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Design and Control of a Low-Cost Ball and Beam Platform for Control System Education

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Abstract. The *Ball & Beam* system is one of the most comprehensive case studies in control engineering, as it is inherently unstable. This paper presents a control approach for this system consisting of a PID controller and a Kalman filter, which are used to estimate the system states and mitigate the effects of noise in position measurements. The control strategy is implemented using Arduino software and an HC-SR04 ultrasonic sensor. Experimental validation demonstrated that the system maintained the ball at the reference position with a maximum error of ±2 cm and an approximate stabilization time of 5 seconds. These results confirm that the proposed control is effective and that the prototype offers a low-cost solution for educational applications.
Keywords: PID control; Kalman filter; Arduino; Stability.

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1 Introduction

Various authors have studied the *Ball & Beam* system using both classical and modern control strategies. For example, Ali et al. (2017) designed a PID controller to regulate the ball's position, tuning the PID parameters through a trial-and-error approach to select values that yielded the best performance. Similarly, Rahul (2018) implemented an LQG controller, demonstrating that this method effectively improved ball tracking. Ahmadi and Khodadadi (2018) proposed a self-tuning PID controller based on fuzzy logic, achieving better dynamic performance than the conventional PID in terms of response time and overshoot reduction.
Control systems are currently fundamental in diverse technological applications, ranging from robotics to industrial automation. However, many of these prototypes are prohibitively expensive, which can pose a barrier for students and educators in low-income educational institutions. The principles of control and applied physics often require specialized equipment and simulators that are not easily accessible, hindering the understanding of fundamental concepts in the control field. This problem involves multiple engineering disciplines, including physics, materials engineering, control systems, system dynamics, mechanical design, and digital signal processing, making it highly suitable for the application of interdisciplinary knowledge in control engineering.

The combined implementation of a PID controller and a Kalman filter in the *Ball & Beam* system makes this prototype distinctive compared to other designs, which typically employ different control schemes. This integration enhances the system's ability to maintain the ball at the desired position while compensating for the accuracy limitations inherent to low-cost components when compared to more expensive alternatives.

The low-cost *Ball & Beam* prototype provides an accessible teaching tool for control systems. This device enables the study of position analysis and stabilization in a tangible and visual manner through the implementation of a PID controller combined with a Kalman filter. Using this prototype, students can actively learn and experiment with fundamental concepts in control engineering. with the fundamental principles of physics, applied mathematics, and control, such as feedback, parameter tuning, and dynamic system response. The design is sufficiently simple for students to understand both the system's operation and the underlying technical principles, while remaining robust enough to demonstrate the practical effects of control techniques. Consequently, the prototype serves as an effective educational tool for teaching and reinforcing key concepts in control systems.

This prototype is intended to provide a low-cost and accessible alternative that can be adapted to different educational levels. Existing prototypes on the market are often prohibitively expensive, making the design and construction of this experimental platform a cost-effective option for teaching control theory (Villafuerte, 2018). The platform enables students to develop skills in Arduino programming, dynamic system analysis, and control techniques. This approach enhances the educational value of the system and justifies the challenge of developing a functional prototype. In control systems, the analysis of the system's temporal response is typically divided into two fundamental stages: the transient response and the steady-state response. The transient response refers to the system's behavior when the reference input changes, during which the output evolves from its initial state to the final state.

The proposed prototype serves as an effective model for teaching control systems in educational settings, highlighting the importance of system observability. A system is considered observable if its state can be determined by monitoring its output over a finite time interval. To facilitate implementation in educational environments, the prototype was constructed using accessible and readily available materials, integrating PID control with a Kalman filter to manage noisy signals. This combination enables precise manipulation of the beam angle to maintain the ball at the desired position, thereby improving the system's accuracy and stability.

The Kalman filter, widely used in fields such as digital image processing, computer vision, and state estimation for stochastic systems, was incorporated to enhance the accuracy of ultrasonic sensor measurements. By filtering out noise, it provides reliable estimates of the ball's position. Unlike other prototypes, this work integrates two advanced control and estimation techniques: PID control and the Kalman filter. The PID controller, based on proportional, integral, and derivative actions, is designed to stabilize the controlled variable, enabling precise adjustment of the beam angle to maintain the ball's position and compensate for disturbances and measurement errors.

The primary objective of the control system is to maintain the ball at a reference position of 25 cm and to achieve rapid stabilization at this operating point despite disturbances. The Kalman filter significantly enhances measurement accuracy by filtering out noise and providing reliable position estimates, thereby optimizing system performance and positioning the prototype as an innovative, functional, and cost-effective alternative to existing solutions.

Finally, this work analyzes and discusses the steps undertaken to develop the *Ball & Beam* prototype for the effective implementation of the control system. The development process typically includes the following stages: problem analysis, prototype design and construction, testing, and validation.

The first step involves conducting a thorough investigation and analysis, clearly defining the objectives. Based on this information, the next stage addresses the techniques and methods implemented to ensure proper system operation through the analysis of the mathematical model. This stage can be the most complex, as it requires translating theoretical concepts into practical implementation while accounting for factors often not considered, such as the materials used, the system's response, and the implemented code. It is important to note that the scope of this study is limited, as disturbances and noise are not taken into account (Cyubahiro, 2022). Finally, testing and validation are performed, enabling a diagnosis of the system to verify the correct functioning of each component and to implement any necessary modifications.

This article is organized as follows: Section 2 presents the mathematical model of the system, the procedure followed for the prototype design, and the implementation of its various components. Section 3 subsequently describes the results obtained, which are compared with the established objectives using diagrams and graphs to illustrate the system's behavior. This section emphasizes the aspects that were successfully achieved, as well as areas requiring improvement or considerations for future enhancements of the prototype.

Finally, Section 4 is dedicated to the conclusions, in which the system's performance is evaluated and the extent to which the objectives were achieved regarding the mitigation of the addressed problem is analyzed. This section clearly presents the actions undertaken during the development of the work. In this way, the article concludes by providing an overview of the progress made and identifying opportunities for future improvements.

2 System design and execution

In this work, a low-cost *Ball & Beam* prototype was designed and controlled, enabling the validation and experimentation of PID control strategies to maintain the ball at the desired position on the beam. This system facilitates the learning and comprehension of fundamental control concepts applied to inherently unstable dynamic systems.

To achieve stability of the ball on the beam, the error between the desired and actual positions was minimized through the implementation of PID control techniques. Additionally, a Kalman filter was employed to reduce noise in the position measurements, thereby improving the accuracy of the signals used by the controller. The dynamic response of the system was further optimized by tuning the PID gains to achieve a fast response with minimal overshoot.

The *Ball & Beam* system is conceptually analogous to the inverted pendulum, an inherently unstable mechanical system that requires active control to maintain balance. While a simple pendulum oscillates harmonically with a frequency determined by its length and gravity, both the inverted pendulum and the *Ball & Beam* system are unstable, necessitating the use of controllers, such as PID, to stabilize them and maintain a balanced position through the adjustment of system variables.

In addition, the Ball and Beam is a "classic laboratory model for the design of control algorithms and engineering education", especially for introducing the analysis of nonlinear and unstable systems, which is fundamental for industrial systems such as unstable platforms, robots and active suspensions (Scharre, J. G. L., 2015).

Accessible materials were used for the construction of the prototype. The main structure was built with wood, which provided a solid and economical foundation. The ball used was made of rigid plastic, selected for its lightness and good response to movement. For the position measurement system, it was made using an ultrasonic sensor model HC-SR04, which detects the distance between the sensor and the ball, as this information is essential for the control of the system. And for the adjustment of the tilt angle, an MG90S servo motor was used, which allows it to be changed precisely.

The PID (Proportional, Integral, and Derivative) controller is one of the most widely used control algorithms in engineering due to its simplicity and effectiveness for linear or slightly nonlinear systems. This type of controller seeks to minimize the error between a desired signal and the actual output of the system, acting accordingly to correct said error in real time.

The PID controller is based on a control law that adds three terms: Proportional Part (KP), this part responds directly to the present error; The integral part (KI), adds the error over time, eliminating the steady-state error that the proportional part cannot correct on its own; Derivative Part (KD), Predicts the future behavior of the error by analyzing its rate of change.

In this project, the Ball and Beam system was controlled by a digital PID, programmed on an Arduino board. It was noted that the Ball and Beam is a naturally unstable system that requires constant control action to keep the ball in the desired position. Through empirical testing, characteristics such as inertia, friction, and delay in servo motor response were identified. The PID algorithm was programmed manually, without using external libraries. For the calculation of the integral and derived parts of the controller, the trapezium rule was used, a numerical technique that allows obtaining a better approximation of the area under the curve (integral), as well as a more stable estimate of the derivative. This contributed to a significant improvement in the system's response, especially in the face of noise and instability.

$$u(t) = \mathrm{Kp}\, e(t) + \mathrm{Ki} \int_0^t e(\tau)d\tau + \mathrm{Kd}\frac{\mathrm{de}(t)}{dt} \tag{1}$$

KP: Proportional Gain
Ki: Integral Gain
Kd: Derived Gain
e(t)=Xdes (t) -X (t)

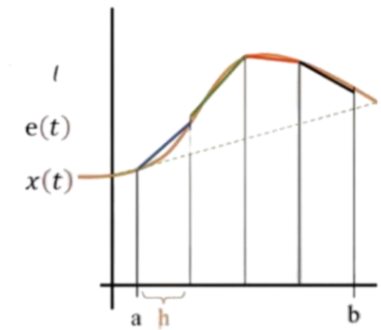

Fig. 1. Trapezius rule.

$$\int_0^t e(\tau)d\tau = \sum_{k=0}^{n} \frac{h\left(e(k)+e(k-1)\right)}{2} \tag{2}$$

The trapezium rule is a method of approximation to a defined integral and is based on approximating the area under the curve of a function by adding the areas of trapezoids formed in the integration interval, which gives us a more accurate estimate.

The adjustment of the KP, KI and KD constants was done by trial and error, seeking a compromise between stability, response time and minimal oscillation. To finish with the adjustment, tests were carried out on the physical system and the responses to different disturbances were analyzed.

The HC-SR04 ultrasonic sensor readings were identified as exhibiting inconsistent variations due to inherent ambient noise, surface type, and ultrasonic echo behavior. To improve the stability and accuracy of the control system, it was decided to integrate a Kalman filter within the PID controller, which allowed the measurements to be filtered in real time before being processed by the control logic.

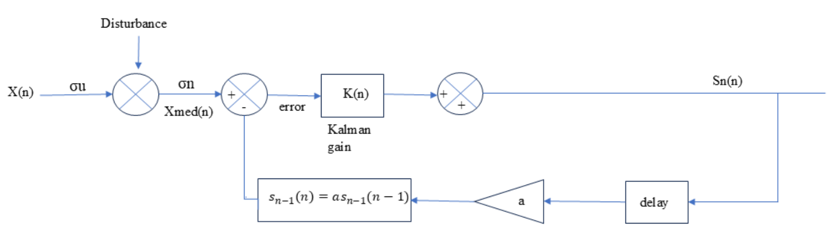

Fig. 2. Diagram of Kalman's filter blocks.

Where:
x(n) noise free signal
$X_{med}(n)$ noisy signal

σ_n standard deviation of the noise free signal
σ_n standard deviation of the signal with noise
Estimation error
$X_{med}(n) - \hat{S}_n(n-1)$
$\hat{S}_n$ number of elements in the sample
$(n-1)$ instant at whitch the estimate is obtained
$\hat{S}_n(n) = \hat{S}_n(n-1) + k(n)[X_{med}(n) - \hat{S}_n(n-1)]$ (first estimate)
$S_{n-1}(n) = a\, S_{n-1}(n-1)$
Least square error
$m_{n-1}(n) = a^2\, m_{n-1}(n-1) + \tau u^2$

$$\tau_u^2 = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(x(k) - \overline{x}_n\right)^2} \quad (3)$$

$$\tau_n = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(x_{\text{med}}(k) - \overline{x}_{\text{med}}\right)^2} \quad (4)$$

$\bar{x}$ mean of the noise $-$ free simples
$\bar{x}$ mean of the noisy simples
Kalman gain

$$k(n) = \frac{m_{n-1}^{(n)}}{m_{n-1}^{(n)} + \tau_n^2} \quad (5)$$

$$\hat{S}_n(n) = \hat{S}_n(n-1) + k(n)\left[X_{\text{med}}(n) - \hat{S}_n(n-1)\right] \quad (6)$$

Least square error update

$$m_n(n) = m_{n-1}(n)\left[1 + k(n)\right] \quad (7)$$

The implementation was done directly within the code, adjusting the filter to work with a single state variable. This implementation allowed the PID controller to work with a more stable and reliable signal, reducing oscillations in the output and avoiding abrupt corrections of the servo motor. In comparative tests, Kalman's filtering system showed less variability in response, greater accuracy in ball positioning, and better overall performance against disturbances.
Finally, the prototype was developed correctly, managing to demonstrate the principle of automatic control in an unstable physical system, in addition to validating the functionality of modern techniques such as the Kalman filter in low-cost embedded systems.
During testing, the system showed a good responsiveness to small displacements and external disturbances, however, it was observed that the behavior of the prototype was influenced by physical factors such as friction

between the ball and the rail, the delay of the sensor and the response speed of the servo motor. By integrating techniques such as the trapezium rule into the calculation of the integral and derived part of the PID, and by filtering the input signals, it was possible to effectively compensate for these effects.

initially, the variables that show the degrees of freedom must be very specific, according to the diagram shown in Figure 3. the first generalized coordinate of the Ball and Beam is the position of the bar relative to the axis of the x-coordinate which becomes an angle and is represented by q_2, while the second generalized coordinate is given by the displacement of the ball along the entire surface of the bar with respect to The crossing of the x and y axes, is represented by q_1.

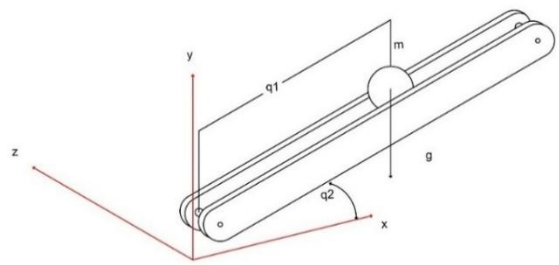

Fig. 3. Variables of the Ball and Beam System (Cuevas, 2008).

In Figure. 3. The diagram of the beam carrying the ball is visualized, you can notice the movement it will make with respect to the x-axes, and the inclination will be controlled by the servo motor so that the ball can move along the beam, hence the kinetic energy that the ball will cause.
Once the degrees of freedom have been defined, which in this case translates into the generalized coordinates, the calculation of the energies of the system arises.
The total kinetic energy of a rigid body is the sum of its translational and rotational kinetic energy, which is expressed as:

$$\kappa = \sum_{i=1}^{n} \frac{1}{2} I_i v_i^2 \tag{8}$$

where *n* represents the number of generalized coordinates, *Ii* is the moment of inertia of the i-th body in motion, and *Vi* is the i-th linear velocity.
As can be seen in Figure 3, the Ball and Beam system has two independent masses that are given by the ball and the bar, respectively, therefore, each of them contributes to the kinetic energy as follows:

Table 1. Parameters and variables of the *ball and beam system*.

Parameter	Name	Value	Units
Question 1	Ball position	-	*m*
Question 2	Bar position	-	*Grad*
ℓ	Bar Length	0.5	*m*
m	Ball Mass	0.02	kg
g	Gravity Constant	9.81	*m/sec2*
R	Ball Radius	0.035	*m*

b	Distance from the axis of rotation to the center of the beam	0.25	m

$$K_{\text{bola}} = \sum_{i=1}^{2} \frac{1}{2} b V_b^2 \tag{9}$$

$$K_{\text{barra}} = \sum_{i=1}^{2} \frac{1}{2} m V_m^2 \tag{10}$$

where Ib, I_m, V_b and V_m are the instants of inertia and velocity of the ball and the bar, respectively. Next, a decomposition of the kinetic energy of the ball is carried out in its linear fragment and in its angular part, since the ball, in addition to rising and falling due to the action of the bar, (angular movement) also moves along it (linear movement).
The breakdown is as follows:

$$K_{\text{bola}} = \frac{1}{2} b \dot{q}_2^2 + \frac{1}{2} m \dot{v}_m^2 \tag{11}$$

Where $\vec{V}_m$ is the velocity vector of the mass.
The total equation of kinetic energy is the sum of two partial equations that show the kinetic energy of the masses.

$$\begin{aligned} K_{\text{total}} &= \frac{1}{2} I_b \dot{v}_b^2 + \frac{1}{2} I_m \dot{q}_2^2 + \frac{1}{2} m \dot{v}_m^2 \\ &= \frac{1}{2} b \dot{q}_2^2 + \frac{1}{2} I_m \dot{q}_2^2 + \frac{1}{2} m \dot{v}_m^2 \\ &= \frac{1}{2}(b + I_m)\dot{q}_2^2 + \frac{1}{2} m \dot{v}_m^2 \end{aligned} \tag{12}$$

To find $\vec{V}_m$, you need the position vector of the ball's center of mass in Figure 4, which is given by:

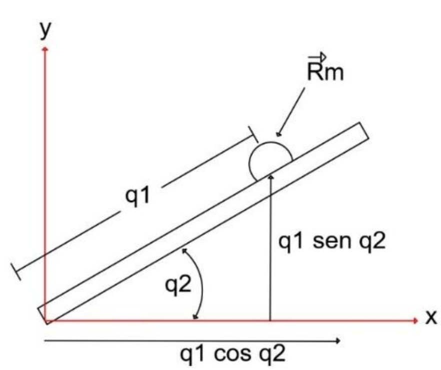

Fig. 4. Vector of position of the center of mass of the ball (Cuevas, 2008).

$$\vec{R}_m = q_1(\cos q_2, \sin q_2) \tag{13}$$

Similarly, the velocity of the center of mass is obtained by differentiating the position vector with respect to time:

$$\vec{V}_m = \frac{d}{dt}(\vec{R}_m) \tag{14}$$

$$\begin{aligned} &= -q_1\dot{q}_2 \sin q_2 + \dot{q}_1 \cos q_2, \quad q_1\dot{q}_2 \cos q_2 + \dot{q}_1 \sin q_2 \\ &= \left(V_{11}, V_{12}\right) \end{aligned} \tag{15}$$

$$\begin{aligned} V_{11} &= -q_1\dot{q}_2 \sin q_2 + \dot{q}_1 \cos q_2 \\ V_{12} &= q_1\dot{q}_2 \cos q_1 + \dot{q}_1 \sin q_1 \end{aligned} \tag{16}$$

To determine the square of the velocity $\vec{V}_m$, you must:

$$\begin{aligned} \vec{V}^2{}_m = \vec{V}_m \vec{V}^T{}_m &= \begin{bmatrix} V_{11} & V_{12} \end{bmatrix}, \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} \\ \vec{V}_m &= \dot{q}_1 + q_1\dot{q}_2 \end{aligned} \tag{17}$$

substituting the values of V_{11} and V_{12}*result:*

$$\begin{aligned} \vec{V}^2_{\mathrm{m}} &= q_1{}^2\dot{q}_2 \sin q_2 - 2q_1q_2 \sin q_2\dot{q}_1 \cos q_2 + q_1^2 \cos^2 q_2 + \dot{q}_2^2\dot{q}_2^2 \cos^2 q_2 + 2q_1q_2 \cos q_2\dot{q}_1\, \mathrm{s}\, en q_2 + \dot{q}_1^2 \mathrm{s}\, en^2 q^2 \\ \vec{V}^2_m &= \dot{q}_1^2 + q^2{}_1\dot{q}_2{}^2 \end{aligned} \tag{18}$$

Finally, if (18) (19) is replaced, we get:

$$K_{\text{total}} = \frac{1}{2}(I_b + I_m)\dot{q}_2^2 + \frac{1}{2}m(\dot{q}_1^2 + q_1^2\dot{q}_2^2) \tag{19}$$

The position of a body with mass *m* that is at a height *h* is recognized as potential energy and is calculated by the following equation:

$$V = mgh \tag{20}$$

If observed in Fig. 3 and in accordance with (20) it can be deduced that:

$$h = q_1 \sin(q_2) \tag{21}$$

Therefore, the equation of the total potential energy of the ball and *beam* is defined by:

$$V_{\text{total}} = mgq_1 \sin q_2 \tag{22}$$

Once the potential and kinetic energies of the Ball and Beam system have been obtained, the next step is to find the Lagrangian by substituting the kinetic (14) and potential (17) energies of the system and according to (14) we have:

$$\begin{aligned} L &= \frac{1}{2}(I_b + I_m)\dot{q}_2^2 + \frac{1}{2}m(\dot{q}_1^2 + q_1^2\dot{q}_2^2) - mgq_1 \sin q_2 \\ &= \frac{1}{2}I_{\text{total}}\dot{q}_2^2 + \frac{1}{2}m(\dot{q}_1^2 + q_1^2\dot{q}_2^2) - mgq_1 \end{aligned} \tag{23}$$

As can be seen in equation (24) the total inertia of ITotal *is* composed of the sum of the inertia *Ib* and *Im*, to find *ITotal first* the inertia of the ball and the inertia of the bar where *Im* is divided into the inertia of the bar *Im1*, plus the inertia of the bond *Im2*. He defined total inertia as.

$$I_{\text{total}} = I_b + I_{m1} + I_{m2} \tag{24}$$

A variable change in the equations of a system refers to the change in the order of the equations of the system, that is, more specifically, going from having a pair of second-order equations to having four first-order equations and thus simplifying the solution process.

For the nonlinear model of the system *of balls and beams* given by equations, the following variables are proposed:

$$\begin{aligned} x_1 &= q_1 \\ x_2 &= q_2 \\ x_3 &= \dot{q}_1 \\ x_4 &= \dot{q}_2 \end{aligned} \quad \textbf{(25)}$$

The derivation of the new variables is maintained

$$\begin{aligned} \dot{x}_1 &= \dot{q}_1 = x_2 \\ \dot{x}_2 &= \ddot{q}_1 \\ \dot{x}_3 &= \dot{q}_2 = x_4 \\ \dot{x}_4 &= \ddot{q}_2 \end{aligned} \quad \textbf{(26)}$$

As can be seen, the value of $\dot{x}_1$ and $\dot{x}_3$ is shown as a function of the variable change, so it remains to find the value of $\dot{x}_2$ and $\dot{x}_4$, to obtain these values it is erased from the equations, for this the variable changes in equation (26) is substituted:

$$m\dot{x}_2 - mx_1x_4^2 + mg\sin(x_3) = 0 \quad \textbf{(27)}$$

from where 2 is deleted, obtaining:$\dot{x}$

$$m\dot{x}_2 - mx_1x_4^2 + mg\sin(x_3) = 0 \quad \textbf{(28)}$$

For x_4, the change of the variable and its resulting derivative:

$$\dot{x}_4(I_T + mx_1) + 2mx_1x_2x_4 + mgx_1\cos x = f \quad \textbf{(29)}$$

from which 4 are eliminated, obtaining:$\dot{x}$

$$\dot{x}_4 = \frac{f}{I_T + mx_1{}^2} - \frac{2mx_1x_2x_4}{I_T + mx_1{}^2} - \frac{mgx_1\cos(x_3)}{I_T + mx_1{}^2} \quad \textbf{(30)}$$

In a mathematical representation of the ball-and-beam system in the matrix state variables, it is as follows:

$$\begin{aligned} x &= f(x) + g(x) \\ y &= u(x) \end{aligned} \quad \textbf{(31)}$$

where *you* = f(x) and $g(x)$ are composed of:

$$f(x)=\begin{bmatrix} x_2 \\ x_1 {x_4}^2 - g\,\text{sen}\left(x_3\right) \\ x_4 \\ -\dfrac{2mx_1x_2x_4}{I_T+m{x_1}^2}-\dfrac{mgx_1\cos x_3}{I_T+m{x_1}^2} \end{bmatrix}\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

(32)

$$\mathrm{g(x)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{\mathrm{I_T + m{x_1}^2}} \end{bmatrix}$$

This state equation summarizes the dynamic model and will serve as the basis for the design and implementation of the controller.

For control purposes, an output of the system is selected, for the ball and beam the output is the position of the ball, that is:

$$y = h(x) = q_1 = x_1$$

(33)

The construction includes a beam on which the ball rolls, representing the desired position, an axis that allows the beam to tilt, a rotary-linear mechanism connected to the servo motor for tilting, a servo motor support, and a base for the prototype made of lightweight wood. The beam was smoothed by sanding its surface, leaving a V-shaped groove to hold the ball. The shaft is secured to the base with screws, and the beam is assembled at the top using a screw coupling to ensure proper alignment. The servo motor is mounted on its stand along with the rotary-linear mechanism, with an initial angle set to 90°, and the sensor is positioned at the end of the beam and the mechanism. The cables and breadboard are used to connect the sensor to pins 6 and 7, and the servo motor to pin 9 of the Arduino, as illustrated in Figure 5.

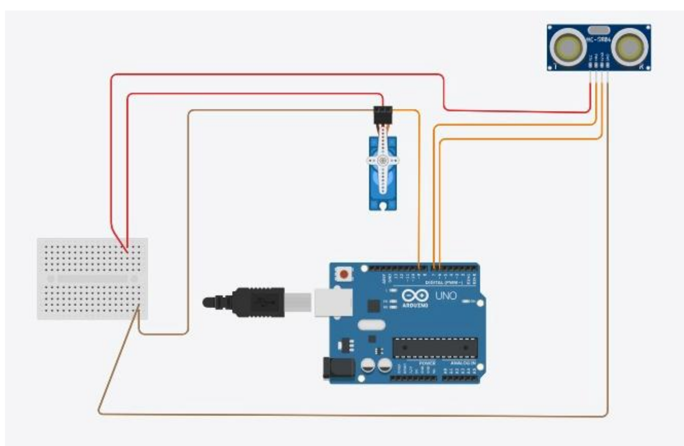

Fig. 5. Connection diagram. Sensor (HC-SR04) and a servo motor (SG90) controlled by the PID algorithm implemented in an Arduino Uno.

PID parameters are adjusted to achieve the desired stability and tracking of the ball, performing initial tests to verify correct mechanical and electronic operation, validating system behavior, and making improvements to PID parameters based on test results. This process allowed the construction of a functional ball-and-beam prototype suitable for the understanding of automatic control and academic experimentation.

Table 2. Ball and Beam Cost Table

COMPONENT	DESCRIPTION	COST
Wood	Light pirul wood	$100.00 MXN
Ball	Plastic	$30.00 MXN
Servomotor	Generates the angular position	$70.00 MXN
Ultrasonic sensor	It measures in real time the angular position of the ball on the bar.	$60.00 MXN
Arduino	Processes information and calculates corrective actions using the PID control algorithm	$100.00 MXN
Cables	Connections	$50.00 MXN
Screws	Hitches	$20.00 MXN
Termofit	Connections	$30.00 MXN
Total		$460.00 MXN

The development of the ball-and-beam prototype along with the focus on costs, the selection of affordable electronic components and low-cost materials. Likewise, with the use of free software, together with the Arduino board, maintaining a low cost, it is possible to implement an efficient control system for educational purposes, with an economic investment of $460.00 MXN.

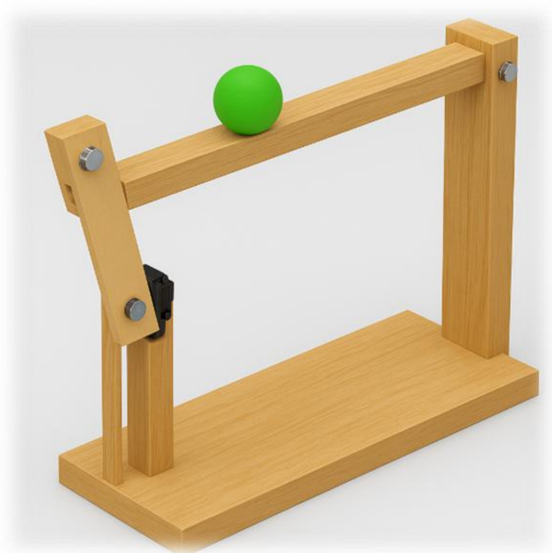

Fig. 6. Design and implementation.

In Figure 6. It illustrates the physical design and implementation of the system, its structure and key components to achieve its correct operation.

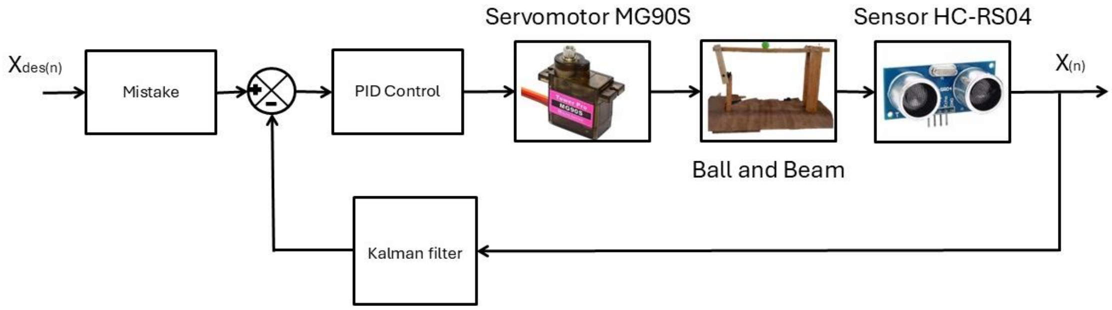

Fig. 7. Block diagram Ball and beam system.

The desired position, Xdes(n), the estimation error, and the PID control of the prototype are illustrated with a Kalman filter applied at the output, producing a noise-free signal, X(n). The PID controller generates the control signal based on the difference between the desired and estimated positions. The MG90S servo motor receives this signal and adjusts the beam angle accordingly. The ball moves along the beam according to physical laws. The HC-SR04 sensor measures the ball's position, and the Kalman filter processes these noisy measurements. The resulting improved position estimate is fed back to the control system.

2 Results and discussion

The *Ball & Beam* system proved to be an effective platform for the application of classical controllers. A PID controller was used to regulate the position of the ball on the beam by comparing the position measured by the ultrasonic sensor with a desired reference. The controller generates a control signal based on the proportional, integral, and derivative components of the error, producing corrective actions applied to the servo motor to tilt the beam and maintain the ball at the desired position.

The PID controller was tuned based on the system response, aiming to achieve stabilization in minimal time with low overshoot and minimal steady-state error. Initially, the system's response to disturbances and reference changes was identified, and the PID parameters were adjusted using a trial-and-error approach guided by performance criteria. The system was configured with the following parameters: proportional gain (K_p) of 20.0, integral gain (K_i) of 0.03, and derivative gain (K_D) of 20.0. The Kalman filter was implemented with a process variance (σ_u) of 0.1 and a measurement variance (σ_n) of 9.0 to enhance the accuracy of the measured position. Tests were performed in a controlled environment, on a flat surface free from external disturbances. The ball was initially positioned 50 cm from the edge of the beam, and mechanical forces were applied relative to the horizontal. The test lasted 60 seconds, with the objective of bringing the ball to 25 cm.

Under these conditions, the results showed that the ball reached an approximate position of ±2 cm relative to the desired reference, demonstrating system stability. However, the response fluctuated depending on the quality of the input signal, justifying the use of the Kalman filter. To evaluate the impact of the Kalman filter, comparative tests were conducted without its implementation under the same PID parameters and experimental conditions.

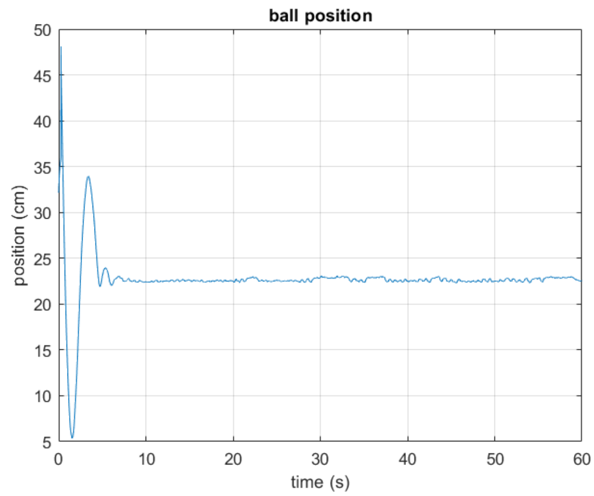

Fig. 8. Position of the ball with respect to Kalman's unfiltered time.

In Figure 8 shows the behavior of the system without the Kalman filter, where an overshoot of approximately 20 cm and a minimum of 6 cm is observed at the beginning and the stabilization time was approximately 10 seconds.

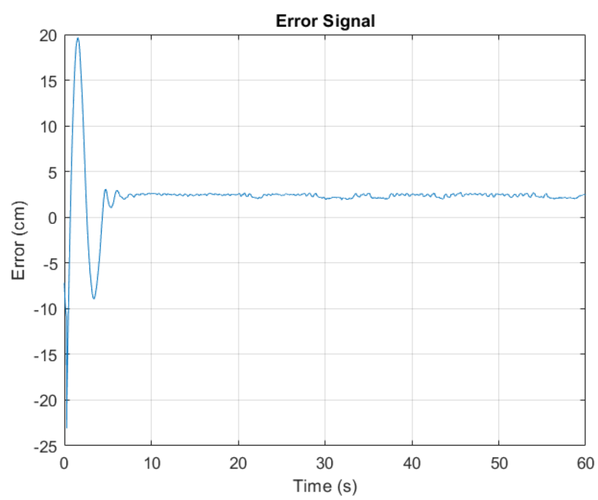

Fig. 9. Kalman's unfiltered error signal.

In Figure. 9 shows more pronounced oscillations over time, causing the system to take longer to stabilize.

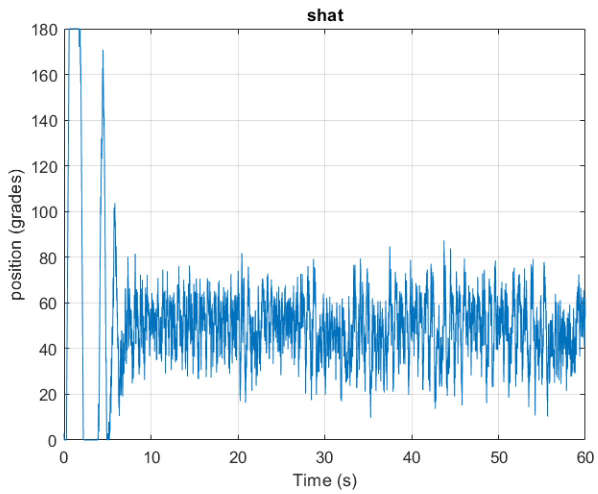

Fig. 10. Kalman's unfiltered signal.

In Figure. 10 shows that without the Kalman filter the signal is very noisy and causes direct consequences with the position of the ball and making the system less stable.

By implementing the Kalman filter, noise in the input signal was reduced, resulting in a more stable and precise control action, which is particularly noticeable when the ball moves to reach its equilibrium position. Without the filter, oscillations were more frequent, and the system responded more aggressively, negatively affecting stability. The Kalman filter enabled the integration of ultrasonic sensor measurements with the predictive model of the ball's motion, providing a more accurate estimate of the actual position. The combination of a PID controller and a Kalman filter in the *Ball & Beam* system achieved high performance in terms of stability and precision in control execution. This validates the approach for applying classical controllers to complex physical systems.

The behavior of the ball is illustrated in the graphs. In the experimental setup, the desired position was set to 25 cm, while the ball stabilized at approximately 23 cm. This deviation of 2 cm can be attributed to factors such as PID tuning and friction from the wooden beam, whose material properties introduce significant resistance, resulting in stabilization within ±2 cm. As shown in Figure 13, an overshoot of approximately 12 cm occurs when the ball's position temporarily exceeds the desired value, primarily due to the ball's inertia. The ball reached a maximum position of 37 cm during the initial transient response, after which it stabilized in approximately 5 seconds.

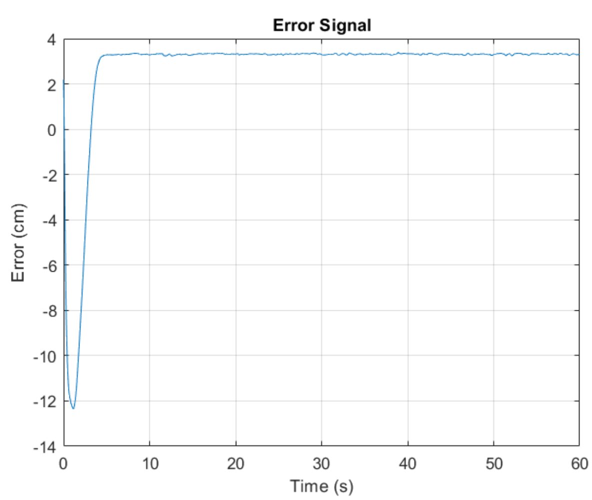

Fig. 11. Error signal.

The estimated position (Shat) is the result of the Kalman filter applied to the sensor measurements; it is a filtered variant of the measured distance that eliminates noise and makes the system more stable for PID control. Figure 11 shows the response of the control loop in the system and the measurement of the feedback behavior of the Shat state, after the noise component described above has been added.

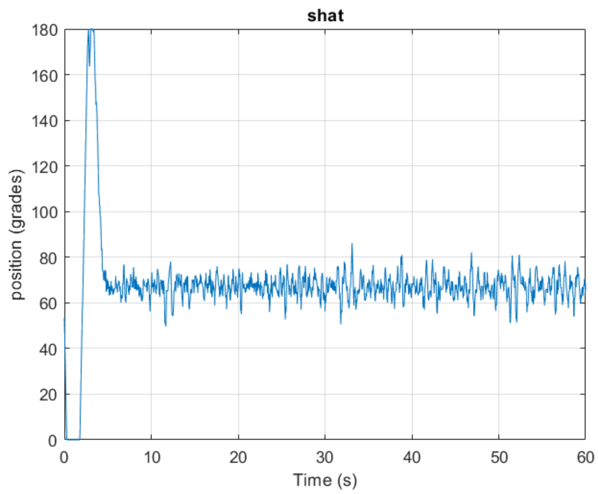

Fig. 12. Filtered signal (Shat).

The PID control signal determines at what angle the servo motor should be positioned to tilt the beam in order for the ball to reach its desired position.

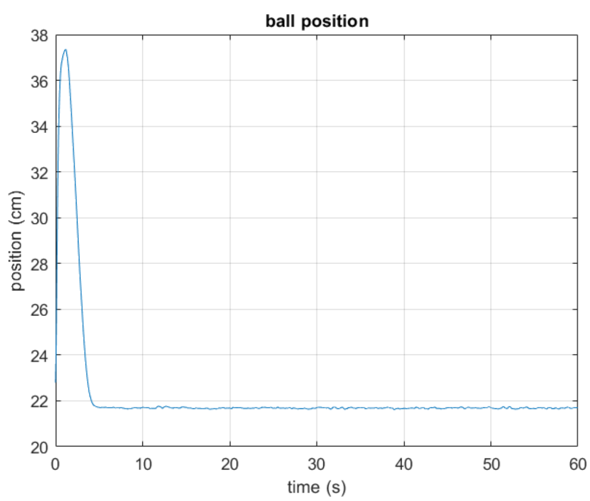

Fig. 13. Position of the ball with respect to time.

In fig. 13 it is visualized that the ball stabilizes in approximately 5 seconds, generating a margin of error effect +/- 2cm. As mentioned in the introduction, it is important to choose the right materials for the prototype, in addition to the cost, criteria such as functionality, availability, safety and compatibility, whether of mechanical or electronic components, must be taken into account, as this can greatly influence the performance and operation of the system and allow the ball to stabilize in the desired position.

The advantage of using Arduino software is that it is very accessible and friendly in its programming language, having the flexibility with the serial port, also with electronic components such as the ultrasonic sensor and servomotor.

One of the disadvantages of the ultrasonic sensor is that it has a range to detect objects, when the ball is very close it does not detect it. During the construction of the prototype, various manufacturing techniques were used, such as carpentry and finishing on a milling machine for the ball gutter. Taking into account all this, the results described in the initial part of this section were achieved, with this it can be said that the objectives to solve the initial problem are achieved.

3 Conclusions

The integration of the Kalman filter into the PID controller in the *Ball and Beam* system significantly reduced the noise in the measured position signal, resulting in more stable control. With this implementation, the average deviation from the desired position of 25 cm decreased from ±4 cm (without the filter) to ±2 cm, and the settling time was reduced from 10s to 5s. Furthermore, the maximum overshoot was reduced from 15 cm to approximately 12 cm, indicating improved handling of the ball's inertia. These results demonstrate that the PID–Kalman combination enhances the system's performance in terms of accuracy and stability, validating its application for controlling physical systems with noisy measurements.

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