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Electricity Sector in Mexico: Dynamic Government Subvention Model

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Abstract. In this paper, we develop a numerical model for imperfect electricity market with governmental subsidies. The model is driven by generalized Cournot model and applied to the current electricity market in Mexico. State-owned electricity producer maximizes its profit taking into account subsidies endowed by government. We establish the problem as mixed complementarity problem [MCP]; and as programming language General Algebraic Modeling System (GAMS) has been chosen.As the solver to obtain numerical solutions we have chosen the solver PATH which is the generalized Newton Method.We have found that generalized Cournot modelbrings quite reasonable and most realistic prices and demand for electricity, compared to classical Cournot case.

Keywords: price subvention profit maximization strategic competition.

1. Introduction

Nowadays we see that the interest in modeling strategic models with subventions is increasing, and not only in the electricity industry. The Mexican electricity sector can be presented as a highly regulated and subsidized market. Foreign and national private producers can participate within Mexican electricity market only in restricted mode.

This study presents our model with subvention function within the model-based analysis. We develop the game-theoretic electricity market model with an ability of governmental support in an oligopolistic market structure. The strategic behavior of the companies is modeled as a generalized Cournot oligopoly game. This allows analyzing strategic storage utilization in an oligopolistic market environment with imperfectly competitive generators.

Every supplier in the model may own different plants and poses different technologies. Further, any company can hold different production plants on the same technology with different variable production cost and different installed capacity. We do not consider investment decisions here as the model, we rather concentrate on the economic effects and make capacity adjustments for the next period, if necessary.

The mathematical model which is presented in Section 2 is an enlargement of the model presented at [10]. During the process of the game, market participants maximize their profits taking into account various factors as variable production costs, network access costs, transportation costs and transportation losses. Also, the strategic behavior of the competitors is considered, and it is not the case when we have perfect competition markets. Generally, within the oligopolistic competition, market price for homogeneous good depends on the market power, market shares and the demand elasticity. In generalized Cournot oligopoly market power is weighted by influence coefficients (e.g. see [2]).

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As presented in [1] and [3]-[5] it is often useful to represent optimization problems of such type as the problem of complementarity. Basically, by solving a mixed complementarity problem (MCP) one needs to specify all the restriction, presented as lower/upper bounds for decision variables in the model; and then define first order conditions for the equilibrium. This procedure is presented in Section 2.

Our paper has the following structure: introduction chapter, then we presents the mathematical model. Third chapter describes the numerical part; and the last chapter presents conclusions.

2. The Model

To model the electricity market in Mexico we aggregate all possible electricity suppliers into two: a state owned and the other one is private and index them with i =1 and 2. Also, we divide Mexico into five different regions (see Section 3); and regions, be denoted by r =1, 2, 3, 4, 5$\in R$.

The production of either firm is denoted by y^i. Next, the subvention l level we define by $e_y^i(y^i)$ and cost production function is noted as $c_y^i(y^i)$.

The upper bound for the production function y^i is presented by installed capacity $\bar{y}^i$. Electricity which produced in certain region r can be used within the same region; or exported to any neighbor region. It is very important to note that such market clearance is quite important for electricity markets, as there is no storage option so that we have $y^i = \sum_{r \in R} S^{i,r}$. Here $s^{i,r}$ denotes the supply of the firm i to the region r.

Next, as $Ex^{r,r*}$ we define total electricity export from region r to any neighboring region r*. Also, there is a restriction for transportation: $\bar{E}^{r,r*} \geq Ex^{r,r*}(\tau^{r,r*})$ where τ is a transportation tariff parameter. We assume that there is no transportation cost within the region, e.g. τ $\tau^{r,r}$ = 0. The state will determine the subvention target σ. The price for electricity offered by producers for every region is defined as P_S^r; the resulting electricity demand in r is given as Q^r.

Now, the actual price payed by consumer $P^r(Q^r)$ can be presented as the sum of equals the sum of P_S^r and the tariff cost as presented in (1):

$$\varsigma : P^r(Q^r) = P_S^r + (\varsigma - P_S^r)\frac{Z^r}{Q^r} \tag{1}$$

The Z^r represents certain part of Q^r for which a special price is offered (normally applies to the heavy industry contracts). Now we rearrange the formulae (1) in order get the expression for P_S^r:

$$P_S^r = P^r(Q^r)\frac{Q^r}{S^r} - \varsigma\frac{Z^r}{Q^r} \tag{2}$$

Revenue maximization Lagrangian for the firm i is given as:

$$L^i = \sum_{r \in R}\left(P^r(Q^r)\frac{Q^r}{S^r} - \varsigma^r\frac{Z^r}{S^r}\right)s^{i,r} - C^i(y^i) - \sigma E^i(y^i) + k^i(\bar{y}^i - y^i) - \sum_{r \in R}\tau^{r,r^*}s^{i,r^*}. \tag{3}$$

The right-hand of (3) can be explained as following:

The first term is revenue from electricity sold in all the regions.
The second term is the cost for production.
The third term is subvention cost.
The fourth term counts for the production capacity shadow price
The fifth term is transmission cost for neighboring regions.

Now, we present the first order condition for optimal solution:

$$\frac{\partial L^i}{\partial s^{i,r}} \le 0, s^{i,r} \ge 0, \frac{\partial L^i}{\partial s^{i,r}} s^{i,r} = 0, \; \frac{\partial L^i}{\partial k^{i,r}} \ge 0, k^i \ge 0, \frac{\partial L^i}{\partial k^i} k^i = 0. \tag{4}$$

The most important factor for our model is $\partial L^i / \partial s^{i,r}$ - the derivative of the lagrangian with respect to the supply. If our firms were price takers, the derivatives would have the following form

$$\frac{\partial L^i}{\partial s^{i,r}} P^r\left(Q^r\right) = \left(P^r\left(Q^r\right) - \varsigma^r\right)\frac{Z^r}{Q^r} - C^i\left(y^i\right) - \sigma E^i\left(y^i\right) - k^i - \tau^{r,r*}. \tag{5}$$

Next, as we have strategic Cournot behavior of our producers, the firms, the effect on the revenue caused by the choice of output is taken into account by the firms. So now we present the elasticity of the demand:

$$\varepsilon^r = \left|\frac{dQ^r}{dP^r}\frac{P_0^r}{Q_0^r}\right| \tag{6}$$

and with the regional share of the firm *i* denoted as $\vartheta^{i,r}$. Thus the derivative of the problem (3) with respect to supply in Nash equilibrium can be expressed in the following way:

$$\frac{\partial L^i}{\partial s^{i,r}} = P^r(Q^r) + (P^r(Q^r) - \varsigma^r)\frac{Z^r}{Q^r} - C^i(y^i) - \sigma E^i(y^i) - k^i - \tau^{r,r^*} - \\ -\vartheta^{i,r}\left((P^{r^*}(Q^{r^*}) - \varsigma^{r^*})\frac{Z^r}{Q^r} + \frac{P^r(Q^r)}{\varepsilon^r}\right) \tag{7}$$

If we compare the optimality conditions under the Cournot-Nash model with a model where all firms are price takers, we find out that only a term which depends on the market share is added in (7). This last term includes the mark-up

$$\vartheta^{i,r}\frac{P^r\left(Q^r\right)}{\varepsilon^r}, \tag{8}$$

known from conventional oligopoly models, and a term induced by the feed-in tariff ς that reduces the markup if the feed-in tariff is greater than the market price:

$$P^r : \vartheta^{i,r}\left(P^r\left(Q^r\right) - \varsigma^r\right)\frac{Z^r}{Q^r}. \tag{9}$$

The result (9) comes the conjecture of the firm about a production volume of the competitor in the Nash-equilibrium with regard to a marginal change in its own production. A formula (10) is the complete model with the price taking behavior of minor actor and strategic behavior of dominant firm (see [8] and [9]). Here, the binary variable l^i, which is zero in the case of price-taking firms, and one in the case of strategic firms. So now the combined condition for optimality both for the price takers and for strategically thinking firms can be presented as:

$$\frac{\partial L^i}{\partial s^{i,r}} = p^r(Q^r) + \left(P^r(Q^r) - \varsigma^r\right)\frac{Z^r}{Q^r} - C^i(y^i) - \sigma E^i(y^i) - k^i - \tau^{r,r^*} - \\ -l^i\vartheta^{i,r}\left((P^{r^*}(Q^{r^*}) - \varsigma^{r^*}\right)\frac{Z^r}{Q^r} + \frac{P^r(Q^r)}{\varepsilon^r} = 0. \tag{10}$$

Also, we assume the production cost function of firm *i* is

$$C^i(y^i) = a^i \exp\left(b^i \frac{y^i}{\overline{y}^i}\right), \tag{11}$$

where a and b are calibrating coefficients,

For the subvention:

$$E^i(y^i) = f^i \exp\left(g^i \frac{y^i}{\bar{y}^i}\right) \qquad (12)$$

Here as well the coefficients *f* and *g* are calibrating coefficients as well.

3. Numerical results and Discussion

As we see from Table 1 presented below, Mexican government gives a substantial subsidy to low energy consumers. But if we try to reflect an average household as 300kWh or above we see that consumer price to pay is reflected in the mixed oligopoly model. This model still includes the subsidies as the CFE's objective function.

TABLE1. Consumer tariffs for Mexican households (Centavos / kWh)

YEAR	CONSUME<75KWH	75KWH<CONSUME<140KWH	CONSUME>140KWH
2002	$46.9	$65.0	$162.3
2004	$52.9	$88.7	$186.2
2006	$59.7	$98.6	$208.0
2008	$63.9	$104.9	$222.7
2010	$68.7	$113.7	$240.9

The Tables 2, 3 and 4 represent results of our experiments for different σ, which can be targeted by the government. The Mexican Republic is divided into five regions as follows: North (Region 1), Baja California (Region 2), Central (Region 3), Distrito Federal (Region 4), and South (Region 5).

TABLE2. Prices per region in Mexico for σ = 9% (Centavos/kWh).

REGIONS	PERFECT COMPETITION	OLIGOPOLY CLASSIC	OLIGOPOLY SUBVENTION
1	***$124.88***	***$192.34***	***$155.56***
2	***$155.45***	***$257.78***	***$222.32***
3	***$126.11***	***$196.80***	***$161.56***
4	***$191.55***	***$302.43***	***$252.77***
5	***$145.11***	***$243.78***	***$165.88***

Now, Table 2 reflects numerical results for subsidy targeted value σ =9%. If we compare Table 4 with the Table 3below we observe an increment in prices. Still, it is compensated by the social effect of subsidies. Next, if we analyze the results for the value σ =24.5% (see the Table 3) we can see that the burden of too high subsidy has a negative impact on the unsubsidized industries/customers.

TABLE3. Prices per region in Mexico for σ =15.5% (Centavos/kWh).

REGIONS	PERFECT COMPETITION	OLIGOPOLY CLASSIC	OLIGOPOLY SUBVENTION
1	***$134.29***	***$193.44***	***$158.01***
2	***$157.44***	***$267.23***	***$234.01***
3	***$128.18***	***$201.71***	***$164.92***
4	***$193.35***	***$305.17***	***$256.62***
5	***$151.12***	***$247.34***	***$183.44***

Table 3 presents the results for σ = 15.5%. The prices for the perfect competition are far below the actual domestic rates (compare with Table 2). The best approximation of real prices is achieved by the Oligopoly with Subvention model. The Central and North regions have the highest demand and, consequently, the price. In reality, CFE is maintaining the same price over regions, which means certain extra subsidies to the North and Central regions. As our calculations show, σ = 15.5% gives us the best possible consumer price. Thus it is reasonable to accept σ = 15.5% as an optimal target.

TABLE4. Prices per region in Mexico for σ = 24.5% (Centavos/kWh).

REGIONS	PERFECT COMPETITION	OLIGOPOLY CLASSIC	OLIGOPOLY SUBVENTION
1	***$511.38***	***$880.16***	***$790.28***
2	***$670.27***	***$1084.44***	***$890.28***
3	***$520.12***	***$847.34***	***$693.14***
4	***$807.14***	***$1311.07***	***$1078.34***
5	***$612.27***	***$994.31***	***$816.24***

Table 4 shows expected electricity prices for σ =24.5%. It is clear, that too heavy support of the poor customer has to be equalized by the higher market price for non-subsidized industries/customers. The official estimate of a subsidy in Mexico is around 5 billion USD per year. It is quite high because of residential and agricultural tariffs. They are setway below the standard industry tariff. The distributional effects of this subsidy are enormous: n2000, residential consumers received 64.1% of the total subsidy; the industrial sector, 17.9%; the agriculture sector,11%; and the commercial sector,5.3%.

4. Conclusions

We have developed a game theoretical electricity market model which includes certain subvention function. Such subvention function depends exogenously on the target stated by the government. It also depends on the firm´s level of production.

Such interpretation permits to be interpreted as generalized Cournot equilibrium where producers maximize their profits. We establish the model in GAMS language as a Mixed Complementarity Problem, and it is solved by nonlinear complementarity and equation system solvers. Our numerical experiments show that a switch from monopoly driven model to generalized Cournot-Nash equilibrium may lead to lower consumer prices and higher consumption demand.

Next steps to improve the model are also quite clear: in order to improve the outcome more advanced, variable and fixed cost function should be added in the model. Also, the model should be enlarged and should use the sequence of the years instead of only one base year. This would allow making the future model more reliable.

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