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Impact of Nonlinear Mutual Magnetic Flux on the Operational Performance of Single-Phase Transformers: A Numerical Approach

Mario Oscar Ordaz Oliver [1], Alberto Ortiz Licona [2], María Angélica Espejel Rivera [1], Amadeo Manuel Hernández Hernández [1]*

[1 *]Departamento de Ingeniería Eléctrica y Electrónica, Tecnológico Nacional de México, Campus Pachuca,
[2] Hitachi Energy USA, Raleigh, NC, USA
mario.oo@pachuca.tecnm.mx, alberto.ortiz-licona@hitachienergy.com, maria.er@pachuca.tecnm.mx, amadeo.hh@pachuca.tecnm.mx

Abstract. In this work, a numerical analysis of the mathematical model of a single-phase transformer is carried out, integrating a complete expression of the mutual magnetic flux to evaluate its impact on energy transfer efficiency. Based on Faraday's and Ampère's equations, the fundamental relationships between magnetic flux, currents, and voltages in the windings are established. Through numerical simulations, the influence of mutual leakage flux is examined, along with structural parameters such as resistances, reactances, and the number of turns. The obtained results validate the model's usefulness in predicting the transformer's behavior under variations in load and network conditions, providing an analytical tool for the design and improvement of these electrical machines. This approach not only enhances the theoretical understanding of transformers but also contributes to the development of more efficient and reliable electrical systems.

Keywords: Computational simulation, electromagnetic analysis, energy transfer, operational efficiency, mathematical modeling.

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1 Introduction

An electrical transformer is a static electromagnetic machine that enables the transfer of electrical energy between two or more circuits through the principle of electromagnetic induction. Its operation is based on the interaction of magnetic fields generated by alternating currents in the primary and secondary windings, allowing voltage and current levels to be modified without altering the frequency. This transformation capability is fundamental in electrical power generation, transmission, and distribution systems, as it enables efficient energy transfer with minimal losses over long distances.

The single-phase transformer plays a crucial role in modern electrical infrastructure, with applications ranging from power distribution systems in residential and industrial environments to strategic infrastructures such as hospitals, data centers, and telecommunications systems, where a stable and reliable power supply is essential. Additionally, its integration into renewable energy systems and smart grids is crucial for improving energy efficiency and facilitating the transition toward more sustainable generation and consumption systems.

Despite its importance, the design and construction of electrical transformers present various technical challenges. Among the most relevant issues are energy losses due to nonlinear phenomena such as hysteresis and eddy currents in the ferromagnetic core, leakage flux losses, variations in transformer performance due to load fluctuations and voltage changes, as well as the identification of its parameters for characterizing transformer losses. Addressing these challenges requires the formulation of rigorous mathematical models that accurately describe the electromagnetic behavior of the transformer, enabling its analysis and optimization under different operating conditions.

Ćalasan et al. (2020) analyzed parameter estimation in single-phase transformers, considering different mathematical approaches. Meanwhile, Yarymbash et al. (2019) developed a unified mathematical model of the single-phase transformer, incorporating effects such as hysteresis and eddy currents. In their work, Ketabi and Naseh (2012) developed a single-phase transformer modeling method for inrush current estimation, considering core saturation and residual flux. Similarly, Wilk (2015) developed a mathematical model to represent the hysteresis phenomenon in a single-phase transformer with multiple windings. His study proposed a set of differential loop equations that account for the common flux linking all windings as a function of magnetomotive force. Huang et al. (2003) developed a harmonic model to represent nonlinearities in a single-phase transformer using descriptive functions. Their study proposed mathematical formulas for excitation current under no-load conditions, allowing the Fourier series to be obtained and, consequently, calculating the magnitudes and phase angles of different harmonic orders. Additionally, they validated the model's accuracy through experimental measurements, highlighting its simplicity in calculations and high precision in harmonic estimation. Furthermore, Martínez Figueroa (2024) developed and validated mathematical models for single-phase and three-phase transformers, estimating their parameters based on laboratory tests and field measurements. His study addressed transformer core saturation and the impact of inrush current on power system protection. Additionally, he explored methodologies to mitigate these effects and emphasized the importance of accurately modeling the nonlinearity of the magnetic circuit, considering models such as Jiles-Atherton and Preisach. Krishan et al. (2016) proposed a real-time algorithm for estimating electrical parameters in single-phase transformers using LabVIEW-FPGA. In their study, they measured and sampled input and output voltages and currents to recalculate series winding parameters through a differential equation algorithm. Furthermore, they employed the Fast Fourier Transform (FFT) with a Hanning window to extract fundamental components and applied the Least Squares Error (LSE) method for core loss curve fitting. The validity of their method was confirmed through simulations in LabVIEW and Multisim, as well as experimental results. In their work, García et al. (2000) presented a state-space model of a single-phase transformer incorporating nonlinear phenomena such as magnetic saturation and hysteresis. Their analysis covered transient and periodically steady state operating conditions, utilizing a novel and simplified formulation to represent these nonlinearities. Additionally, they applied Newton algorithms to accelerate time-domain computations, achieving rapid steady-state solutions. They compared their method with conventional numerical approaches for solving ordinary differential equations (ODEs), evaluating efficiency in terms of complete cycles and CPU times required. Similarly, Hernández-Romero et al. (2020) addressed the optimization problem in designing the active part of a 10 MVA, 115/13.8 kV three-phase power transformer, aiming to minimize total acquisition cost considering losses. To achieve this, they implemented a method based on Genetic Algorithms (GA), enabling the calculation of dimensions, core mass, windings, losses, and transformer impedances. The obtained results demonstrated that the optimized model meets specified constraints and minimizes cost function, which is crucial during the bidding stage for transformer manufacturers.

The mathematical model of the single-phase transformer presented in this study is formulated based on the fundamental principles of electromagnetism, as established by Faraday, Ampère, Lenz, and Maxwell. These principles define the linear relationships governing voltages, currents, and magnetic flux in the transformer windings. A key variable in this model is the mutual magnetic flux, which quantifies the portion of flux generated by one winding that links with the other, thereby directly influencing energy transfer efficiency. A precise mathematical representation of this parameter enables the minimization of losses, enhancement of operational efficiency, and optimization of electromagnetic coupling between windings.

Irrespective of the specific transformer configuration, its operation relies on energy transfer through magnetic coupling between the primary and secondary windings, both wound around a ferromagnetic core. Typically composed of laminated silicon steel, this core is essential in mitigating eddy current losses induced by time-varying magnetic fields. Lamination increases the core's effective electrical resistance, thereby reducing eddy currents and enhancing overall efficiency.

Structurally, the primary and secondary windings are arranged concentrically, with the lower-voltage winding positioned closer to the core. This design facilitates electrical isolation from high voltage winding and minimizes stray magnetic flux, thereby improving magnetic coupling and energy transfer. Stray flux, representing the portion of the generated flux that does not effectively link both windings, contributes to leakage reactance and impacts overall transformer performance.

Mathematical modeling serves as a crucial tool for analyzing and optimizing transformer operation. This model integrates Maxwell's equations with circuit theory to describe electromagnetic interactions. Faraday's law dictates that the induced voltage in a coil is proportional to the temporal variation of the magnetic flux passing through it, forming the foundation of electromagnetic energy conversion. Ampère's law establishes the relationship between current in the windings and the resulting magnetic field, while Lenz's law ensures energy conservation by enforcing opposition between the induced current and the flux variation that generates it.

An accurate transformer model must incorporate mutual and stray flux components. Magnetizing inductance, governed by core permeability and winding geometry, determines the extent of coupled flux, whereas leakage inductances represent flux components that fail to link both windings. These leakage effects are modeled using series inductances in the equivalent circuit.

Core losses, arising from hysteresis and eddy currents, are typically represented by a parallel resistor in the magnetizing branch, capturing energy dissipation due to alternating magnetization cycles.

Mathematical formulation results in a system of coupled differential equations that describes the temporal evolution of voltages and currents within the windings. In the time domain, these equations characterize the dynamic interplay between electrical and magnetic parameters, offering a robust analytical framework for assessing transformer behavior under varying operating conditions, such as load fluctuations, connection transients, and short-circuit events.

By implementing this mathematical model in computational simulation environments, transformer response can be predicted across a broad spectrum of scenarios, reducing reliance on physical testing and thereby minimizing costs. Such simulations facilitate the quantification of efficiency, evaluation of loss reduction strategies, and design optimization for enhanced performance. This study develops a model based on fundamental physical laws governing electrical and magnetic circuits. The resulting system of differential equations is implemented in a block diagram simulation framework, enabling comprehensive analysis of transformer behavior under diverse operating conditions and design configurations. This approach not only deepens the theoretical understanding of transformer operation but also serves as a valuable tool for the development of more efficient and reliable electrical systems.

Advanced numerical simulations are indispensable for evaluating transformer performance under various conditions, allowing the examination of core saturation, load variations, and supply voltage fluctuations. This study introduces a mathematical model incorporating a rigorous formulation of mutual flux to achieve a more accurate representation of the transformer's electromagnetic behavior. The model is subsequently implemented in a numerical simulation, providing a robust analytical tool for transformer design and optimization, ultimately contributing to the development of high-efficiency and high-reliability electrical power systems.

2 Methodology

The mathematical modeling analyzed in this work, which is related to the behavior of a single-phase transformer, is based on the fundamental laws of electromagnetism formulated from Maxwell's principles, as well as the electrical circuit equations applied to electrical machines. These laws allow the description of magnetic flux behavior, electromagnetic induction, and the relationships between current and voltage (Chapman, 2012). The operation of the transformer is based on Faraday's law, which states that the induced voltage in a coil is proportional to the time variation of the magnetic flux passing through it, enabling energy transfer through the variation of magnetic flux in the primary and secondary windings magnetically coupled through the ferromagnetic core. The relationship between the current in the windings and the magnitude of the magnetic field is described by Ampère's law, which links the magnetic field to the current that generates it through the number of turns in the winding, thereby allowing the modeling of the electromagnetic coupling of the transformer (Kosow, 2021). The polarity of the induced voltage in the secondary winding is determined by Lenz's law, which states that the induced current opposes the variation of the magnetic flux that causes it, ensuring the conservation of energy in the system. Maxwell's equations provide a more general basis for describing the propagation of the electromagnetic field within the transformer, particularly the Maxwell-Faraday equation, which explains how a time-varying electric field generates a time-varying magnetic field, thus grounding electromagnetic induction in the transformer. Since the magnetic flux coupling both windings is not completely ideal, a portion of the flux is dispersed and is called leakage flux, in addition to the fact that the ferromagnetic core exhibits losses due to hysteresis and eddy currents, which affect the transformer's efficiency and are often modeled by an equivalent resistance in the magnetic circuit and a frequency-dependent loss term (Fraile Mora, 2008). In mathematical terms, the transformer's T-model represents these effects through parameters such as winding resistance, self-inductances, and magnetizing inductance, allowing the formulation of a system of coupled differential equations that describe the time evolution of voltages and currents in the transformer. The time-domain equations express the relationship between input and output voltages, currents in the windings, and variations of the magnetic flux in the core, thereby providing a fundamental analytical tool for the simulation and optimization of the transformer's behavior under different operating conditions.

Faraday's law is a fundamental law describing how voltage is generated in transformer windings due to changes in magnetic flux. It states that the magnitude of the induced voltage in a conductor is proportional to the rate of change of the magnetic flux passing through it:

$$e_{ind}(t) = \frac{d\lambda(t)}{dt}$$

Where λ(t) is the linked flux of the coil through which the voltage $e_{ind}(t)$ is induced. The linked flux is the sum of the flux passing through each turn in all the turns of a coil.

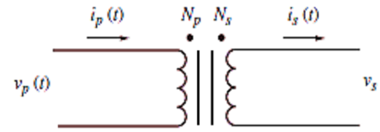

Fig. 1. Diagram of an ideal electrical transformer.

The transformation ratio is key to modeling how the transformer adjusts the voltage levels between primary and secondary and is obtained from Ampere's law as (Kosow, 2021):

$$V_p = N_p \frac{d\varphi(t)}{dt}, \qquad V_s = N_s \frac{d\varphi(t)}{dt}$$

If the variation of the magnetic flux density φ is isolated in both expressions, we have:

$$\frac{d\varphi(t)}{dt} = \frac{V_p}{N_p}, \qquad \frac{d\varphi(t)}{dt} = \frac{V_s}{N_s}$$

Although an ideal transformer model assumes that the magnetic flux is uniformly distributed throughout the core at every instant, this condition is not strictly met in practical scenarios. Nevertheless, such an assumption proves sufficiently accurate for steady-state analysis. In real-world operation, a transient response in the magnetic flux arises when the transformer transitions from one load condition to another. The current drawn by the load induces an increase in magnetic flux, which also links to the primary winding, necessitating a corresponding rise in the primary current. Therefore, under steady-state conditions, the time-dependent magnetic flux φ(t) can be reasonably considered uniform throughout the transformer core, leading to the following conclusion:

$$\frac{V_p}{N_p} = \frac{V_s}{N_s}$$

From this expression we can obtain the next assumption:

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

Now, if we consider that the power in primary and secondary is equal, given the assumption that the losses in the electrical machine are zero, said power can be represented according to the expressions:

$$P_p = V_p I_p, \qquad P_s = V_s I_s$$

$$P_p = P_s, \qquad V_p I_p = V_s I_s$$

Therefore, the current ratio is, as seen in Figure (1):

$$\frac{I_s}{I_p} = \frac{V_s}{V_p} = \frac{N_s}{N_p}$$

Since $\varphi_1(t)$ and $\varphi_2(t)$ are the leakage fluxs in the primary and secondary circuits, respectively. Under this consideration and using Kirchhoff's voltage law applied to the primary and secondary electrical circuit, the following is obtained:

$$V_p = R_p I_p + \frac{d\lambda_p(t)}{dt} = R_p I_p + L_{pp}\frac{di_p(t)}{dt} + L_{ps}\frac{di_s(t)}{dt}$$

With:

$$L_p \frac{di_p(t)}{dt} = \frac{d\lambda_p(t)}{dt}$$

and for the secondary side

$$V_s = R_s I_s + \frac{d\lambda_s(t)}{dt} = R_s I_s + L_{ss}\frac{di_s(t)}{dt} + L_{sp}\frac{di_p(t)}{dt}$$

Where:

$$L_s \frac{di_s(t)}{dt} = \frac{d\lambda_s(t)}{dt}$$

Where $\lambda_p(t)$ is the primary linked flux, defined as:

$$\lambda_p(t) = \lambda_{pp}(t) + \lambda_{ps}(t)$$
$$\lambda_p(t) = L_{pp} i_p(t) + L_{ps} i_s(t)$$

and $\lambda_s(t)$ is the secondary linked flux, defined as:

$$\lambda_s(t) = \lambda_{ss}(t) + \lambda_{sp}(t)$$
$$\lambda_s(t) = L_{ss} i_s(t) + L_{sp} i_p(t)$$

From these expressions, a phasor form associated with the single-phase transformer model can be rewritten. To do this, the Laplace transform is applied to the set of differential equations obtained above, assuming that the initial conditions are zero, where s = jw, with angular frequency w = 2πf and f known as the power supply frequency in Hertz. Therefore, we have:

$$V_p = \left(R_p + jwL_{pp}\right) I_p + jwL_{ps} I_s$$

$$V_s = jwL_{sp} I_1 + \left(R_s + jwL_{ps}\right) I_s$$

where the impedances can be defined as:

$$Z_{pp} = R_p + jwL_p,\ Z_{ps} = jwL_{ps}$$
$$Z_{sp} = jwL_{sp},\ Z_{ss} = R_s + jwL_{ss}$$

Now, from the previous phasor expressions and with the proposed change of variable, it is possible to represent the phasor dynamics of the single-phase electrical transformer in a matrix form, which allows the currents I_p, I_s to be evaluated in a phasor form, as follows:

$$\begin{bmatrix} V_p \\ V_s \end{bmatrix} = \begin{bmatrix} Z_{pp} & Z_{ps} \\ Z_{sp} & Z_{ss} \end{bmatrix} \begin{bmatrix} I_p \\ I_s \end{bmatrix}$$

In this sense, Cramer's rule can be applied to said phasor representation, to obtain the equations of I_p, I_s' as:

$$I_p = \frac{V_p Z_{ss} - V_s Z_{ps}}{Z_{pp} Z_{ss} - Z_{sp} Z_{ps}}$$

$$I_s = \frac{V_p Z_{pp} - V_s Z_{sp}}{Z_{pp} Z_{ss} - Z_{sp} Z_{ps}}$$

Now, if the equations of the single-phase transformer are rewritten in terms of the leakage flux, we have the following:

$$V_p = R_p i_p + \frac{1}{w} \frac{d\varphi_p(t)}{dt}$$

$$V_s' = R_s' i_s' + \frac{1}{w} \frac{d\varphi_s'(t)}{dt}$$

The above equations represent the voltages of the primary and secondary windings of the transformer, as a function of the currents and the variations of the leakage flux. The term (1/w) (dφ(t))/dt is due to inductance, and represents the change in magnetic flux φ(t), which affects the behavior of a transformer, where:

$$\varphi_p(t) = w\lambda_p(t) = X_p i_p(t) + \varphi_m(t)$$

$$\varphi'_s(t) = w\lambda_s'(t) = X_s' i_s'(t) + \varphi_m(t)$$

The currents $i_1(t)$ and $i_s'(t)$ are:

$$i_p(t) = \frac{\varphi_p(t) - \varphi_m(t)}{X_p}$$

$$i_s'(t) = \frac{\varphi_s'(t) - \varphi_m(t)}{X_s'}$$

Where: $\varphi_p(t)$ and $\varphi_s(t)$ define the magnetic flux as a function of the currents i_p and i_s' in the transformer windings. X_p and X_s' represent the primary and secondary reactances, respectively. φ_m (t) is the mutual flux between the primary and secondary windings due to magnetic coupling. This is responsible for the electromagnetic induction that allows the transformer to operate. The magnetic link between the primary and secondary windings ensures efficient energy transfer between them. It is defined as:

$$\varphi_m(t) = wL_m(i_p + i_s') = X_m(i_p + i_s')$$

The above equation describes the mutual magnetic flux $\varphi_m(t)$, where we have the mutual reactance X_m, the mutual inductance L_m in addition to the currents and the frequency

$$\frac{d\varphi_p(t)}{dt} = wV_p - wR_p i_p$$

Now, it is represented by φ_p (t) which is the magnetic flux in the primary winding influenced by the voltage applied to the primary winding V_p, in addition it is known that the resistive voltage drop losses are $R_p i_p$, with b defined as the coefficient that indicates the proportion of the voltage in the change of flux.

The following equation is the derivative of the flux $\varphi_p(t)$ with the mutual component:

$$\frac{d\varphi_{\text{p}}(t)}{dt} = wV_{\text{p}} - wR_{\text{p}}\left[\frac{\varphi_{\text{p}}(t) - \varphi_m(t)}{X_{\text{p}}}\right] \tag{1}$$

Then the derivative of the flux $\varphi_s(t)'$ is represented:

$$\frac{d\varphi'_s(t)}{dt} = wV_s' - wR_s' i_s'$$

in the same way:

$$\frac{d\varphi'_s(t)}{dt} = wV_s' - wR_s'\left[\frac{\varphi'_s(t) - \varphi_m(t)}{X_s'}\right] \tag{2}$$

Given $\varphi_m(t)$ in the form:

$$\varphi_m(t) = X_m\left(i_p + i_s'\right)$$

The primary and secondary currents are replaced, as follows:

$$\varphi_m(t) = X_m\left[\frac{\varphi_p(t) - \varphi_m(t)}{X_p} + \frac{\varphi'_s(t) - \varphi_m(t)}{X_s'}\right]$$

$$\varphi_m(t) = \left[\frac{X_m(\varphi_p(t) - \varphi_m(t))}{X_p} + \frac{X_m\left(\varphi'_s(t) - \varphi_m(t)\right)}{X_s'}\right]$$

These equations describe the behavior of a single-phase transformer under the influence of voltage in the primary and the construction parameters of the electrical machine (Fraile Mora, 2008).

Since the currents are a function of the mutual flux $\varphi_m(t)$, we proceed to find the expression for the mutual flux linkage $\varphi_m(t)$, as a function of the leakage fluxs in the primary winding and the secondary winding $\varphi_p(t)$ and $\varphi_s(t)$, respectively:

$$\varphi_m(t) = \frac{X_m(\varphi_p(t) - \varphi_m(t))}{X_p} + \frac{X_m\left(\varphi'_s(t) - \varphi_m(t)\right)}{X_s'}$$

subsequently

$$\varphi_m(t) = \frac{X_s' X_m(\varphi_p(t) - \varphi_m(t)) + X_p X_m\left(\varphi'_s(t) - \varphi_m(t)\right)}{X_p X_s'}$$

then it is obtained that:

$$X_p X_s' \varphi_m(t) = X_s' X_m \varphi_p(t) - X_s' X_m \varphi_m(t) + X_p X_m \varphi'_s(t) - X_p X_m \varphi_m(t)$$

$$X_p X_s' \varphi_m(t) + X_s' X_m \varphi_m(t) + X_p X_m \varphi_m(t) = X_s' X_m \varphi_p(t) + X_p X_m \varphi'_s(t)$$

grouping common terms, we obtain that

$$\varphi_m(t)(X_pX_s^{'}+X_s^{'}X_m+X_pX_m)=X_s^{'}X_m\varphi_p(t)+X_pX_m\varphi'_s(t)$$

In this way if the mutual dispersion flux $\varphi_m(t)$ is:

$$\varphi_m(t)=\frac{X_s^{'}X_m\varphi_p(t)+X_pX_m\varphi'_s(t)}{X_pX_s^{'}+X_s^{'}X_m+X_pX_m} \quad \textbf{(3)}$$

This system of equations characterizes the behavior of a single-phase transformer as a function of mutual and leakage fluxs, enabling the reconstruction of primary and secondary currents. The mathematical model, incorporating leakage fluxs and their derivatives, exhibits critical attributes that are essential for analyzing and comprehending transformer operation. By integrating electromagnetic and electrical aspects, this model serves as an indispensable tool for transformer design, simulation, and control across various applications. In a transformer, the voltages induced in the primary and secondary windings are directly governed by the time variation of the mutual flux $\varphi_m(t)$, as dictated by Faraday's law. Incorporating $\varphi_m(t)$ into the mathematical model enhances the accuracy of induced voltage predictions under diverse operating conditions. This is essential for determining the correct transformation ratios between the primary and secondary windings.

3 Results

The development of a mathematical model for a single-phase transformer necessitates the rigorous application of fundamental principles of electromagnetism, circuit theory, and the behavior of magnetic materials. These principles enable the precise representation of transformer operation, providing analytical tools for understanding its performance under various operating conditions and optimizing its design based on energy efficiency and stability criteria.

Simulating the mathematical model of a single-phase transformer offers significant advantages in design, analysis, and operational optimization. By implementing differential equations that describe the relationships among voltages, currents, and magnetic flux in the windings, detailed studies can be conducted without the need for costly and repetitive experimental testing. This approach facilitates the prediction of transformer behavior under diverse operating scenarios, enhances energy efficiency, and improves operational safety. Moreover, mathematical modeling allows for the anticipation of nonlinear effects such as core saturation, hysteresis losses, and eddy currents—key considerations for the development of more efficient and reliable electrical machines.

Incorporating mutual magnetic flux into the transformer model is essential for evaluating the magnetic coupling between the primary and secondary windings. The efficiency of this coupling depends on factors such as core geometry, ferromagnetic material properties, and winding configuration. A precise mathematical representation of mutual flux enables the optimization of core design to minimize leakage losses, thereby improving energy transfer and mitigating the adverse effects of stray flux.

Moreover, the mutual magnetic flux plays a critical role in accurately simulating transient phenomena, such as transformer energization, abrupt load changes, and responses to electrical faults. Rapid variations in magnetic flux during these events induce overvoltage's and transient currents that can compromise both the stability of the transformer and the integrity of the power system. Incorporating the core's nonlinear behavior into the mathematical model significantly enhances the accuracy of transient response predictions and observations, including internal capacitive effects between windings and between windings and ground. This modeling approach supports the development of advanced control and protection strategies that ensure reliable and efficient transformer operation under dynamic conditions.

Thus, the mathematical formulation of the single-phase transformer not only serves as a fundamental analytical tool for the design of these static electrical machines but also contributes to the advancement of more efficient and sustainable power systems. Based on equations (1), (2), and (3), an analog machine model is constructed, where the system is solved using integrators implemented through block-based representations, as illustrated in Figure (2).

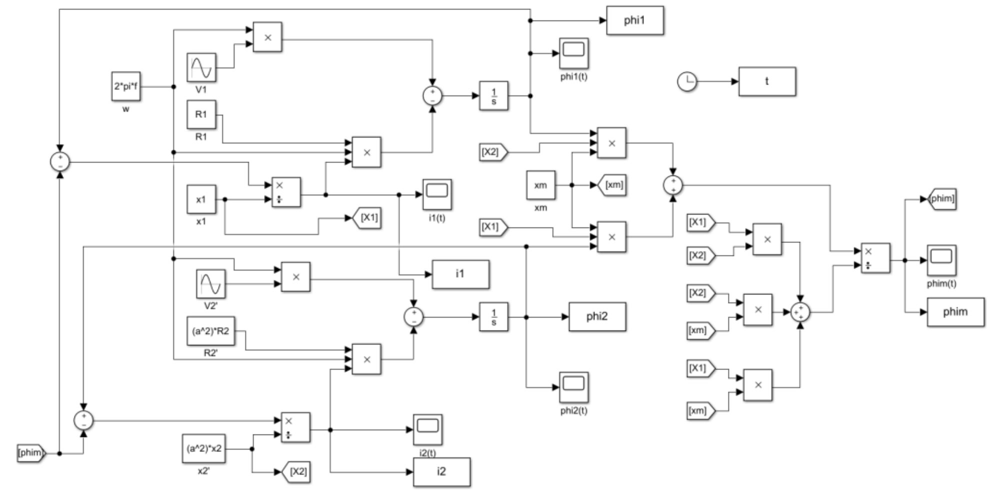

Fig. 2. Block diagram of the single-phase transformer model.

To define the transformer parameters, a MATLAB command-line interface is developed to assign the corresponding values to the single-phase transformer parameters. By simulating the mathematical model, it is possible to determine the secondary current and analyze the leakage fluxes within the machine.

The following section details the parameters, and their respective values used in the simulation of the single-phase transformer model. Each parameter is crucial, as it directly influences the transformer's performance and efficiency under various design and operating conditions

The simulation of the single-phase transformer model is conducted under specific electrical and operational parameters that significantly influence its performance and efficiency. The primary winding resistance R_p=0.25Ω determines the extent of energy loss in the form of heat, where a lower resistance reduces thermal dissipation and enhances overall efficiency. Similarly, the secondary winding resistance R_s=0.25Ω plays a crucial role in minimizing power losses and optimizing energy transfer to the load side.

The reactance of the primary winding X_p=0.056Ω contributes to the total impedance of the transformer, affecting voltage regulation and its response to variations in load conditions. Likewise, the secondary winding reactance X_s=0.056Ω influences the impedance characteristics of the transformer, making its precise determination essential for the proper design of circuits that incorporate the transformer. Mutual reactance X_m=0.028Ω, which quantifies the electromagnetic coupling between the windings, plays a fundamental role in improving energy transfer efficiency and ensuring system stability.

The number of turns in the primary winding N_p=1000 is a critical factor in defining the transformation ratio and electromagnetic induction. An increase in the number of turns generally leads to a higher voltage on the secondary side. Similarly, the number of turns in the secondary winding N_s=2000 directly affects the transformation ratio, with a greater number of turns resulting in an increased secondary voltage. The applied voltage on the primary winding V_p=120V determines the output voltage of the secondary, making it essential to accurately define this parameter to evaluate the transformer's ability to handle different load conditions.

The frequency of the primary voltage waveform has a direct impact on reactance and, consequently, on the overall performance of the transformer. In this simulation, a frequency of 60 Hz is considered, corresponding to standard industrial applications. Additionally, multiple cycles are graphed in the analysis to observe the transformer’s dynamic behavior, enabling a comprehensive evaluation of its performance over time. These conditions establish a rigorous framework for analyzing the electrical and magnetic characteristics of the transformer, ensuring the accuracy and reliability of the simulation results.

The comparative analysis is presented through a series of graphs that illustrate the currents in both the primary and secondary windings, along with their corresponding magnetic flux, including mutual magnetic flux. These graphical representations are derived from the differential equations formulated in the previous section, incorporating an explicitly nonlinear (recursive) formulation of mutual flux as well as its conventional representation found in the literature (Ong, C. M. 1998, Grainger et al., 1996), defined as:

$$\varphi_m(t) = X_m\left(\frac{\varphi_p(t)}{X_p} + \frac{\varphi'_s(t)}{X'_s}\right) \quad \mathbf{(4)}$$

The block diagram representing the dynamics of the single-phase transformer, incorporating a linear formulation of mutual magnetic flux as defined by equation (4), is shown in Figure (4). This diagram clearly illustrates the non-recursive nature of the mutual magnetic flux representation.

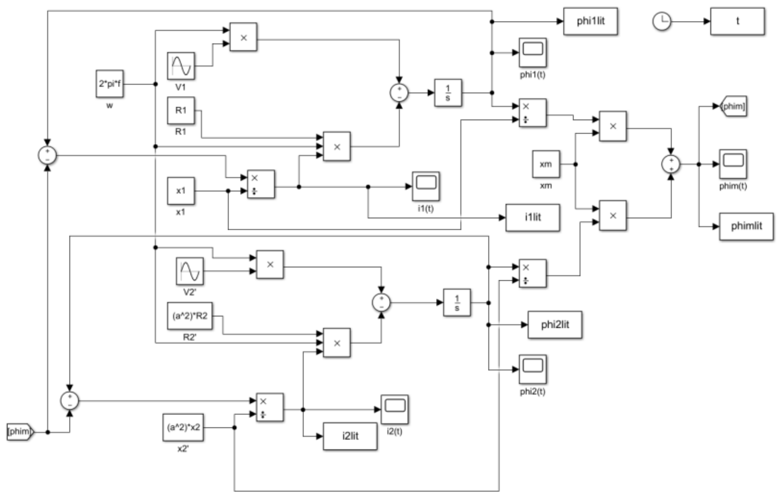

Fig. 3. Block diagram of the single-phase transformer model, with linear mutual flux.

Figure (4) presents the primary winding current, obtained by solving differential equations (1) and (2). The numerical approximation is performed using the Runge-Kutta algorithm with an integration step of 0.1×10^{-6} seconds, ensuring high computational accuracy in the simulation. This graph illustrates a higher magnitude of electric current in the primary winding when influenced by a non-explicitly linear mutual magnetic flux, compared to its counterpart induced by a linearly shaped mutual flux. This phenomenon indicates that the transformer is operating in a regime where core saturation reduces the magnetization impedance, thereby forcing a higher current flux. Such behavior provides a more accurate representation of the static electrical machine's performance under varying operational conditions.

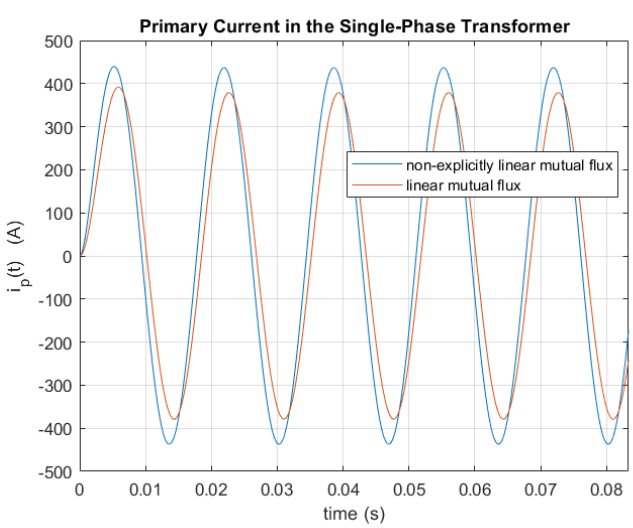

Fig. 4. Primary winding current in single-phase transformer.

Figure (5) presents the secondary winding current, obtained by solving differential equations (1) and (2). The numerical approximation is performed using the Runge-Kutta algorithm with an integration step of 0.1×10^{-6} seconds. This Figure demonstrates a higher magnitude of electric current in the secondary winding when influenced by a non-explicitly linear mutual magnetic flux, compared to its counterpart induced by a linear mutual flux. This phenomenon can be attributed to the transformer operating in a regime where core saturation reduces the magnetization impedance, thereby increasing the current flux. Such behavior provides a more accurate representation of the performance of the static electrical machine under varying operational conditions.

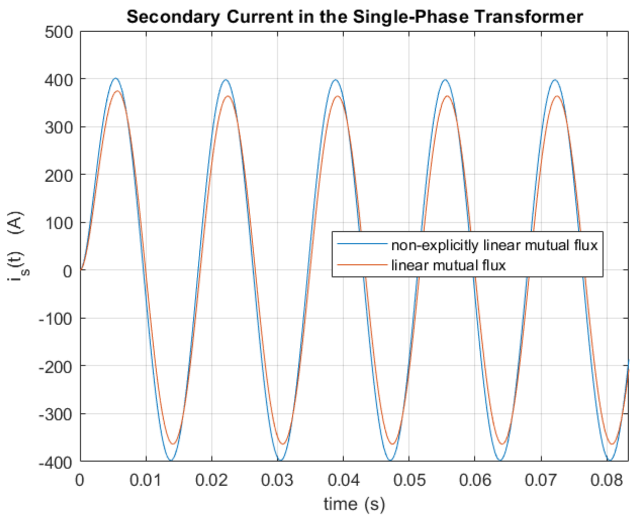

Fig. 5. Secondary winding current in single-phase transformer.

As illustrated in Figure (6), the magnetic fluxes within the transformer windings, along with the mutual magnetic flux, exhibit relatively lower magnitudes in the simulated dynamics when a non-explicitly linear representation of the mutual magnetic flux is employed. This contrasts with the behavior observed under a linear mutual flux assumption. This difference highlights the influence of flux nonlinearity on the electromagnetic interactions within the transformer, impacting its overall performance and operational characteristics.

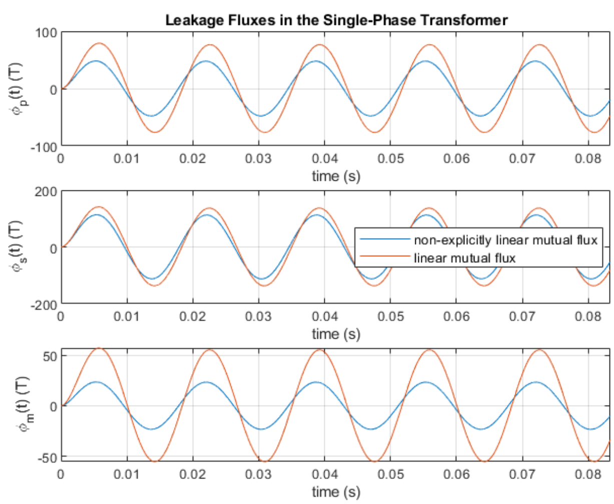

Fig. 6. Magnetic dispersion fluxes.

Among the key effects, a reduction in core losses stands out, as lower-magnitude magnetic fluxes decrease the likelihood of core saturation. This mitigates hysteresis and eddy current losses, thereby enhancing the transformer's energy efficiency. Additionally, reduced magnetic induction leads to lower thermal stress, minimizing heat-related losses in the core. This extends the lifespan of the ferromagnetic material and reduces the need for supplementary cooling systems. Furthermore, maintaining an optimally controlled magnetic flux improves voltage regulation under normal operating conditions, preventing excessive fluctuations in output voltage and ensuring stable performance across varying load demands. In contrast, certain drawbacks become evident when the magnetic flux is insufficient. Reducing energy transfer efficiency can result from inadequate coupling between the primary and secondary windings, limiting the power transmission and decreasing the overall efficiency of energy conversion. Additionally, an increased influence of leakage reactances arises when leakage flux becomes significant compared to mutual flux, leading to a higher transformer impedance that restricts energy transfer and negatively impacts dynamic performance. Furthermore, an insufficient magnetic flux constrains the transformer's load-handling capability, preventing it from operating at maximum power without experiencing substantial voltage drops (Chapman, 2012, Kosow, 2021 and Fraile Mora, 2008).

4 Conclusions

This study presents a numerical evaluation of the mathematical model of a single-phase transformer, emphasizing the critical role of incorporating the inherent nonlinearities of its operation, particularly the mutual magnetic flux. Through simulation, it has been demonstrated that conventional models, which often assume a simplified linear representation of the magnetic circuit, may underestimate key effects that significantly influence transformer performance, especially under variable load conditions.

The findings highlight that integrating nonlinear terms into mathematical formulation provides a more accurate representation of the transformer's real behavior. This approach enhances the precision of design, diagnostics, and operational analyses in practical applications. Furthermore, the methodology employed in this research can be extended to the modeling and simulation of other electrical machines, contributing to the development of more robust characterization strategies.

In conclusion, incorporating a nonlinear representation of the mutual magnetic flux, along with other inherent nonlinearities, into transformer models is not only theoretically relevant but also essential for practical analysis and design. The findings highlight the critical need to develop more refined and accurate models that provide a faithful representation of the electromagnetic behavior of transformers under a wide range of operating conditions.

References

Ćalasan, M. P., Jovanović, A., Rubežić, V., Mujičić, D., & Deriszadeh, A. (2020). Notes on parameter estimation for single-phase transformer. *IEEE Transactions on Industry Applications, 56*(4), 3710–3718. https://doi.org/10.1109/TIA.2020.2992667

Yarymbash, D., Kotsur, M., Kulanina, Y., & Divchuk, T. (2019, September). Idling mode simulation of single-phase transformer. In *2019 IEEE International Conference on Modern Electrical and Energy Systems (MEES)* (pp. 118–121). IEEE. https://doi.org/10.1109/MEES.2019.8896637

Ketabi, A., & Naseh, M. (2012). Single-phase transformer modeling for inrush currents simulation using differential evolution. *European Transactions on Electrical Power, 22*(3), 402–411. https://doi.org/10.1002/etep.614

Wilk, A. (2015). Representation of magnetic hysteresis in a circuit model of a single-phase transformer. *COMPEL - The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, 34*(3), 778–791. https://doi.org/10.1108/COMPEL-10-2014-0282

Huang, S. R., Chung, S. C., Chen, B. N., & Chen, Y. H. (2003). A harmonic model for the nonlinearities of single-phase transformer with describing functions. *IEEE Transactions on Power Delivery, 18*(3), 815–820. https://doi.org/10.1109/TPWRD.2003.813878

Martínez Figueroa, G. de J. (2024). *Modeling and characterization of single-phase and three-phase transformers based on minimum information* [Tesis doctoral, Universitat Politècnica de Catalunya]. UPC, Departament d'Enginyeria Elèctrica. https://doi.org/10.5821/dissertation-2117-406724

Krishan, R., Mishra, A. K., & Rajpurohit, B. S. (2016, November). Real-time parameter estimation of single-phase transformer. In *2016 IEEE 7th Power India International Conference (PIICON)* (pp. 1–6). IEEE. https://doi.org/10.1109/POWERI.2016.807731

García, S., Medina, A., & Pérez, C. (2000, July). A state space single-phase transformer model incorporating nonlinear phenomena of magnetic saturation and hysteresis for transient and periodic steady-state analysis. In *2000 Power Engineering Society Summer Meeting (Cat. No. 00CH37134)* (Vol. 4, pp. 2417–2421). IEEE. https://doi.org/10.1109/PESS.2000.867369

Hernández-Romero, N., Ortíz-Licona, A., Tuoh-Mora, J. C. S., Lagos-Eulogio, P., Marín, J. M., & Rosas-Ortíz, G. (2020). Diseño de la parte activa de un transformador de potencia mediante algoritmos genéticos para minimizar las pérdidas. *Pädi Boletín Científico de Ciencias Básicas e Ingenierías del ICBI, 7*(14), 52–58. https://doi.org/10.29057/icbi.v7i14.4424

Chapman, S. J. (2012). *Máquinas eléctricas* (5ª ed.). McGraw-Hill.
Kosow, I. L. (2021). *Máquinas eléctricas y transformadores*. Reverté.
Fraile Mora, J. (2008). *Máquinas eléctricas*. McGraw-Hill.
Ong, C. M. (1998). *Dynamic simulation of electric machinery: Using MATLAB/SIMULINK*. Prentice Hall.
Grainger, J. J., & Stevenson, W. D. (1996). *Análisis de sistemas de potencia*. McGraw-Hill.