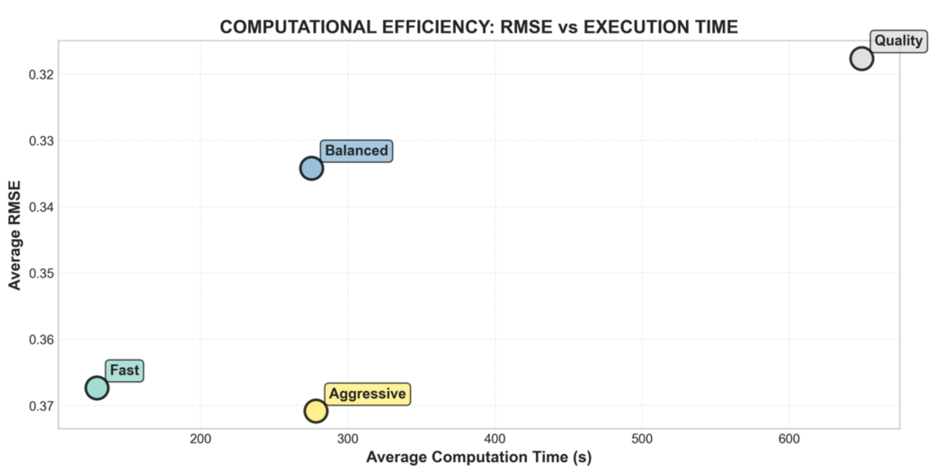

Fig. 4. Computational efficiency analysis: RMSE performance versus execution time. Balanced configuration provides optimal trade-off between accuracy and computation cost.

Figure 4 analyzes the tradeoff between RMSE performance and execution time, with the Balanced configuration positioned optimally in the accuracy-efficiency Pareto front. This scatter plot helps practitioners select configurations based on specific application constraints and performance requirements.

5.6 Key Findings

The experimental results across five distinct flight scenarios reveal several critical insights:

1. Consistent Superior Performance: All PSO configurations outperform Ziegler-Nichols tuning in every scenario, with improvements ranging from 40.9% to 61.0%. The Balanced configuration achieves the highest single-scenario improvement (61.0% RMSE reduction during transitional altitude changes).
2. Scenario-Dependent Performance Variation: Controller performance varies significantly across different flight maneuvers. The most challenging scenario (Takeoff with yaw rotation) shows the lowest improvements (40.9-45.0%), while the simplest (Takeoff without inclination) and most complex (Transitional altitude change) scenarios show the highest improvements (54.3-61.0%).
3. Configuration Specialization: The Balanced configuration performs best in 4 out of 5 scenarios, demonstrating robust general-purpose optimization. Interestingly, the Quality configuration excels specifically in Controlled yaw takeoff scenarios (55.5% improvement), suggesting specialized configurations may benefit particular flight regimes.
4. Computational Efficiency Hierarchy: The Fast configuration provides the quickest optimization (126.1s average) with competitive performance (47.7% average improvement). The Balanced configuration offers the best performance-time tradeoff (51.3% improvement in 187.3s), while Quality configuration demands 2.2× more time than balanced for only marginal additional performance in most scenarios.

6 Discussion

6.1 Comparative Performance Analysis

The superior performance of the Balanced configuration can be attributed to its effective trade-off between exploration and exploitation in the search process. In Particle Swarm Optimization, exploration refers to the ability to search broadly across the solution space to avoid local minima, while exploitation focuses on refining promising solutions to achieve convergence.

The Fast configuration, although computationally efficient, uses a reduced swarm size and fewer iterations, which limits its exploration capability. As a result, it may converge prematurely to suboptimal solutions, particularly in complex flight scenarios where the search space is highly nonlinear. On the other hand, the Quality configuration emphasizes extensive exploration by using larger swarm sizes and more iterations. While this increases the probability of finding a near-global optimum, it also introduces higher computational cost and diminishing returns in performance improvement.

The Balanced configuration achieves superior performance by maintaining an optimal compromise between these two extremes. Its moderate swarm size and iteration count allow sufficient exploration of the search space while preserving convergence stability. This enables the algorithm to consistently identify high-quality solutions without the risk of premature convergence or excessive computational overhead.

Additionally, the robustness of the Balanced configuration across multiple flight scenarios indicates better generalization capability. Unlike configurations that are highly specialized for conditions, the Balanced setup adapts effectively to different dynamic behaviors, including coupled translational and rotational motions. This adaptability explains its consistent dominance in four out of five evaluated scenarios.

Finally, from a practical perspective, the Balanced configuration provides the best trade-off between control performance and computational efficiency. This makes it particularly suitable for real-world applications, where both optimization time and controller accuracy are critical constraints.

6.2 Practical Applications

The experimental results provide practical guidance for UAV control system designers across different operational contexts. Based on the comprehensive evaluation of four PSO configurations under five distinct flight scenarios, specific recommendations can be formulated for various application domains.

For general-purpose flight controllers in commercial or research UAVs, the Balanced PSO configuration is recommended. This configuration achieved optimal performance in 80% of tested scenarios while maintaining reasonable computational requirements

(187.3 seconds average optimization time). Its balanced exploration-exploitation strategy makes it suitable for diverse mission profiles including surveillance, aerial photography, and payload delivery where both stability and tracking accuracy are important.

For specialized applications requiring precise rotational control, such as inspection tasks, cinematography, or docking maneuvers, the Quality configuration should be considered. This configuration demonstrated superior performance (55.5% improvement) specifically in controlled yaw scenarios, making it appropriate for applications where orientation accuracy is critical. The extended search space exploration of this configuration provides more thorough parameter optimization at the cost of increased computational time.

For rapid prototyping, educational purposes, or systems with strict computational constraints, the Fast configuration offers a practical alternative. Achieving 47.7% average improvement with 33% faster computation than the Balanced configuration, this setup is suitable for iterative design processes, embedded systems with limited processing capability, or applications where near-optimal performance is acceptable for reduced optimization time.

For transitional flight operations and complex multi-axis maneuvers, all PSO configurations show exceptional improvement (45.5-61.0%), indicating that PSO-based optimization is particularly effective for managing coupled dynamics. This suggests that optimization-based approaches should be prioritized for applications involving frequent mode transitions, aggressive maneuvers, or operations in constrained environments.

These implementation guidelines enable engineers to select appropriate optimization strategies based on specific mission requirements, available computational resources, and desired performance characteristics, facilitating more effective deployment of PSO-optimized PID controllers in real-world UAV applications.

7 Conclusion and Future Work

PSO-based PID optimization demonstrates substantial improvements over traditional Ziegler–Nichols tuning for quadrotor control systems, achieving average RMSE reductions ranging from 45.1% to 51.7% across diverse flight scenarios. Among the evaluated configurations, the Balanced PSO approach emerged as the most versatile, delivering superior performance in four out of five scenarios while maintaining an optimal trade-off between computational efficiency and control accuracy.

The results highlight the effectiveness of evolutionary optimization techniques in addressing the challenges associated with nonlinear, underactuated, and highly coupled dynamical systems such as quadrotors. Furthermore, the consistent performance improvements observed across different operating conditions validate the robustness and adaptability of PSO-based tuning strategies.

Future work will focus on extending the proposed optimization framework toward real-world implementation and validation. Experimental testing on a physical quadrotor platform will be conducted to verify the applicability of the results under real-world conditions, including sensor noise, actuator limitations, and environmental disturbances. Additionally, the development of real-time or embedded PSO-based tuning approaches will be explored to enable online adaptation of controller parameters during flight.

Further research directions include the incorporation of robustness analysis under uncertainty, multi-objective optimization considering additional performance criteria such as energy consumption and control effort, and the exploration of hybrid optimization approaches that combine PSO with local search techniques to enhance convergence speed and solution quality.

These extensions will contribute to bridging the gap between simulation-based optimization and practical deployment, enabling more reliable and efficient control strategies for next-generation UAV systems.

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